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Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887



PRELIMINARY EXAMINATION 2023 Secondary 4

TUESDAY

15 August 2023

MATHEMATICS Paper 1

4052/01

2 hours 15 minutes

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your class, name and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

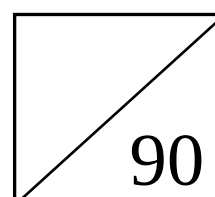
The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For p , use either your calculator value or 3.142, unless the question requires the answer in terms of p .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.



Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1** It is given that M is a prime number and is a digit of the number 508 M 45.
If the number, correct to 3 significant figures, is 509 000, find the maximum value of M .

Answer $M = \dots\dots\dots$ [1]

- 2** Simplify $3 + 2(3x - 2)^2$.

Answer $\dots\dots\dots$ [2]

- 3** Solve the equation $\left(\frac{1}{7}\right)^{2x} = \sqrt[3]{49}$.

Answer $x = \dots\dots\dots$ [2]

- 4 (a) Factorise $3y^{2a} + 6y^a$ completely.

Answer [1]

- (b) Factorise $20pq - 15sq + 3sr - 4pr$ completely.

Answer [2]

- 5 Express $\frac{3n^2 - 8n - 3}{9 - n^2} + \frac{5n - 3}{n + 3}$ as a single fraction in its simplest form.

Answer [3]

- 6 Given that x satisfies the inequality $\frac{1}{2}x - (x - 2) \leq \frac{3}{4}(2x - 3) - 5$, find the smallest value of x if x is an integer.

Answer $x = \dots\dots\dots$ [3]

- 7 It is given that $90^\circ < A < 180^\circ$ and $\sin A = \frac{7}{25}$.
(a) Find the value of A .

Answer $\dots\dots\dots$ [1]

- (b) Without the use of the calculator, find the numerical value of $\cos A$.

Answer $\dots\dots\dots$ [2]

- 8** The time, t seconds, taken to download a file from a computer is inversely proportional to the internet connect speed, v Mbps. When the speed is 250 Mbps, the time taken is 4 seconds.

(a) Sketch the graph of t against v in the axes below. [1]



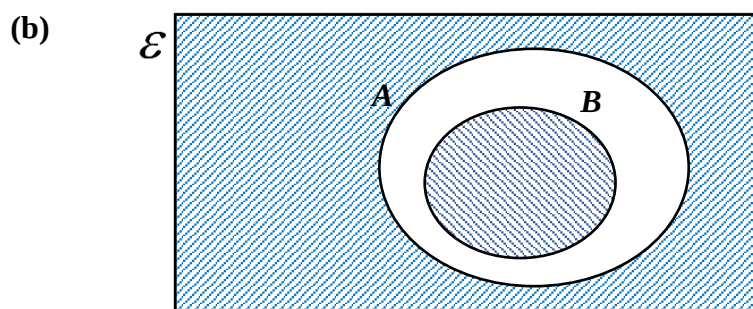
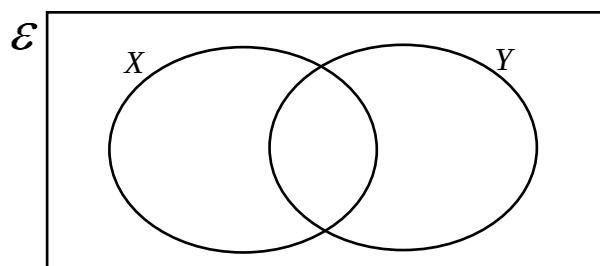
(b) Find the time taken to download a file when the speed is 160 Mbps.

Answer sec [2]

(c) If the speed is increased by 25%, find the percentage change in the time taken.

Answer% [2]

- 9 (a) On the Venn Diagram, shade the region which represents $X' \cup Y'$. [1]



From the Venn diagram above,

- (i) write, in set notation that illustrates the relationship between A and B .

Answer $B \subseteq A$ [1]

- (ii) write down the set notation that represents the shaded region.

Answer [1]

- 10 It is given that $\xi = \{x : x \text{ is an integer and } 0 < x \leq 15\}$,

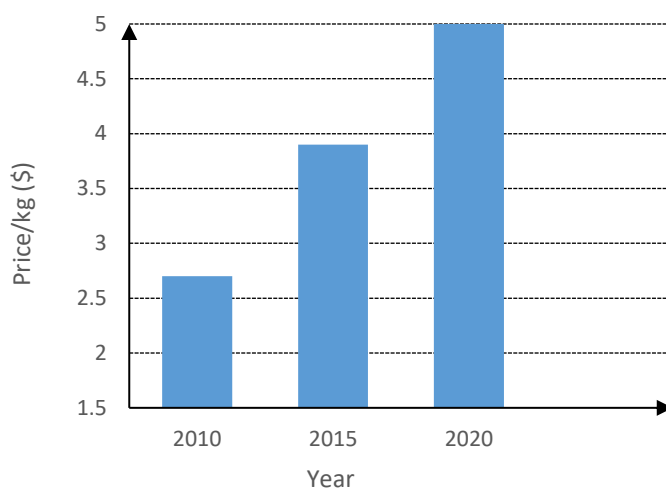
$P = \{x : x \text{ is a prime number}\}$ and

$Q = \{x : x \text{ is divisible by 3 or 5}\}$.

List the elements in $P' \cap Q$.

Answer {.....} [2]

- 11** The graph shows the price of 1 kg of apples in Country *X* from 2010 – 2020.



- (a)** State the misleading feature of the graph.

Answer

 [1]

- (b)** Explain how this feature affects the reader's interpretation of the graph.

Answer

 [1]

12 Consider the pattern,

$$L_1: 3^2 - 2^2 = 3 + 2 = 5$$

$$L_2: 4^2 - 3^2 = 4 + 3 = 7$$

$$L_3: 5^2 - 4^2 = 5 + 4 = 9$$

$$L_4: 6^2 - 5^2 = 6 + 5 = 11$$

: : :

: : :

$$L_n: \dots \dots \dots$$

(i) Write down L_n of the pattern. [1]

(ii) Explain, with calculations, why 1137 is a term of the sequence 5, 7, 9, 11, ... [2]

(iii) Find the exact value of $L_1 + L_2 + L_3 + \dots + L_{99}$.

Answer[1]

- 13** The numbers 720 and 1575, written as the products of their prime factors are $720 = 2^4 \times 3^2 \times 5$ and $1575 = 3^2 \times 5^2 \times 7$.

(i) Find the largest common factor of both 720 and 1575.

Answer[1]

(ii) Find the smallest whole number that is divisible by both 720 and 1575.

Answer[1]

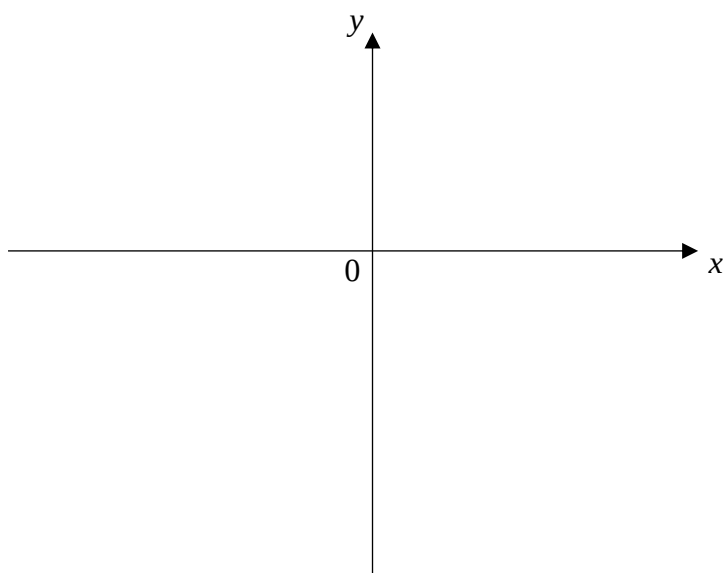
(iii) When the product of 720 and 1575 is divided by p , the result is a perfect square.
Find the smallest integer value of p , where $p > 0$.

Answer $p =$ [1]

- 14 (i)** Express $-x^2 - 6x - 10$ in the form $a(x - p)^2 + q$, where a , p and q are integers.

Answer [2]

- (ii)** Sketch the graph of $y = -x^2 - 6x - 10$ on the axes below.
Indicate clearly the coordinates of the point where the graph crosses the y -axis and the turning point. [1]



- (iii)** Hence, explain, without solving, why there are no solutions of x when $-x^2 - 6x = 10$.

Answer

.....

 [1]

- 15** The following stem-and-leaf diagram shows the duration of each of the 20 calls, in minutes, made by Tammy using her mobile phone on a certain day.

<u>Stem</u>	<u>Leaves</u>
1	3 4 4 5 7 9
2	0 1 2 5 6 6 7 9
3	1
4	4 6
5	1
6	2 R

Key: 1 | 3 means 13 minutes

- (i)** Find the percentage of calls that are shorter than 31 minutes.

Answer% [1]

- (ii)** Find the median call duration.

Answer min [1]

- (iii)** Given that the range of the duration of the calls made is 52 minutes.
Find the value of R .

Answer [1]

- 16** The table below summarises the number of spam calls received by 80 elderly males of a community club in a month.

No. of spam calls	$5 \leq n < 10$	$10 \leq n < 15$	$15 \leq n < 20$	$20 \leq n < 25$
Frequency	30	24	15	11

- (i) Calculate an estimate for
 (a) the mean number of spam calls received,

Answer [1]

- (b) the standard deviation.

Answer [1]

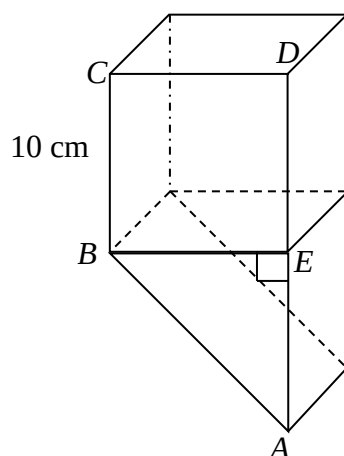
- (ii) The survey is also conducted for 80 elderly females at the community club. The mean number of spam calls received is 16 calls and the standard deviation is 9 calls.

Make two comparisons between the results of the survey conducted for the elderly males and females.

Answer.....

[2]

- 17 A container is made up of a cube and a right-angled triangular prism. Each side of the cube is 10 cm and $EB = EA$.

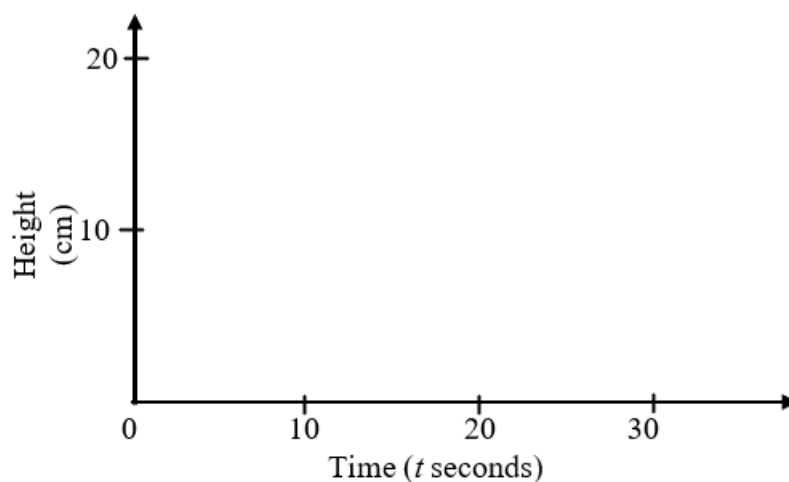


- (a) Find the volume of the container.

Answer cm³ [2]

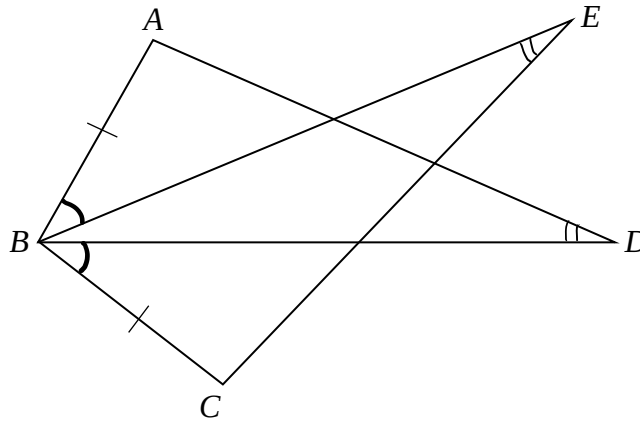
- (b) Water is poured from the top at a constant rate and fills the container in 30 seconds.

Sketch the graph showing how the height of the water varies with time. [1]



- 18** In the diagram, $BA = BC$, $\angle ABE = \angle DBC$ and $\angle BEC = \angle BDA$.
Prove that the triangles ABD and CBE are congruent.

[3]



Answer

- 19** The points A , B and C are vertices of a triangle. The point A is $(2, 7)$ and the gradient of the line AB is -3 .

(a) Find the equation of the line AB .

Answer [2]

(b) The equation of the line BC is $2x + y = 17$.

Find the coordinates of B .

Answer [2]

(c) Find the length of AB .

Answer units [2]

- 20** 2 regular polygons A and B have n sides and $2n$ sides respectively.
The size of each interior angle in polygon A is 300% greater than the size of each exterior angle in polygon B .

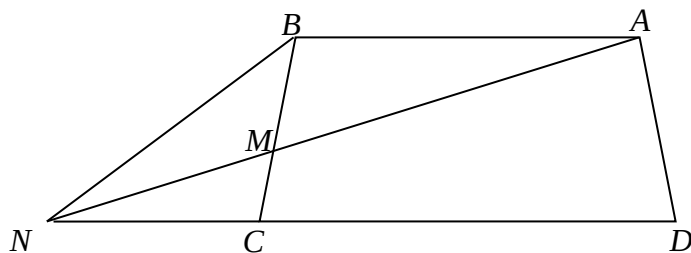
(i) Find the number of sides polygon A has.

Answer..... [2]

(ii) Hence, find the size of an interior angle in B .

Answer.....° [1]

- 21** $ABCD$ is a trapezium, with side AB parallel to side DC . M is a point on BC such that $3CM = BC$. The lines DC and AM produced meet at N .



- (a)** Show that $\triangle ABM$ and $\triangle NCM$ are similar. [2]

- (b)** Show that $2NC = AB$. [1]

Answer

- (c)** Calculate the value of

(i) $\frac{\text{area of } \triangle ABM}{\text{area of } \triangle NCM},$

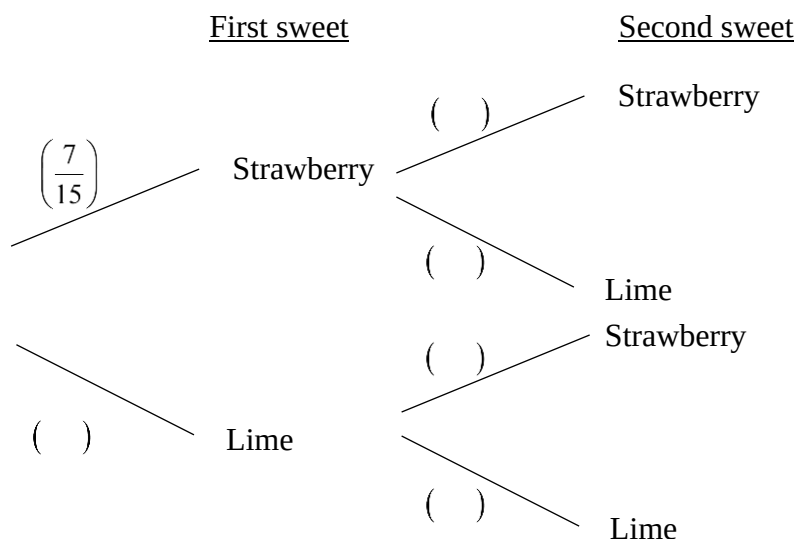
Answer [1]

(ii) $\frac{\text{area of } \triangle NBC}{\text{area of } \triangle NBA}.$

Answer [1]

- 22** A packet of 15 sweets contains 2 different flavours. There are 7 strawberry-flavoured sweets and 8 lime-flavoured sweets in the packet. Two sweets are taken from the packet at random, without replacement.

(i) Complete the tree diagram to show the probability of the possible outcomes. [2]



(ii) Hence, find the probability as a fraction in its simplest form, that

(a) both sweets drawn are the same flavours,

Answer [2]

(b) at least one sweet is strawberry-flavoured.

Answer [2]

23 Point A has the position vector $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and point B has position vector $\begin{pmatrix} 0 \\ 11 \end{pmatrix}$.

(i) Calculate $|\overrightarrow{AB}|$.

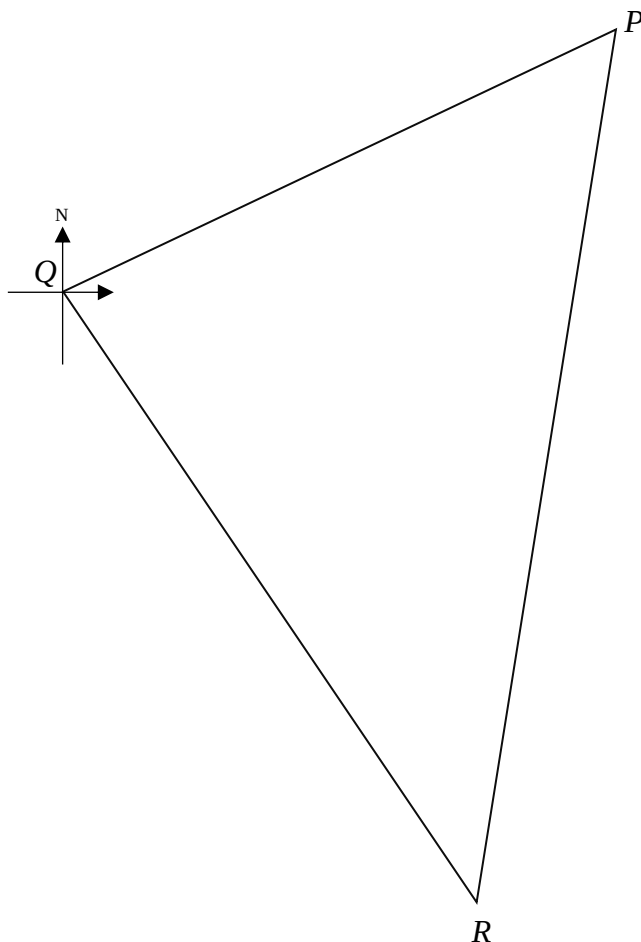
Answer units [2]

(ii) Given that the point C is $(-3, 15)$, using vectors, explain why B is the midpoint of AC . [3]

Answer

- 24** In the scale drawing, triangle PQR represents the positions of three shops.

Scale: 1 : 10 000



- (a)** A new shop is to be built at A where it is equidistant from the points P and R and equidistant from the line segments PR and QR .
- (i)** On the scale drawing, mark the point A . [2]
- (ii)** Find, in square metres, the actual area of triangle APR .

Answer m² [2]

- (b)** A new community centre, C is to be built inside the triangle PQR such that the bearing of C is 075° and 500 m from Q .
Mark the point C on the diagram above. [1]

- 25 A car dealer priced a car at \$189 500 and offered the following 2 payment schemes:

Scheme A: Cash Payment with 10% discount.

Scheme B: 30% deposit and \$2519 monthly instalments for 5 years.

Peter bought the car under scheme B.

- (a) Calculate the total amount of money Peter paid for the car.

Answer \$..... [2]

- (b) Peter claims that he had paid more buying the car using scheme B as compared to scheme A. Do you agree with Peter? Justify your answer.

Answer I with Peter because.....

[2]

- (c) Calculate the simple interest rate per annum Peter was charged for scheme B.

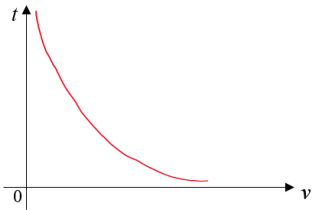
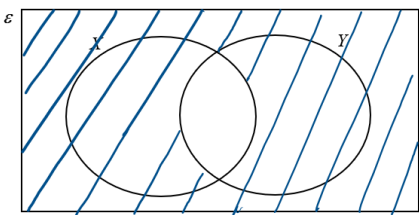
Answer% [3]

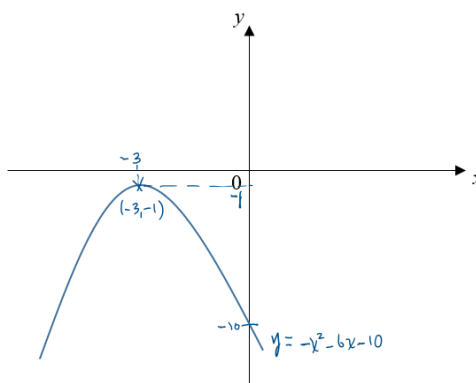
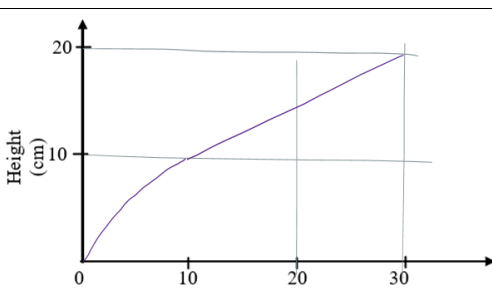
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Answer Key for 2023 Sec 4 Preliminary Examinations
Paper 1

Qn	Answers
1	7
2	$18x^2 - 24x + 11$
3	$-\frac{1}{3}$
4a	$3y^a(y^a + 2)$
4b	$(4p - 3s)(5q - r)$
5	$\frac{2n - 4}{3 + n}$
6	5
7a	163.7°
7b	$-\frac{24}{25}$
8a	
8b	6.25 s
8c	-20
9a	
9b	$B \subset A$
10	{6, 9, 10, 12, 15}
11a	The vertical axis does not start at zero
11b	The graph misleads the reader thinking that the price of apples in 2020 is thrice that of 2010's when it is only about twice.
12i	$L_n : (n+2)^2 - (n+1)^2 = n+2 + n+1 = 2n+3$

12ii	$2n + 3 = 1137$ $n = 567$ <u>n is an integer.</u> Hence 1137 is the 567 th term of the sequence.
112iii	10197
13i	45
13ii	25200
13iii	35
14i	$-(x+3)^2 - 1$
14ii	 A Cartesian coordinate system showing a parabola opening downwards. The vertex is marked with a blue 'x' at $(-3, -1)$. The y-intercept is marked at $(0, -10)$. The equation $y = -x^2 - 6x - 10$ is written next to the curve. The x-axis and y-axis are labeled, and the origin is marked with 0.
14iii	$-x^2 - 6x - 10 = 0$ There are no solutions as the curve does not intersect the x-axis as shown above.
15i	70%
15ii	25.5 min
15iii	5
16i(a)	12.9375
16i(b)	5.26
16ii	The elderly females received more spam calls in the month as compared to the elderly males as they received a mean number of 16 calls. However, the results from the elderly males are more consistent as it has a standard deviation which is about 3.74 calls lower than that of the elderly females.
17a	1500 cm^3
17b	 A graph showing Height (cm) on the y-axis versus Time (t seconds) on the x-axis. The y-axis has markings at 10 and 20. The x-axis has markings at 0, 10, 20, and 30. A curve starts at the origin (0,0) and passes through the points (10,10) and (30,20). Dashed lines connect these points to the axes.

18	$\angle ABE = \angle DBC$ (given) $\angle ABD = \angle ABE + \angle EBD$ $= \angle CBD + \angle EBD$ $= \angle CBE$ $AB = CB$ (given) $\angle BEC = \angle BDA$ (given) $\therefore \triangle ABD \cong \triangle CBE$ (AAS)
19a	$y = -3x + 13$
19b	$(-4, 25)$
19c	19.0 units
20i	6
20ii	150°
21a	$\angle BAM = \angle CNM$ (alt. $\angle$ s) $\angle BMA = \angle CMN$ (vert. opp. $\angle$ s) $\therefore \triangle ABM$ is similar to $\triangle NCM$
21b	Since $\triangle ABM$ is similar to $\triangle NCM$ $\frac{NC}{AB} = \frac{CM}{MB} = \frac{1}{2}$ Hence $2NC = AB$ (shown)
21ci	4
21cii	$\frac{1}{2}$
22i	<u>First sweet</u> $\left(\frac{7}{15}\right)$ Strawberry $\left(\frac{8}{15}\right)$ Lime <u>Second sweet</u> $\left(\frac{3}{7}\right)$ Strawberry $\left(\frac{4}{7}\right)$ Lime $\left(\frac{1}{2}\right)$ Strawberry $\left(\frac{1}{2}\right)$ Lime
22ii(a)	$\frac{7}{15}$
22ii(b)	$\frac{11}{15}$
23(i)	5 units
23ii	$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$ $= \begin{pmatrix} -3 \\ -7 \end{pmatrix} + \begin{pmatrix} -3 \\ 15 \end{pmatrix}$ $= \begin{pmatrix} -6 \\ 8 \end{pmatrix}$

Name : _____

SOLUTIONS

Class

Index Number

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TUESDAY

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MATHEMATICS Paper 1

4052/01

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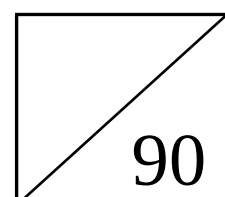
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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1** It is given that M is a prime number and is a digit of the number 508 M 45.
If the number, correct to 3 significant figures, is 509 000, find the maximum value of M .

Answer $M = \dots\dots\dots 7 \dots\dots\dots$ [1]

- 2** Simplify $3 + 2(3x - 2)^2$.

$$\begin{aligned} & 3 + 2(3x - 2)^2 \\ &= 3 + 2(9x^2 - 12x + 4) \\ &= 3 + 18x^2 - 24x + 8 \\ &= 18x^2 - 24x + 11 \end{aligned}$$

Answer $\dots\dots\dots 18x^2 - 24x + 11 \dots\dots\dots$ [2]

- 3** Solve the equation $\left(\frac{1}{7}\right)^{2x} = \sqrt[3]{49}$.

$$\begin{aligned} \left(\frac{1}{7}\right)^{2x} &= \sqrt[3]{49} \\ 7^{-2x} &= \sqrt[3]{7^2} \\ 7^{-2x} &= 7^{\frac{2}{3}} \\ -2x &= \frac{2}{3} \\ x &= -\frac{1}{3} \end{aligned}$$

Answer $x = \dots\dots -\frac{1}{3} \dots\dots\dots$ [2]

- 4 (a) Factorise $3y^{2a} + 6y^a$ completely.

$$\begin{aligned} & 3y^{2a} + 6y^a \\ &= 3y^a(y^a + 2) \end{aligned}$$

Answer $3y^a(y^a + 2)$ [1]

- (b) Factorise $20pq - 15sq + 3sr - 4pr$ completely.

$$\begin{aligned} & 20pq - 15sq + 3sr - 4pr \\ &= 5q(4p - 3s) + r(3s - 4p) \\ &= 5q(4p - 3s) - r(4p - 3s) \\ &= (4p - 3s)(5q - r) \end{aligned}$$

Answer $(4p - 3s)(5q - r)$ [2]

- 5 Express $\frac{3n^2 - 8n - 3}{9 - n^2} + \frac{5n - 3}{n + 3}$ as a single fraction in its simplest form.

$$\begin{aligned} & \frac{3n^2 - 8n - 3}{9 - n^2} + \frac{5n - 3}{n + 3} \\ &= \frac{(3n + 1)(n - 3)}{(3 - n)(3 + n)} + \frac{5n - 3}{n + 3} \\ &= \frac{-(3n + 1)(3 - n)}{(3 - n)(3 + n)} + \frac{5n - 3}{n + 3} \\ &= \frac{-3n - 1 + 5n - 3}{3 + n} \\ &= \frac{2n - 4}{3 + n} \end{aligned}$$

Answer $\frac{2n - 4}{3 + n}$ [3]

- 6 Given that x satisfies the inequality $\frac{1}{2}x - (x - 2) \leq \frac{3}{4}(2x - 3) - 5$, find the smallest value of x if x is an integer.

$$\begin{aligned}\frac{1}{2}x - (x - 2) &\leq \frac{3}{4}(2x - 3) - 5 \\ 2x - 4(x - 2) &\leq 3(2x - 3) - 20 \\ 2x - 4x + 8 &\leq 6x - 9 - 20 \\ -2x - 6x &\leq -29 - 8 \\ -8x &\leq -37 \\ x &\geq 4.625\end{aligned}$$

Answer $x = \dots\dots\dots 5 \dots\dots\dots$ [3]

- 7 It is given that $90^\circ < A < 180^\circ$ and $\sin A = \frac{7}{25}$.
(a) Find the value of A .

$$\begin{aligned}A &= 180^\circ - \sin^{-1}\left(\frac{7}{25}\right) \\ &= 180^\circ - 16.260\dots^\circ \\ &= 163.739\dots^\circ \\ &= 163.7^\circ \text{ (1dp)}\end{aligned}$$

Answer $\dots\dots\dots 163.7 \dots\dots\dots$ [1]

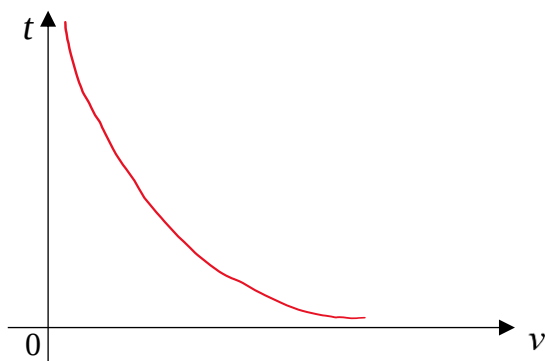
- (b) Without the use of the calculator, find the numerical value of $\cos A$.

$$\begin{aligned}x &= \sqrt{25^2 - 7^2} \\ &= 24 \\ \cos A &= -\frac{24}{25}\end{aligned}$$

Answer $\dots\dots\dots -\frac{24}{25} \dots\dots\dots$ [2]

- 8** The time, t seconds, taken to download a file from a computer is inversely proportional to the internet connect speed, v Mbps. When the speed is 250 Mbps, the time taken is 4 seconds.

(a) Sketch the graph of t against v in the axes below. [1]



(b) Find the time taken to download a file when the speed is 160 Mbps.

$$\begin{aligned}
 t &= \frac{k}{v} \\
 k &= 4 \times 250 \\
 &= 1000 \\
 t &= \frac{1000}{v} \\
 &= \frac{1000}{160} \\
 &= 6.25
 \end{aligned}$$

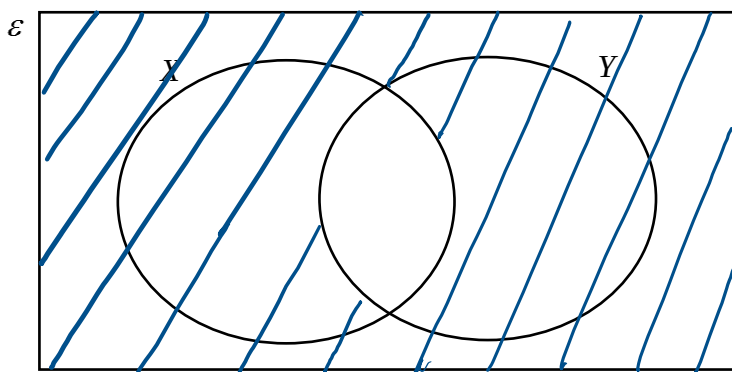
Answer 6.25..... sec [2]

(c) If the speed is increased by 25%, find the percentage change in the time taken.

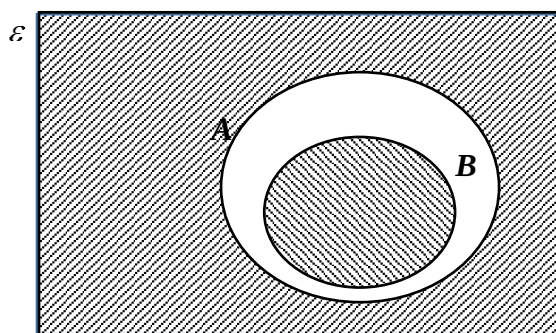
$$\begin{aligned}
 t &= \frac{k}{v} \\
 t_1 &= \frac{4k}{5v} \\
 t_1 &= \frac{4}{5}t \\
 \text{Percentage change} &= -\frac{1}{5} \times 100\% \\
 &= -20\%
 \end{aligned}$$

Answer-20% [2]

- 9 (a) On the Venn Diagram, shade the the region which represents $X' \cup Y'$. [1]



(b)



From the Venn diagram above,

- (i) write, in set notation that illustrates the relationship between A and B.

Answer $B \subset A$ [1]

- (ii) write down the set notation that represents the shaded region.

Answer $A' \cup B$ [1]

- 10 It is given that $\xi = \{x : x \text{ is an integer and } 0 < x \leq 15\}$,

$P = \{x : x \text{ is a prime number}\}$ and

$Q = \{x : x \text{ is divisible by 3 or 5}\}.$

List the elements in $P' \cap Q$.

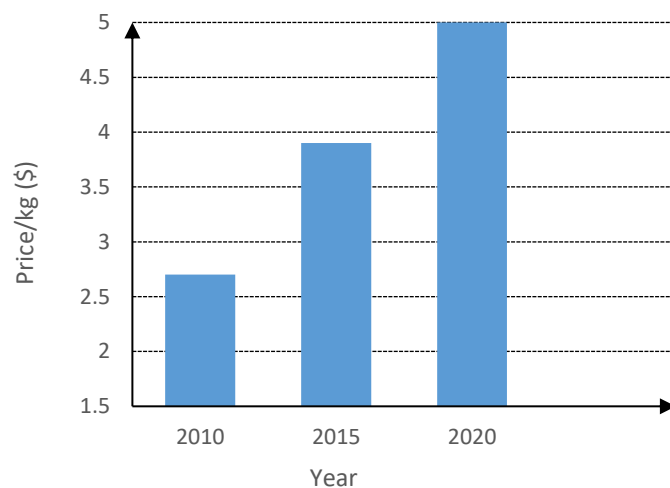
$$P' = \{1, 4, 6, 8, 9, 10, 12, 14, 15\}$$

$$Q = \{3, 5, 6, 9, 10, 12, 15\}$$

$$P' \cap Q = \{6, 9, 10, 12, 15\}$$

Answer $\{.....6, 9, 10, 12, 15.....\}$ [2]

- 11 The graph shows the price of 1 kg of apples in Country X from 2010 – 2020.



- (a) State the misleading feature of the graph.

Answer The **vertical axis** does not start at zero [1]

- (b) Explain how this feature affects the reader's interpretation of the graph.

Answer

The graph misleads the reader thinking that the price of apples in 2020 is thrice that of 2010's when it is only about twice. [1]

12 Consider the pattern,

$$L_1: 3^2 - 2^2 = 3 + 2 = 5$$

$$L_2: 4^2 - 3^2 = 4 + 3 = 7$$

$$L_3: 5^2 - 4^2 = 5 + 4 = 9$$

$$L_4: 6^2 - 5^2 = 6 + 5 = 11$$

$$:$$

$$:$$

$$L_n: \dots\dots\dots$$

(i) Write down L_n of the pattern. [1]

$$L_n: (n+2)^2 - (n+1)^2 = n+2+n+1 = 2n+3$$

(ii) Explain, with calculations, why 1137 is a term of the sequence 5, 7, 9, 11, ... [2]

$$2n+3=1137$$

$$n=567$$

n is an integer. Hence 1137 is the 567th term of the sequence.

(iii) Find the exact value of $L_1 + L_2 + L_3 + \dots + L_{99}$.

$$101^2 - 2^2 = 10197$$

Answer 10197.....[1]

- 13** The numbers 720 and 1575, written as the products of their prime factors are
 $720 = 2^4 \times 3^2 \times 5$ and $1575 = 3^2 \times 5^2 \times 7$.

(i) Find the largest common factor of both 720 and 1575.

$$\text{HCF} = 3^2 \times 5 = 45$$

Answer 45.....[1]

(ii) Find the smallest whole number that is divisible by both 720 and 1575.

$$\text{LCM} = 2^4 \times 3^2 \times 5^2 \times 7 = 25200$$

Answer 25200.....[1]

(iii) When the product of 720 and 1575 is divided by p , the result is a perfect square.

Find the smallest integer value of p , where $p > 0$.

$$\frac{2^4 \times 3^2 \times 5 \times 3^2 \times 5^2 \times 7}{p} = \frac{2^4 \times 3^4 \times 5^3 \times 7}{p}$$

$$\therefore p = 5 \times 7 = 35$$

Answer $p =$ 35..... [1]

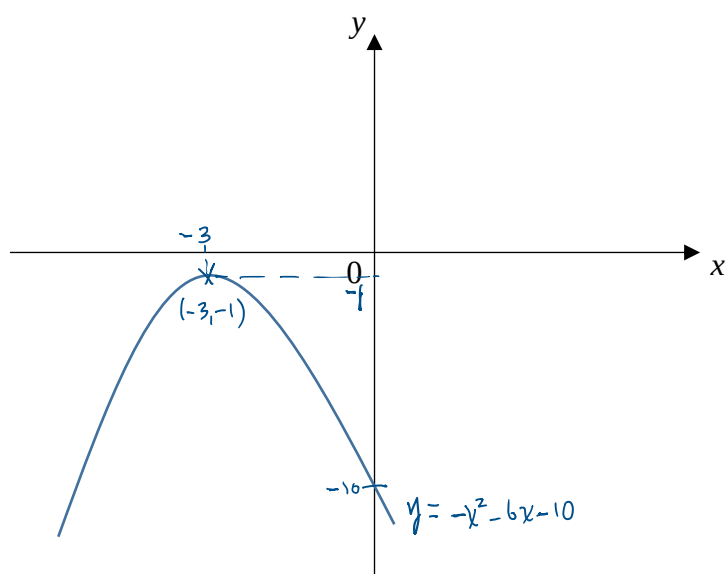
- 14 (i) Express $-x^2 - 6x - 10$ in the form $a(x - p)^2 + q$, where a , p and q are integers

$$\begin{aligned}
 -x^2 - 6x - 10 &= -(x^2 + 6x) - 10 \\
 &= -[(x + 3)^2 - 9] - 10 \\
 &= -(x + 3)^2 + 9 - 10 \\
 &= -(x + 3)^2 - 1
 \end{aligned}$$

Answer $-(x + 3)^2 - 1$ [2]

- (ii) Sketch the graph of $y = -x^2 - 6x - 10$ on the axes below.

Indicate clearly the coordinates of the point where the graph crosses the y -axis and the turning point. [1]



- (iii) Hence, explain, without solving, why there are no solutions of x when $-x^2 - 6x = 10$.

Answer $-x^2 - 6x - 10 = 0$

There are no solutions as the curve does not intersect the x -axis as shown above. [1]

- 15** The following stem-and-leaf diagram shows the duration of each of the 20 calls, in minutes, made by Tammy using her mobile phone on a certain day.

<u>Stem</u>	<u>Leaves</u>
1	3 4 4 5 7 9
2	0 1 2 5 6 6 7 9
3	1
4	4 6
5	1
6	2 R

Key: 1 | 3 means 13 minutes

- (i)** Find the percentage of calls that are shorter than 31 minutes.

Percentage

$$= \frac{14}{20} \times 100\%$$

$$= 70\%$$

Answer 70...% [1]

- (ii)** Find the median call duration.

median call duration

$$= \frac{25 + 26}{2}$$

$$= 25.5 \text{ min}$$

Answer 25.5.....min [1]

- (iii)** Given that the range of the duration of the calls made is 52 minutes.

Find the value of R.

Answer 5..... [1]

- 16** The table below summarises the number of spam calls received by 80 elderly males of a community club in a month.

No. of spam calls	$5 \leq n < 10$	$10 \leq n < 15$	$15 \leq n < 20$	$20 \leq n < 25$
Frequency	30	24	15	11

- (i) Calculate an estimate for
 (a) the mean number of spam calls received,

Answer 12.9375..... [1]

- (b) the standard deviation.

Answer 5.26 (3sf)..... [1]

- (ii) The survey is also conducted for 80 elderly females at the community club. The mean number of spam calls received is 16 calls and the standard deviation is 9 calls.

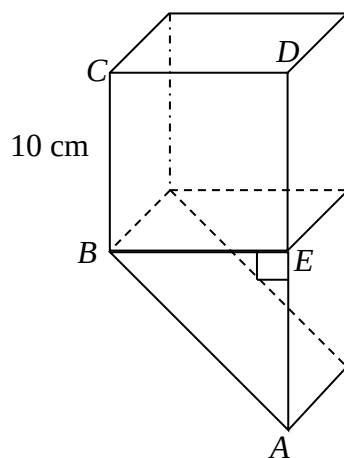
Make two comparisons between the results of the survey conducted for the elderly males and females.

Answer

The elderly **females received more spam calls** in the month as compared to the elderly males as they received a mean number of 16 calls.

However, the results from the elderly **males are more consistent as it has a standard deviation which is about 3.74 calls lower** than that of the elderly females. [2]

- 17 A container is made up of a cube and a right-angled triangular prism. Each side of the cube is 10 cm and $EB = EA$.



- (a) Find the volume of the container.

Volume of container

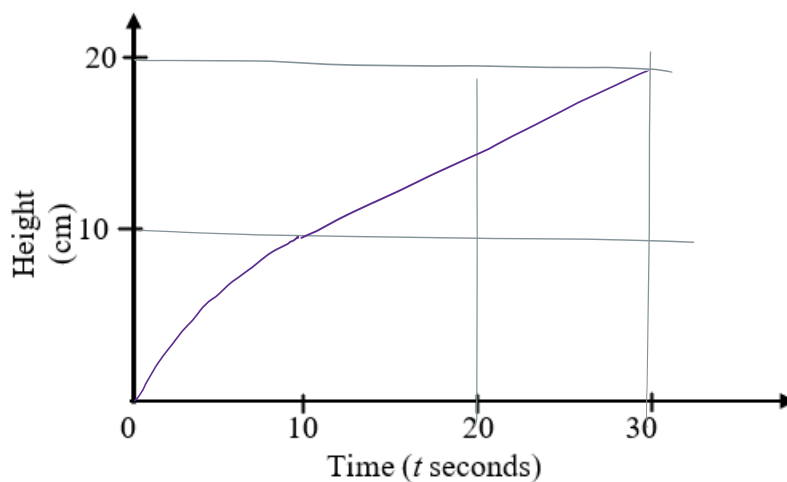
$$= \frac{1}{2}(10 + 20)(10) \times 10$$

$$= 1500 \text{ cm}^3$$

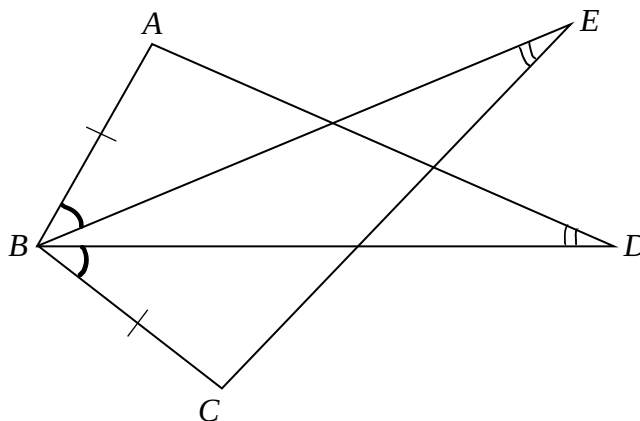
Answer 1500..... cm^3 [2]

- (b) Water is poured from the top at a constant rate and fills the container in 30 seconds.

Sketch the graph showing how the depth of the water varies with time. [1]



- 18** In the diagram, $BA = BC$, $\angle ABE = \angle DBC$ and $\angle BEC = \angle BDA$.
Prove that the triangles ABD and CBE are congruent.



Answer

$$\angle ABE = \angle DBC \text{ (given)}$$

$$\angle ABD = \angle ABE + \angle EBD$$

$$= \angle CBD + \angle EBD$$

$$= \angle CBE$$

$$AB = CB \text{ (given)}$$

$$\angle BEC = \angle BDA \text{ (given)}$$

$$\therefore \triangle ABD \equiv \triangle CBE \text{ (AAS)}$$

[3]

- 19** The points A , B and C are vertices of a triangle. The point A is $(2, 7)$ and the gradient of the line AB is -3 .

(a) Find the equation of the line AB .

$$y - 7 = -3(x - 2)$$

$$y = -3x + 6 + 7$$

$$y = -3x + 13$$

Answer $y = -3x + 13$ [2]

(b) The equation of the line BC is $2x + y = 17$.

Find the coordinates of B .

$$-3x + 13 = 17 - 2x$$

$$-x = 4$$

$$x = -4$$

$$y = 17 - 2(-4)$$

$$= 25$$

$$B(-4, 25)$$

Answer $(-4, 25)$ [2]

(c) Find the length of AB .

$$AB = \sqrt{(2 + 4)^2 + (7 - 25)^2}$$

$$= 18.9736...$$

$$= 19.0 \text{ units (3sf)}$$

Answer 19.0 units [2]

- 20** 2 regular polygons A and B have n sides and $2n$ sides respectively.

The size of each interior angle in polygon A is 300% greater than the size of each exterior angle in polygon B .

- (i)** Find the number of sides polygon A has.

$$\frac{180(n-2)}{n} = 4 \left(\frac{360}{2n} \right)$$

$$180(n-2) = 720$$

$$n-2 = 4$$

$$n = 6$$

Polygon A has 6 sides.

Answer.....6..... [2]

- (ii)** Hence, find the size of an interior angle in B .

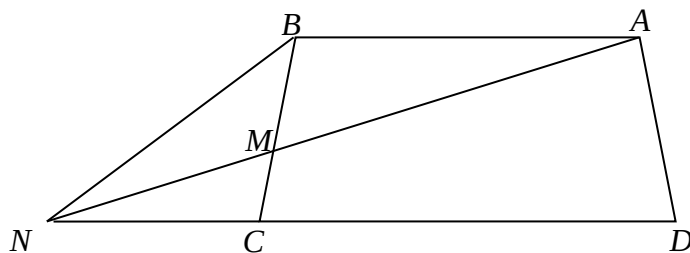
Size of interior angle in B

$$= \frac{180^\circ(12-2)}{12}$$

$$= 150^\circ$$

Answer.....150..... ° [1]

- 21 $ABCD$ is a trapezium, with side AB parallel to side DC . M is a point on BC such that $3CM = BC$. The lines DC and AM produced meet at N .



- (a) Show that $\triangle ABM$ and $\triangle NCM$ are similar. [2]

$$\angle BAM = \angle CNM \text{ (alt. } \angle \text{s)}$$

$$\angle BMA = \angle CMN \text{ (vert. opp. } \angle \text{s)}$$

$$\therefore \triangle ABM \text{ is similar to } \triangle NCM$$

- (b) Show that $2NC = AB$. [1]

Answer

Since $\triangle ABM$ is similar to $\triangle NCM$

$$\frac{NC}{AB} = \frac{CM}{MB} = \frac{1}{2}$$

Hence $2NC = AB$ (shown)

- (c) Calculate the value of

(i) $\frac{\text{area of } \triangle ABM}{\text{area of } \triangle NCM},$

$$\frac{\text{area of } \triangle ABM}{\text{area of } \triangle NCM} = \left(\frac{2}{1}\right)^2 = 4$$

Answer 4..... [1]

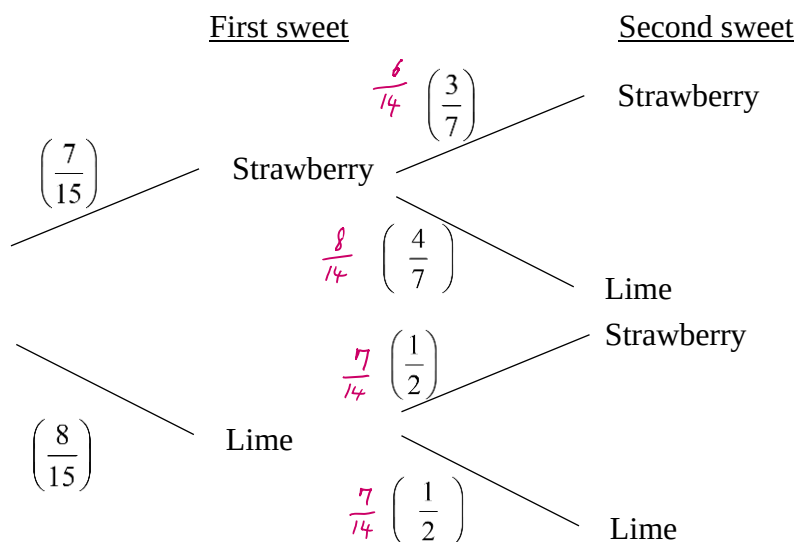
(ii) $\frac{\text{area of } \triangle NBC}{\text{area of } \triangle NBA}.$

$$\begin{aligned} \frac{\text{area of } \triangle NBC}{\text{area of } \triangle NBA} &= \frac{\text{area of } \triangle NBC}{\text{area of } \triangle BNM} \times \frac{\text{area of } \triangle BNM}{\text{area of } \triangle NBA} \\ &= \frac{3}{2} \times \frac{1}{3} \\ &= \frac{1}{2} \end{aligned}$$

Answer $\frac{1}{2}$ [1]

- 22** A packet of 15 sweets contains 2 different flavours. There are 7 strawberry-flavoured sweets and 8 lime-flavoured sweets in the packet. Two sweets are taken from the packet at random, without replacement.

(i) Complete the tree diagram to show the probability of the possible outcomes. [2]



(ii) Hence, find the probability as a fraction in its simplest form, that

(a) both sweets drawn are the same flavours,

$$\begin{aligned}
 &P(\text{same flavours}) \\
 &= \frac{7}{15} \left(\frac{3}{7}\right) + \frac{8}{15} \left(\frac{1}{2}\right) \\
 &= \frac{7}{15}
 \end{aligned}$$

Answer $\frac{7}{15}$ [2]

(b) at least one sweet is strawberry-flavoured.

$$\begin{aligned}
 &P(\text{at least one sweet is strawberry}) \\
 &= 1 - \frac{8}{15} \left(\frac{1}{2}\right) \\
 &= \frac{11}{15}
 \end{aligned}$$

Answer $\frac{11}{15}$ [2]

- 23 Point A has the position vector $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and point B has position vector $\begin{pmatrix} 0 \\ 11 \end{pmatrix}$.

(i) Calculate $|\overrightarrow{AB}|$.

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= \begin{pmatrix} -3 \\ -7 \end{pmatrix} + \begin{pmatrix} 0 \\ 11 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ 4 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}|\overrightarrow{AB}| &= \sqrt{(-3)^2 + 4^2} \\ &= 5 \text{ units}\end{aligned}$$

Answer 5.....units [2]

- (ii) Given that the point C is $(-3, 15)$, using vectors, explain why B is the midpoint of AC. [3]

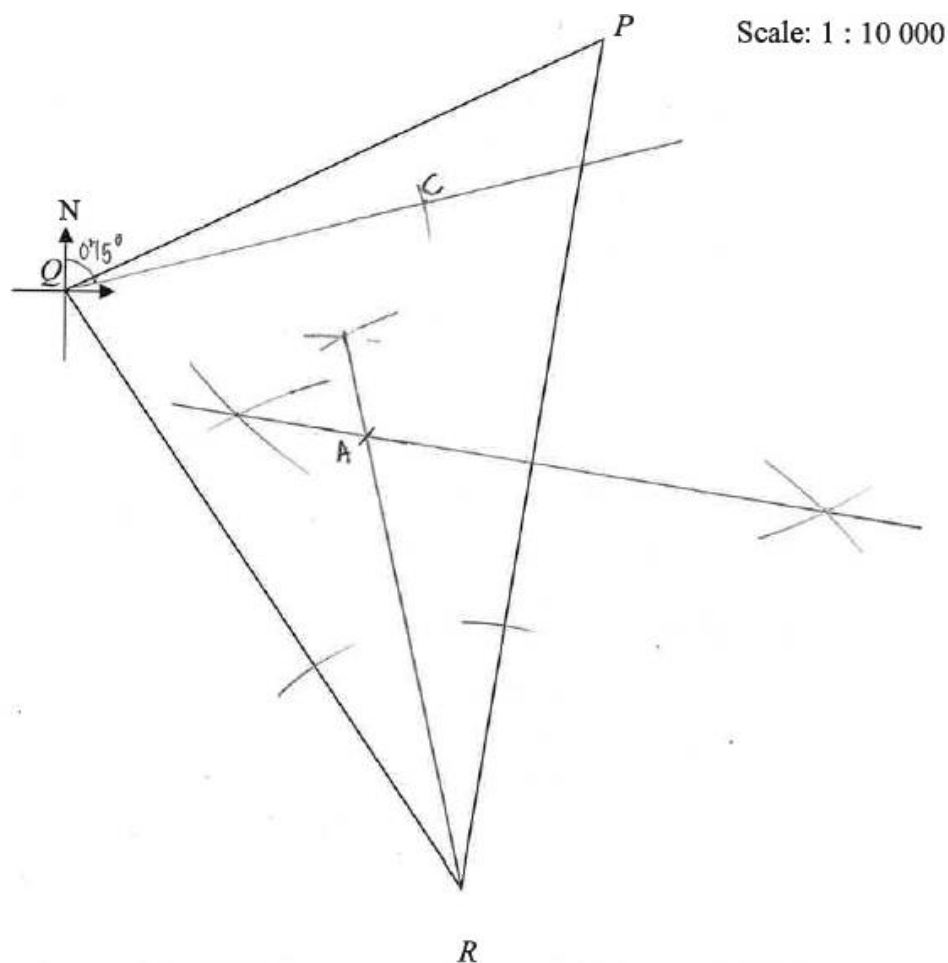
$$\begin{aligned}\overrightarrow{AC} &= \overrightarrow{AO} + \overrightarrow{OC} \\ &= \begin{pmatrix} -3 \\ -7 \end{pmatrix} + \begin{pmatrix} -3 \\ 15 \end{pmatrix} \\ &= \begin{pmatrix} -6 \\ 8 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\overrightarrow{AB} &= \begin{pmatrix} -3 \\ 4 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} -6 \\ 8 \end{pmatrix} \\ &= \frac{1}{2} \overrightarrow{AC}\end{aligned}$$

Since $\frac{AB}{AC} = \frac{1}{2}$, and A is a common point, A, B, C are collinear.

Hence B is the midpoint of AC.

- 24 In the scale drawing, triangle PQR represents the positions of three shops.



- (a) A new shop is to be built at A where it is equidistant from the points P and R and equidistant from the line segments PR and QR .
- (i) On the scale drawing, mark the point A . [2]
- (ii) Find, in square metres, the actual area of triangle APR .

$$1 \text{ cm} : 100 \text{ m}$$

Size of $\triangle APR$

$$= \frac{1}{2} (2.3 \times 100) (11.7 \times 100)$$

$$= 134550 \text{ m}^2$$

$127600 \leq x \text{ m}^2 \leq 141600$

Answer 134550.....m² [2]

- (b) A new community centre, C is to be built inside the triangle PQR such that the bearing of C is 075° and 500 m from Q .
- Mark the point C on the diagram above. [1]

- 25 A car dealer priced a car at \$189 500 and offered the following 2 payment schemes:

Scheme A: Cash Payment with 10% discount.

Scheme B: 30% deposit and \$2519 monthly instalments for 5 years.

Peter bought the car under scheme B.

- (a) Calculate the total amount of money Peter paid for the car.

$$\begin{aligned}\text{amt. paid} &= \frac{30}{100}(189500) + 2519(5)(12) \\ &= \$207990\end{aligned}$$

Answer \$.....207990..... [2]

- (b) Peter claims that he had paid more buying the car using scheme B as compared to scheme A. Do you agree with Peter? Justify your answer.

$$\begin{aligned}\text{amt. (Scheme A)} &= \frac{90}{100}(189500) \\ &= \$170550 \\ \text{Difference} &= 207990 - 170550 \\ &= \$37440\end{aligned}$$

Answer

I **agree** with Peter because he would have spent \$37440 more as shown above.

[2]

- (c) Calculate the simple interest rate per annum Peter was charged for scheme B.

$$\begin{aligned}\text{Total instament} &= \$ (2519 \times 5 \times 12) \\ &= \$151140 \\ 151140 - \frac{70}{100}(189500) &= \frac{\frac{70(189500)}{100} \times R \times 5}{100} \\ \frac{18490 \times 100}{5 \times 132650} &= R \\ R &= 2.7877... = 2.79 \text{ (3sf)}\end{aligned}$$

Answer 2.79.....% [3]

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Class Index Number

Name : _____

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METHODIST GIRLS' SCHOOL

Founded in 1887



PRELIMINARY EXAMINATION 2023 Secondary 4

Thursday

17 August 2023

MATHEMATICS Paper 2

4052/02

2 hours 15 minutes

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

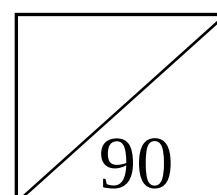
The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For p , use either your calculator value or 3.142, unless the question requires the answer in terms of p

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.



This question paper consists of 21 printed pages and 3 blank pages.

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

- 1 (a) It is given that $y = \frac{1}{4}z(w^2 - x^2)$.
 (i) Find z such that $y = 4$, $w = -1$ and $x = 3$.

Answer[2]

- (ii) Make x the subject of the formula $y = \frac{1}{4}z(w^2 - x^2)$.

Answer $x =$ [2]

(b) Simplify $\frac{3}{x+2} + \frac{6x}{4-x^2}$.

Answer[3]

- (c) In 2022, Singapore has a population of 5.64 million.
The land area of Singapore is 728.6km².
Find the population of Singapore per km², leaving your answer as standard form.

Answer [2]

- 2 (a) A closed cone, of height h cm, is filled with water to half its height as shown in Diagram A. It is then inverted as shown in Diagram B. Find the height of the water level, x cm, in terms of h , in Diagram B.

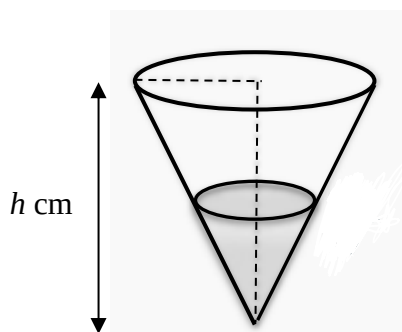


Diagram A

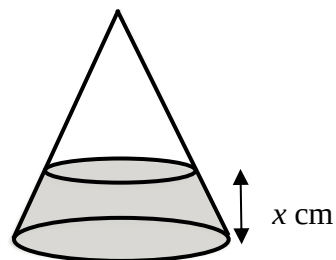


Diagram B

Answer [3]

- (b) Janet began her round-island 145 km cycling route from Point A at 6 am, cycling at an average speed of x km/h.
- (i) Write down the expression, in terms of x , for the time taken by Janet to complete the cycling route.

Answer h [1]

Angel started on the same route 10 minutes later. She cycled at a speed of 2 km/h faster than Janet for the first 20 km and cycled x km/h for the remaining route.

- (ii) Write down the expression, in terms of x , for the time taken by Angel to complete the cycling route.

Answer h [1]

Angel reached the end of the route at the same time as Janet.

- (iii) Form an equation in x and show that it reduces to $x^2 + 2x - 240 = 0$.

Answer [3]

- (iv) Solve the equation $x^2 + 2x - 240 = 0$.

Answer $x = \dots\dots\dots$ [3]

- (v) What time did Janet reach the ending point?

Answer $\dots\dots\dots$ [2]

- 3 (a) In a logistics company, each delivery man is paid \$ x for every correct delivery made, \$0 for no delivery but has to pay a penalty of \$1 for each damaged delivery.

Tom and Jerry made the following deliveries on a particular day.

$$\mathbf{A} = \begin{array}{c} \text{Tom} \\ \text{Jerry} \end{array} \begin{array}{ccc} \text{correct} & \text{no delivery} & \text{damaged} \\ \begin{pmatrix} 13 & 4 & 3 \\ 10 & 8 & y \end{pmatrix} \end{array} \quad \mathbf{B} = \begin{pmatrix} x \\ 0 \\ -1 \end{pmatrix}$$

- (i) Evaluate $\mathbf{AB}$ in terms of x and y .

Answer [2]

- (ii) Explain what your answer in (a)(i) represents.

.....

 [1]

It is given that Tom earned \$18 more than Jerry despite both making the same number of deliveries.

- (iii) Find the values of x and y .

Answer $x = \dots\dots\dots y = \dots\dots\dots$ [3]

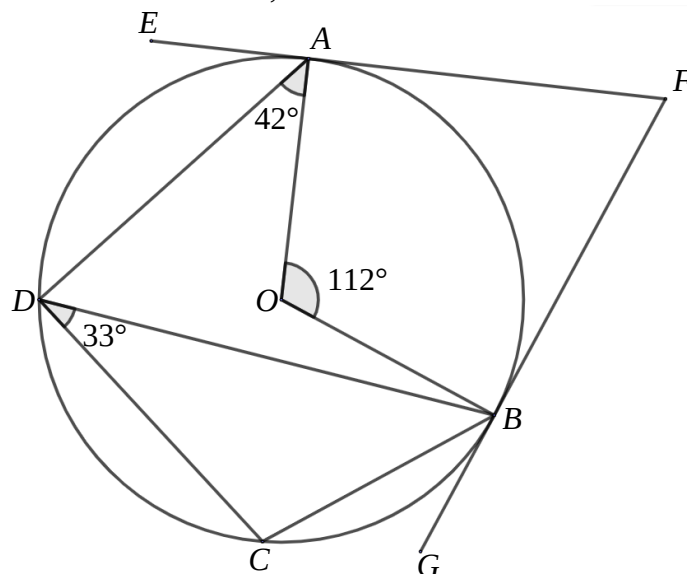
- (iv) By using matrix multiplication, calculate the total amount of money the company needs to pay both Tom and Jerry on that particular day.

Answer \$[2]

- (b) Tom is looking for a new job. He targets to earn an annual salary of \$150 000 at the end of 5 years. The job that he is interviewing for is offering an annual salary increment of 3% per annum.
What is the minimum starting annual salary should Tom request for to meet his target. He needs to set his salary request in multiples of \$1000.

Answer \$ [3]

- 4 (a) In the diagram, A , B , C and D lie on the circle with centre O . EF and GF are tangents to circles at A and B respectively. It is given that $\angle DAO = 42^\circ$, $\angle BDC = 33^\circ$ and $\angle AOB = 112^\circ$



Find

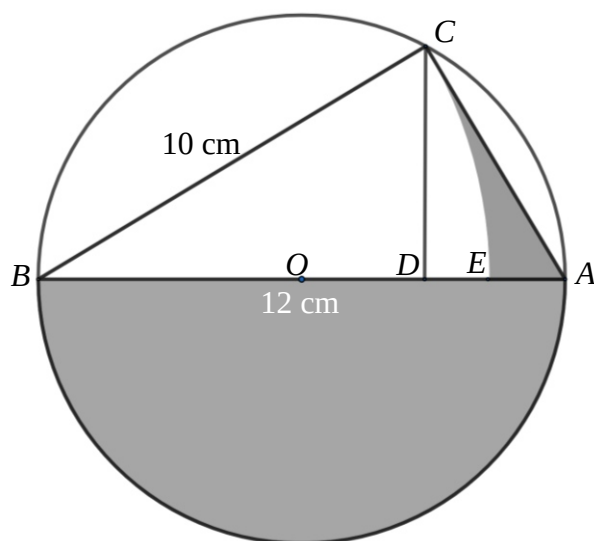
- (i) $\angle CBD$,

Answer $\angle CBD = \dots\dots\dots$ [3]

- (ii) $\angle ABF$.

Answer $\angle ABF = \dots\dots\dots$ [2]

- (b) The diagram shows a circle, centre O , with a diameter AB of 12 cm. CD is perpendicular to AB . CE is an arc of a circle with B as the centre and radius 10 cm.



- (i) Show that $\angle CBD = 0.586$ when corrected to 3 significant figures.

Answer

[1]

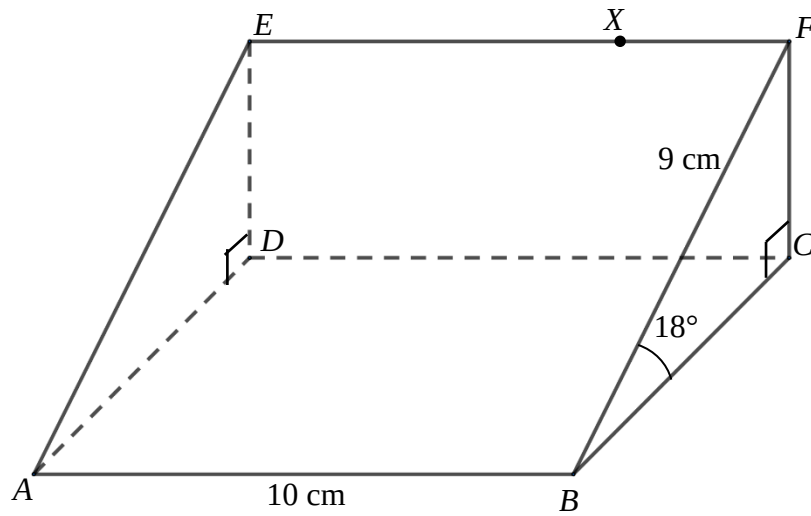
- (ii) Find the arc length CE .

Answer cm [1]

- (iii) Calculate the area of the shaded region.

Answer cm^2 [3]

- 5 A wooden triangular wedge has a rectangular base as shown.
 $AB = 10$ cm and $BF = 9$ cm and $\angle FBC = 18^\circ$.
 X is on EF such that $EX : XF = 3 : 1$.



- (i) Find the length of AC .

Answer cm [3]

- (ii) Find the length of CX .

Answer cm [2]

- (iii) Find the greatest angle of elevation of X from BC .

Answer [2]

- (iv) Find the area of triangle BXC .

Answer cm^2 [2]

- 6** A solid cuboid has a square base of x cm, height h cm and a total surface area of 24 cm^2 .

(a) Find h in terms of x .

Answer $h = \dots\dots\dots$ [2]

(b) Show that the volume of the cuboid, $y \text{ cm}^3$, is given by the formula

$$y = \frac{1}{2}x(12 - x^2).$$

Answer

[1]

The values of x and y for $y = \frac{1}{2}x(12 - x^2)$ are given in the table below:

x	0.5	0.8	1	1.5	2	2.5	3
y	2.9	4.5	5.5	7.3	8	7.2	4.5

(c) On the given grid, draw the graph of $y = \frac{1}{2}x(12 - x^2)$ for $0.5 \leq x \leq 3$. [2]

(d) Use your graph to find the volume of the cuboid when the base area is 4.41 cm^2 .

Answer $\dots\dots\dots \text{cm}^3$ [2]

(e) (i) By drawing a tangent, find the gradient of the curve when $x = 1.7 \text{ cm}$.

Answer $\dots\dots\dots$ [2]

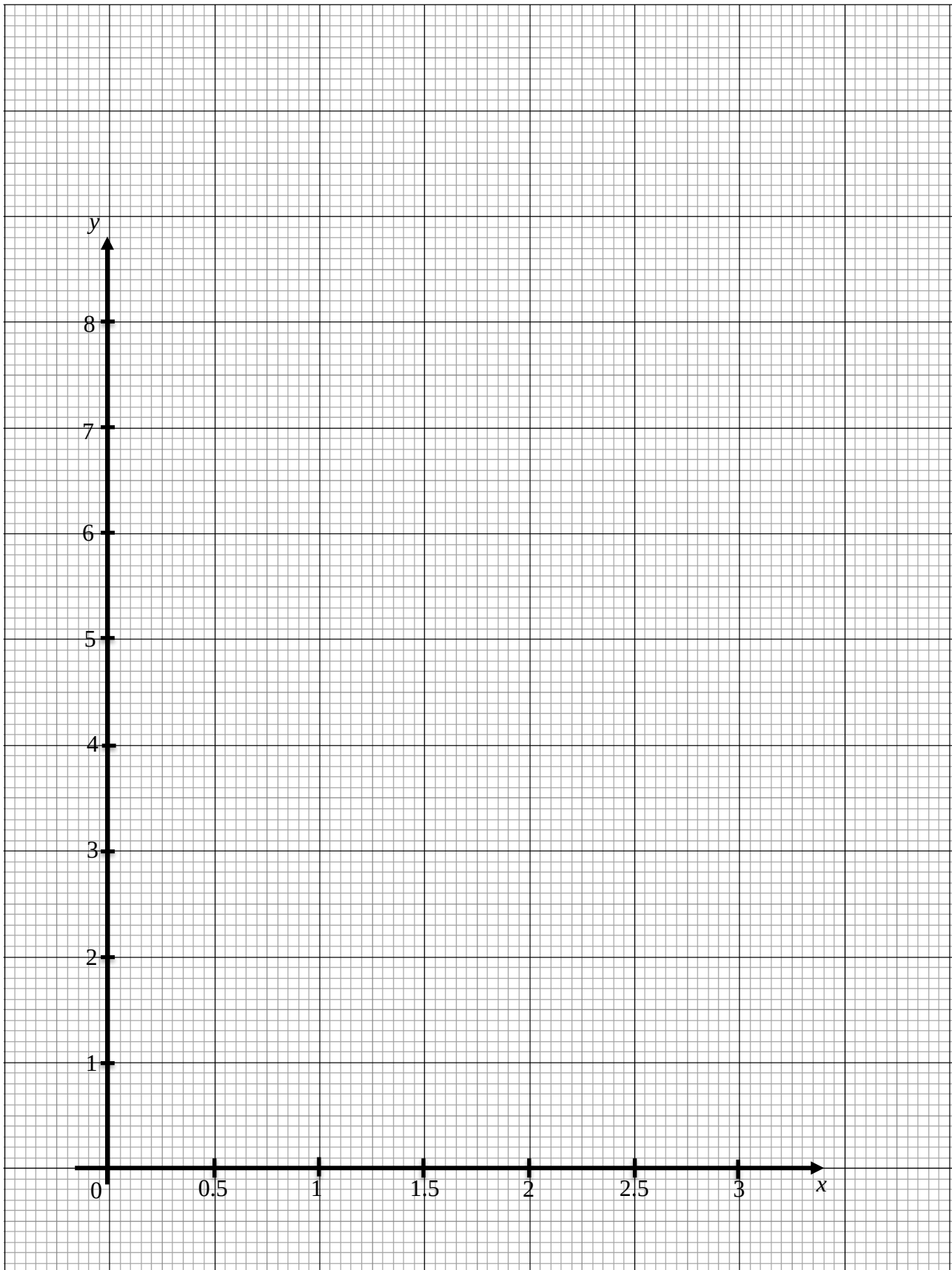
(ii) What does this gradient represent?

Answer $\dots\dots\dots$

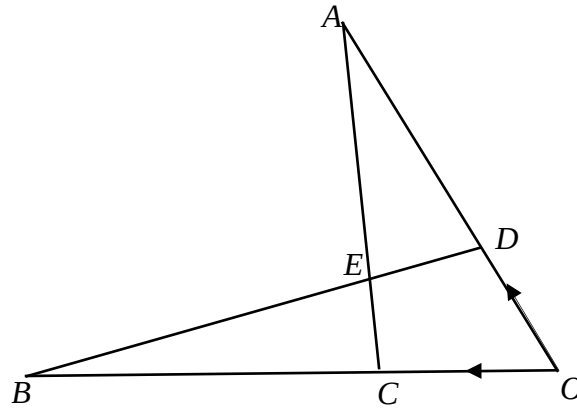
$\dots\dots\dots$ [1]

(f) State the maximum volume of the cuboid.

Answer $\dots\dots\dots \text{cm}^3$ [1]



- 7 In the diagram, $\overrightarrow{OD} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$. It is given that $OD = \frac{1}{2}DA$ and $OC = \frac{1}{3}OB$.



- (a) Express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{AC}$,

Answer [1]

(ii) $\overrightarrow{DB}$.

Answer [1]

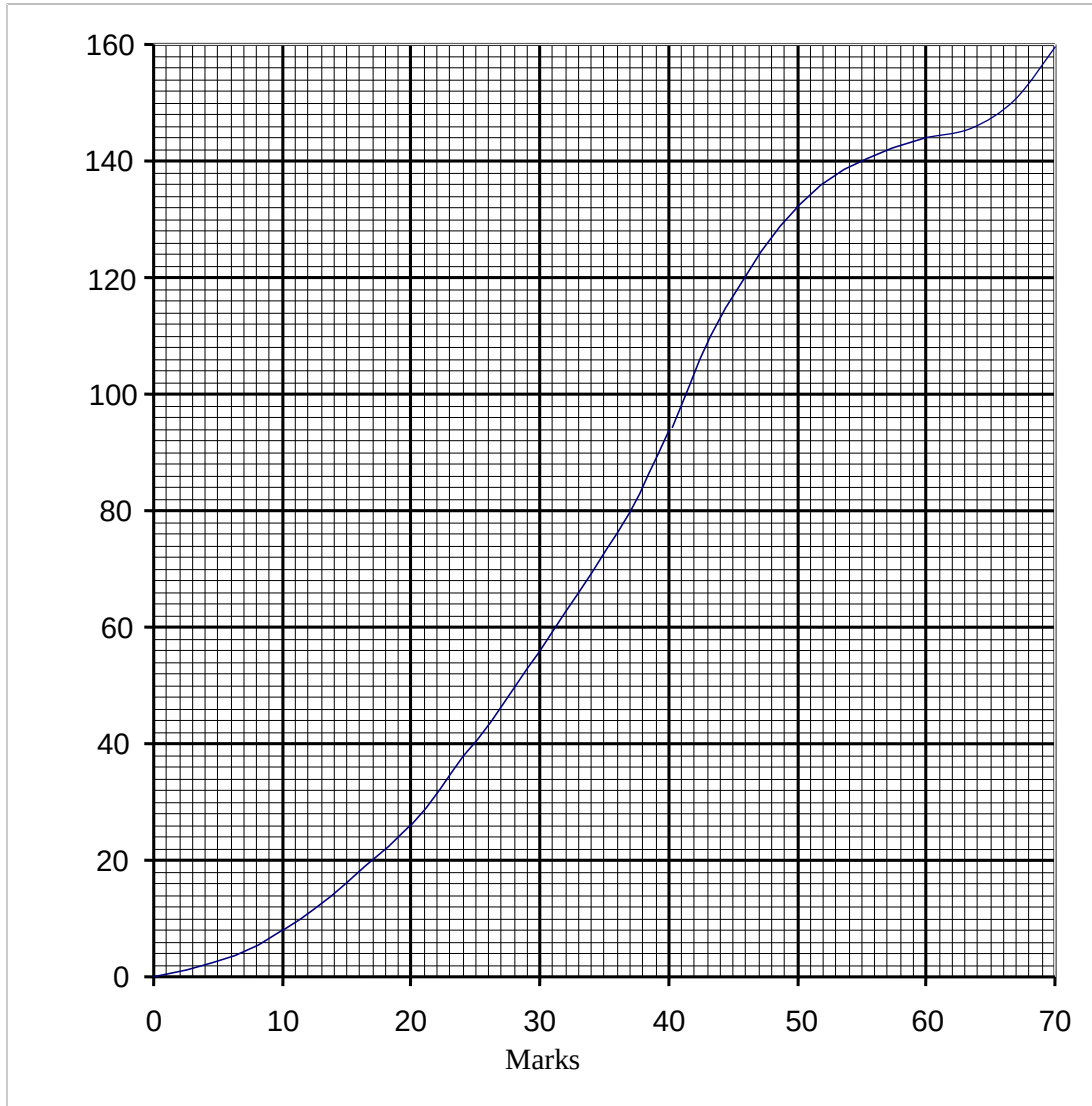
- (b) Given that $\frac{DE}{DB} = \frac{1}{4}$, express $\overrightarrow{OE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answer [2]

- (c) Calculate the numerical value of $\frac{EC}{AE}$.

Answer[2]

- 8 The cumulative frequency curve below shows the distribution of the marks obtained by 160 students for Mathematics Test 1.



- (a) Estimate from the graph,
(i) the median,

Answer[1]

- (ii) the interquartile range,

Answer[1]

- (iii) the percentage of students who obtained more than 50 marks.

Answer[1]

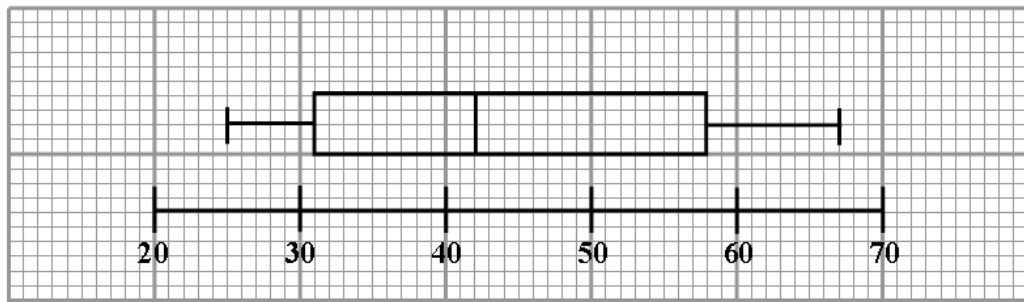
- (b) Given that 15% of the students will be awarded a distinction grade, use the graph to find the lowest mark scored by this group of students.

Answer[1]

- (c) Find the probability of choosing 2 students whose marks are in the 90th percentile and above.

Answer[2]

- (d) The marks obtained by the same group of 160 students who sat for Mathematics Test 2 are also noted. The results are summarised by a box-and-whisker plot shown below.



- (i) State the median mark.

Answer [1]

- (ii) Find the interquartile range of the marks.

Answer[1]

- (e) Make 2 comparisons between the marks obtained by the students in the two tests.

- 1 [1]
- 2 [1]

- 9 An event company is holding a 5-day food exhibition for 200 exhibitors to promote their food items. It collects a registration fee of \$88 and a **daily** booth rental of \$200 from each exhibitor.
- (a) Calculate the total amount of money collected from each exhibitor.

Answer \$..... [1]

The event company needs to rent **one entire exhibition space** for the food exhibition.
The space required for the exhibition is as follows:

Size of each exhibition booth	20m ²
Space for visitors' movement and dining area	60% of the total area occupied by the exhibition booths

The layout of the 5 available exhibition halls is as shown:

Hall A	Hall B	Hall C	Hall D	Hall E
2000 m ²	3000 m ²	3500 m ²	4500 m ²	2500 m ²
\$6000 per day	\$8500 per day	\$9000 per day	\$10000 per day	\$7000 per day

Note: Adjacent exhibition halls have removable partitions to expand into a bigger exhibition hall.

- (b) Calculate the **minimum** amount payable by the event company for the daily rental of the exhibition hall(s) needed.

Answer \$..... [2]

The event company will set up the entire exhibition one day before the actual exhibition starts. They need to include the rental of the entire exhibition space, tables and chairs for this extra day of set up.

50 long tables and 350 chairs will be set up in the dining area.
Each exhibition booth will require 4 square tables and 4 chairs.

Items available for rental for the exhibition:

Square tables:	Every 100 tables at \$30 per day
Long tables:	\$2 each table per day
Chairs:	Every 50 chairs at \$10 per day

The event company employs 3 security guards and 2 part-time staff for 12 hours on each exhibition day.

Information of other costs:

Security Guards: Company A: \$10 per hour for each guard	
Company B: \$8 per hour for first 8 hours per day for each guard and	
\$12 per subsequent hour for each guard	
Part-time staff:	\$9 per hour for each staff

- (c) It is estimated that there will be a total of 48 000 visitors for the 5-day exhibition. The event company wishes to earn a total of \$200 000 profit from organizing the event. Using your answers in parts (a) and (b), calculate and suggest a suitable amount of entrance fee that the event company should charge for each visitor. Explain your answer.

Answer \$..... because

..... [8]

End of Paper

Answer Key

$$1(a)(i) -2 \quad (ii) x = \pm \sqrt{w^2 - \frac{4y}{z}} \quad (b) \frac{3}{(2-x)} \quad (c) 7.74 \times 10^3$$

$$2(a) x = h - \frac{\sqrt[3]{7}}{2}h \quad 2(b)(i) \frac{145}{x} \quad (ii) \frac{20}{x+2} + \frac{125}{x} \quad (iii) 14.5 \text{ or } -16.5 \quad (iv) 3.59\text{pm}$$

$$3(a)(i) \frac{20}{x+2} + \frac{125}{x} \quad (ii) \text{rep amount of money Tom and Jerry will be paid respectively for their}$$

deliveries on a particular day. (iii) $x = 5, y = 6$ (iv) \$106 (b) \$130 000

$$4(a) (i) 43^\circ \quad (ii) 56^\circ \quad (b)(ii) 5.86 \quad (iii) 60.4\text{cm}^2$$

$$5(i) 13.2\text{cm} \quad (ii) 3.74 \text{ cm} \quad (iii) 48.0^\circ \quad (iv) 16.0\text{cm}^2$$

$$6(a) h = \frac{6}{x} - \frac{x}{2} \quad (d) 7.9\text{cm}^3 \quad \text{€(ii) Rep the change of volume. (f) } 8 \text{ cm}^3$$

$$7(a)(i) -3\mathbf{a} + \mathbf{b} \quad (ii) -\mathbf{a} + 3\mathbf{b} \quad (\mathbf{b}) = \frac{3}{4}\mathbf{a} + \frac{3}{4}\mathbf{b} \quad (c) \frac{1}{3}$$

$$8(a)(i) 37 \quad (ii) 21 \quad (iii) 17.5\% \quad (b) 52 \quad (c) \frac{1}{106} \quad (d)(i) 42 \quad (ii) 27$$

(e) did better in test 2 as median for test 2 is higher. More consistent in Test 1 as IQR for Test 1 is smaller.

9(a) \$1088 (b) \$17000 (c) \$1.887.. So suitable amount is \$2 because this amount can cover all costs and profit needed and has an excess amount in case there are less than 480000 visitors.

Class Index Number

Name : Answers

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METHODIST GIRLS' SCHOOL

Founded in 1887



PRELIMINARY EXAMINATION 2023 Secondary 4

Thursday
17 August 2023

MATHEMATICS Paper 2

4052/02
2 h 15 min

Candidates answer on the Question Paper.

INSTRUCTIONS TO CANDIDATES

Write your name, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

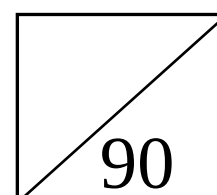
The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For p , use either your calculator value or 3.142, unless the question requires the answer in terms of p

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 90.



This question paper consists of 21 printed pages and 3 blank pages.

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

- 1 (a) It is given that $y = \frac{1}{4}z(w^2 - x^2)$.
 (i) Find z such that $y = 4, w = -1$ and $x = 3$.

$$4 = \frac{1}{4}z[(-1)^2 - 3^2]$$

$$16 = z[-8]$$

$$z = -2$$

Answer- 2.....[2]

- (ii) Make x the subject of the formula $y = \frac{1}{4}z(w^2 - x^2)$.

$$y = \frac{1}{4}z(w^2 - x^2)$$

$$4y = z(w^2 - x^2)$$

$$w^2 - x^2 = \frac{4y}{z}$$

$$x^2 = w^2 - \frac{4y}{z}$$

$$x = \pm \sqrt{w^2 - \frac{4y}{z}}$$

Answer $x = \pm \sqrt{w^2 - \frac{4y}{z}}$ [2]

(b) Simplify $\frac{3}{x+2} + \frac{6x}{4-x^2}$.

$$\begin{aligned}
 & \frac{3}{x+2} + \frac{6x}{4-x^2} \\
 &= \frac{3}{x+2} + \frac{6x}{(2-x)(2+x)} \\
 &= \frac{3(2-x) + 6x}{(2-x)(2+x)} \\
 &= \frac{6-3x+6x}{(2-x)(2+x)} \\
 &= \frac{3(2+x)}{(2-x)(2+x)} \\
 &= \frac{3}{(2-x)}
 \end{aligned}$$

Answer $\frac{3}{(2-x)}$ [3]

- (c) In 2022, Singapore has a population of 5.64 million.
 The land area of Singapore is 728.6km².
 Find the population of Singapore per km², leaving your answer as standard form.

$$\begin{aligned}
 \text{Population} &= \frac{5.64 \times 10^6}{728.6} \\
 &= 7.74 \times 10^3 \text{ (3sf)}
 \end{aligned}$$

Answer 7.74×10^3 [2]

- 2 (a) A closed cone, of height h cm, is filled with water to half its height as shown in Diagram A. It is then inverted as shown in Diagram B. Find the height of the water level, x cm, in terms of h , in Diagram B.

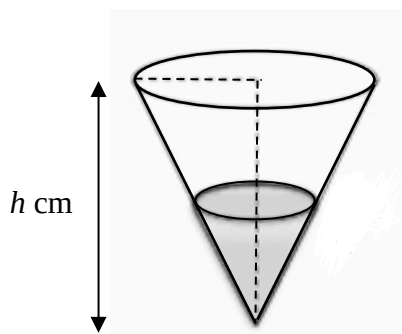


Diagram A

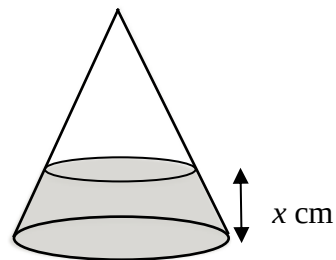


Diagram B

$$\frac{V_{\text{water}}}{V_{\text{cone}}} = \left(\frac{1}{2}\right)^3$$

$$= \frac{1}{8}$$

$$\frac{V_{\text{air in B}}}{V_{\text{cone}}} = \frac{7}{8}$$

$$\left(\frac{h-x}{h}\right)^3 = \frac{7}{8}$$

$$\frac{h-x}{h} = \sqrt[3]{\frac{7}{8}}$$

$$h-x = \frac{\sqrt[3]{7}}{2} h$$

$$x = h - \frac{\sqrt[3]{7}}{2} h$$

Answer [3]

- (b) Janet began her round-island 145 km cycling route from Point A at 6 am, cycling at an average speed of x km/h.
- (i) Write down the expression, in terms of x , for the time taken by Janet to complete the cycling route.

$$\frac{145}{x}$$

Answer $\frac{145}{x}$ h [1]

Angel started on the same route 10 minutes later. She cycled at a speed of 2 km/h faster than Janet for the first 20 km and cycled x km/h for the remaining route.

- (ii) Write down the expression, in terms of x , for the time taken by Angel to complete the cycling route.

Answer $\frac{20}{x+2} + \frac{125}{x}$ h [1]

Angel reached the end of the route at the same time as Janet.

- (iii) Form an equation in x and show that it reduces to $x^2 + 2x - 240 = 0$.

Answer [3]

$$\frac{20}{x+2} + \frac{125}{x} + \frac{1}{6} = \frac{145}{x}$$

$$\frac{20}{x+2} + \frac{1}{6} = \frac{20}{x}$$

$$\frac{120 + x + 2}{6(x+2)} = \frac{20}{x}$$

$$122x + x^2 = 120x + 240$$

$$x^2 + 2x - 240 = 0$$

- (iv) Solve the equation $x^2 + 2x - 240 = 0$.

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-240)}}{2(1)}$$

$$= 14.5 \text{ or } -16.5 \text{ (3sf)}$$

Answer $x = \dots\dots\dots$ [3]

- (v) What time did Janet reach the ending point?

$$\text{Time taken} = \frac{145}{14.524} = 9 \text{ h } 59 \text{ min}$$

$$\text{Time arrived} = 3.59 \text{ pm}$$

Answer $\dots\dots\dots$ [2]

- 3 (a) In a logistics company, each delivery man is paid \$x for every correct delivery made, \$0 for no delivery but has to pay a penalty of \$1 for each damaged delivery.

Tom and Jerry made the following deliveries on a particular day.

$$\begin{array}{ccccc} & \text{correct} & \text{no delivery} & \text{damaged} & \\ \mathbf{A} = \begin{array}{l} \text{Tom} \\ \text{Jerry} \end{array} & \begin{pmatrix} 13 & 4 & 3 \\ 10 & 8 & y \end{pmatrix} & & \mathbf{B} = \begin{pmatrix} x \\ 0 \\ -1 \end{pmatrix} \end{array}$$

- (i) Evaluate $\mathbf{AB}$ in terms of x and y.

$$\begin{pmatrix} 13 & 4 & 3 \\ 10 & 8 & y \end{pmatrix} \begin{pmatrix} x \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 13x - 3 \\ 10x - y \end{pmatrix}$$

Answer [2]

- (ii) Explain what your answer in (a)(i) represents.

Rep amount of money Tom and Kerry will be paid respectively for their deliveries on a particular day..... [1]

It is given that Tom earned \$18 more than Jerry despite both making the same number of deliveries.

- (iii) Find the values of x and y.

$$13 + 3 = 10 + y$$

$$y = 6$$

$$13x - 3 = 10x - 6 + 18$$

$$x = 5$$

Answer $x = 5$ $y = 6$ [3]

- (iv) By using matrix multiplication, calculate the total amount of money the company needs to pay both Tom and Jerry on that particular day.

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 13(5) - 3 \\ 10(5) - 6 \end{pmatrix} = (106)$$

Answer \$106.....[2]

- (b) Tom is looking for a new job. He targets to earn an annual salary of \$150 000 at the end of 5 years. The job that he is interviewing for is offering an annual salary increment of 3% per annum.
What is the minimum starting annual salary should Tom request for to meet his target. He needs to set his salary request in multiples of \$1000.

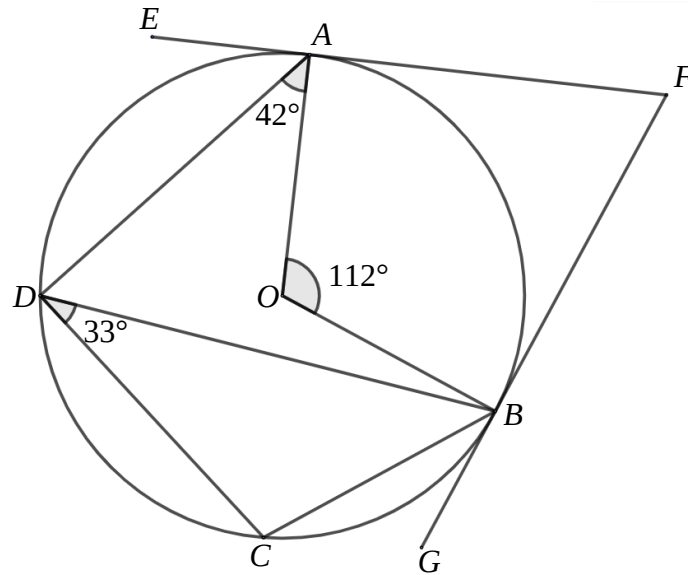
Let P be the annual starting salary.

$$150000 = P \left(1 + \frac{3}{100} \right)^5$$

$$P = 129391.377$$

Answer \$ 130 000... [3]

- 4 (a) In the diagram, A, B, C and D lie on the circle with centre O .
 EF and GF are tangents to circles at A and B respectively.
 It is given that $\angle DAO = 42^\circ$, $\angle BDC = 33^\circ$ and $\angle AOB = 112^\circ$



Find

- (i) $\angle CBD$,

$$\begin{aligned}\angle OAB &= \frac{180^\circ - 112^\circ}{2} \text{ (base angle of isos } \Delta \text{)} \\ &= 34^\circ\end{aligned}$$

$$\begin{aligned}\angle BCD &= 180^\circ - 34^\circ - 42^\circ \text{ (angles in opp segment)} \\ &= 104^\circ\end{aligned}$$

$$\begin{aligned}\angle CBD &= 180^\circ - 104^\circ - 33^\circ \text{ (angles sum of } \Delta \text{)} \\ &= 43^\circ\end{aligned}$$

Answer $\angle CBD = \dots\dots\dots$ [3]

- (ii) $\angle ABF$.

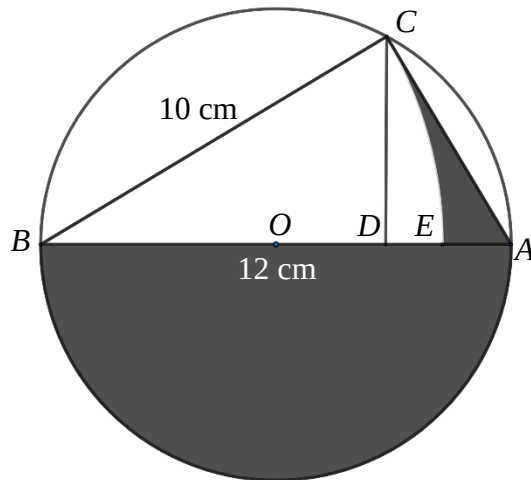
Method 1

Method 2

$\angle ABD = 112^\circ \div 2$ (angle at centre = 2 angle at circumference) $= 56^\circ$ $\angle ABF = 56^\circ$ (alternate segment theorem)	$\angle OBF = 90^\circ$ (tan perpendicular to rad) $\angle ABF = 90^\circ - 34^\circ = 56^\circ$
---	---

Answer $\angle ABF = \dots\dots\dots$ [2]

- (b) The diagram shows a circle, centre O , with a diameter AB of 12 cm. CD is perpendicular to AB . CE is an arc of a circle with B as the centre and radius 10 cm.



- (i) Show that $\angle CBD = 0.586$ when corrected to 3 significant figures.

Answer

[1]

$$\begin{aligned}\cos \angle CBD &= \frac{10}{12} \\ \angle CBD &= 0.58568 \dots \\ \angle CBD &= 0.586\end{aligned}$$

- (ii) Find the arc length CE .

$$10 \times 0.586 = 5.86$$

Answer cm [1]

- (iii) Calculate the area of the shaded region.

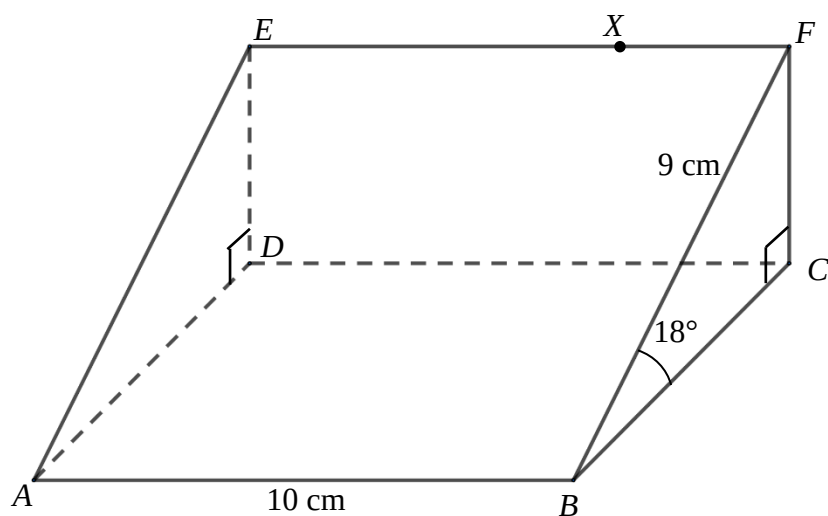
$$\begin{aligned}\text{Shaded area} &= \frac{1}{2}\pi(6)^2 + \left[\frac{1}{2}(10)(12)\sin 0.586 - \frac{1}{2}(10)^2(0.586)\right] \\ &= 60.4 \text{ cm}^2\end{aligned}$$

Answer cm² [3]

- 5 A wooden triangular wedge has a rectangular base as shown in the diagram.

$AB = 10$ cm and $BF = 9$ cm and $\angle FBC = 18^\circ$.

X is on EF such that $EX : XF = 3 : 1$.



- (i) Find the length of AC .

$$\cos 18^\circ = \frac{BC}{9}$$

$$BC = 9 \cos 18^\circ$$

$$AC = \sqrt{10^2 + (9 \cos 18^\circ)^2}$$

$$= 13.163$$

$$= 13.2 \text{ (3sf)}$$

Answer cm [3]

- (ii) Find the length of CX .

$$XF = \frac{1}{4}(10) = 2.5$$

$$\sin 18^\circ = \frac{FC}{9}$$

$$FC = 9 \sin 18^\circ$$

$$XC = \sqrt{2.5^2 + (9 \sin 18^\circ)^2}$$

$$= 3.7396$$

$$= 3.74 \text{ (3 sf)}$$

Answer cm [2]

- (iii) Find the greatest angle of elevation of X from BC .

Let the angle of elevation be θ

$$\begin{aligned}\cos \theta &= \frac{2.5}{3.3796} \\ &= 48.0^\circ\end{aligned}$$

Answer [2]

- (iv) Find the area of triangle BXC .

Method 1

$$BX = \sqrt{(2.5)^2 + 9^2} = \sqrt{\frac{349}{4}}$$

$$(9\cos 18^\circ)^2 = (9\sin 18^\circ)^2 + \left(\sqrt{\frac{349}{4}}\right)^2 - 2(9\sin 18^\circ)\left(\sqrt{\frac{349}{4}}\right)\cos \angle BXC$$

$$\angle BXC = 65.289$$

$$\begin{aligned}\text{Area} &= \frac{1}{2}\left(\sqrt{\frac{349}{4}}\right)(3.7396)\sin 65.289^\circ \\ &= 15.866 = 15.9 \text{ cm}^2\end{aligned}$$

Method 2

$$\text{Area} = \frac{1}{2}(BX)(XC)$$

$$\begin{aligned}&= \frac{1}{2}[9\cos 18^\circ](3.7396) \\ &= 16.0 \text{ cm}^2\end{aligned}$$

Answer cm^2 [2]

- 6 A solid cuboid has a square base of x cm, height h cm and a total surface area of 24 cm^2 .

(a) Find h in terms of x .

$$2x^2 + 4xh = 24$$

$$4xh = 24 - 2x^2$$

$$h = \frac{6}{x} - \frac{x}{2}$$

Answer $h = \dots\dots\dots$ [2]

(b) Show that the volume of the cuboid, $y \text{ cm}^3$, is given by the formula

$$y = \frac{1}{2}x(12 - x^2).$$

Answer

[1]

$$\begin{aligned} y &= x^2h \\ &= x^2\left(\frac{6}{x} - \frac{x}{2}\right) \\ &= 6x - \frac{x^3}{2} \\ &= \frac{1}{2}x(12 - x^2) \end{aligned}$$

The values of x and y for $y = \frac{1}{2}x(12 - x^2)$ are given in the table below:

x	0.5	0.8	1	1.5	2	2.5	3
y	2.9	4.5	5.5	7.3	8	7.2	4.5

(c) On the given grid, draw the graph of $y = \frac{1}{2}x(12 - x^2)$ for $0.5 \leq x \leq 3$. [2]

(d) Use your graph to find the volume of the cuboid when the base area is 4.41 cm^2 .

$$x = \sqrt{4.41} = 2.1$$

Answer 7.9 cm^3 [2]

(e) (i) By drawing a tangent, find the gradient of the curve when $x = 1.7 \text{ cm}$.

Working for gradient

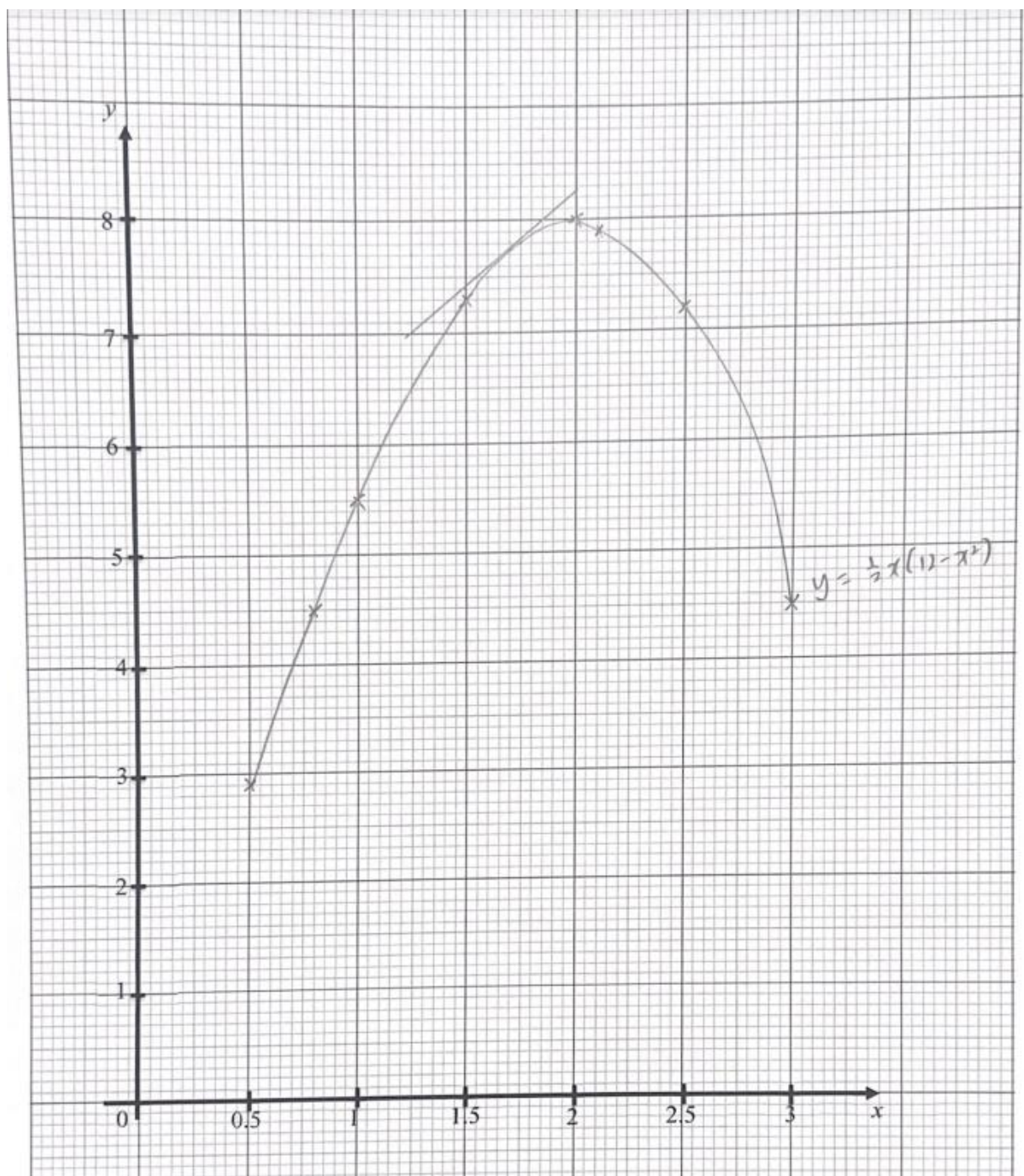
Answer accept 1-1 to 2.2.....[2]

(ii) What does this gradient represent?

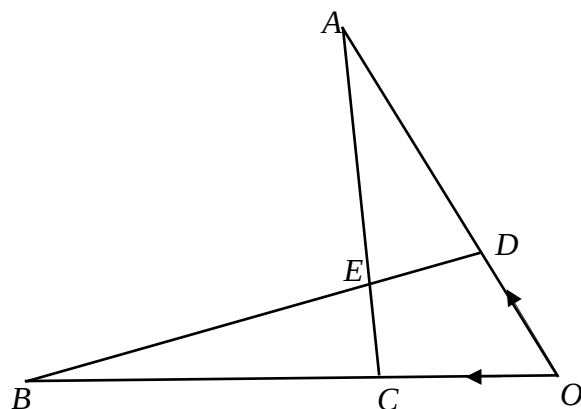
Answer Rep the rate of change of volume of changing. [1]

(f) State the maximum volume of the cuboid.

Answer 8 cm^3 [1]



- 7 In the diagram, $\overrightarrow{OD} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$. It is given that $OD = \frac{1}{2}DA$ and $OC = \frac{1}{3}OB$.



- (a) Express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{AC}$,

Answer $-3\mathbf{a} + \mathbf{b}$ [1]

(ii) $\overrightarrow{DB}$.

Answer $-\mathbf{a} + 3\mathbf{b}$.. [1]

- (b) Given that $\frac{DE}{DB} = \frac{1}{4}$, express $\overrightarrow{OE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$$\overrightarrow{OE} = \overrightarrow{OD} + \overrightarrow{DE}$$

$$= \mathbf{a} + \frac{1}{4}\overrightarrow{DB}$$

$$= \mathbf{a} - \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$$

$$= \frac{3}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$$

Answer $\frac{3}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$ [2]

- (c) Calculate the numerical value of $\frac{EC}{AE}$.

$$\begin{aligned}\overrightarrow{AE} &= \overrightarrow{AO} + \overrightarrow{OE} \\ &= -3\mathbf{a} + \frac{3}{4}\mathbf{a} + \frac{3}{4}\mathbf{b} \\ &= -\frac{9}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\end{aligned}$$

$$= \frac{3}{4}(\mathbf{b} - 3\mathbf{a})$$

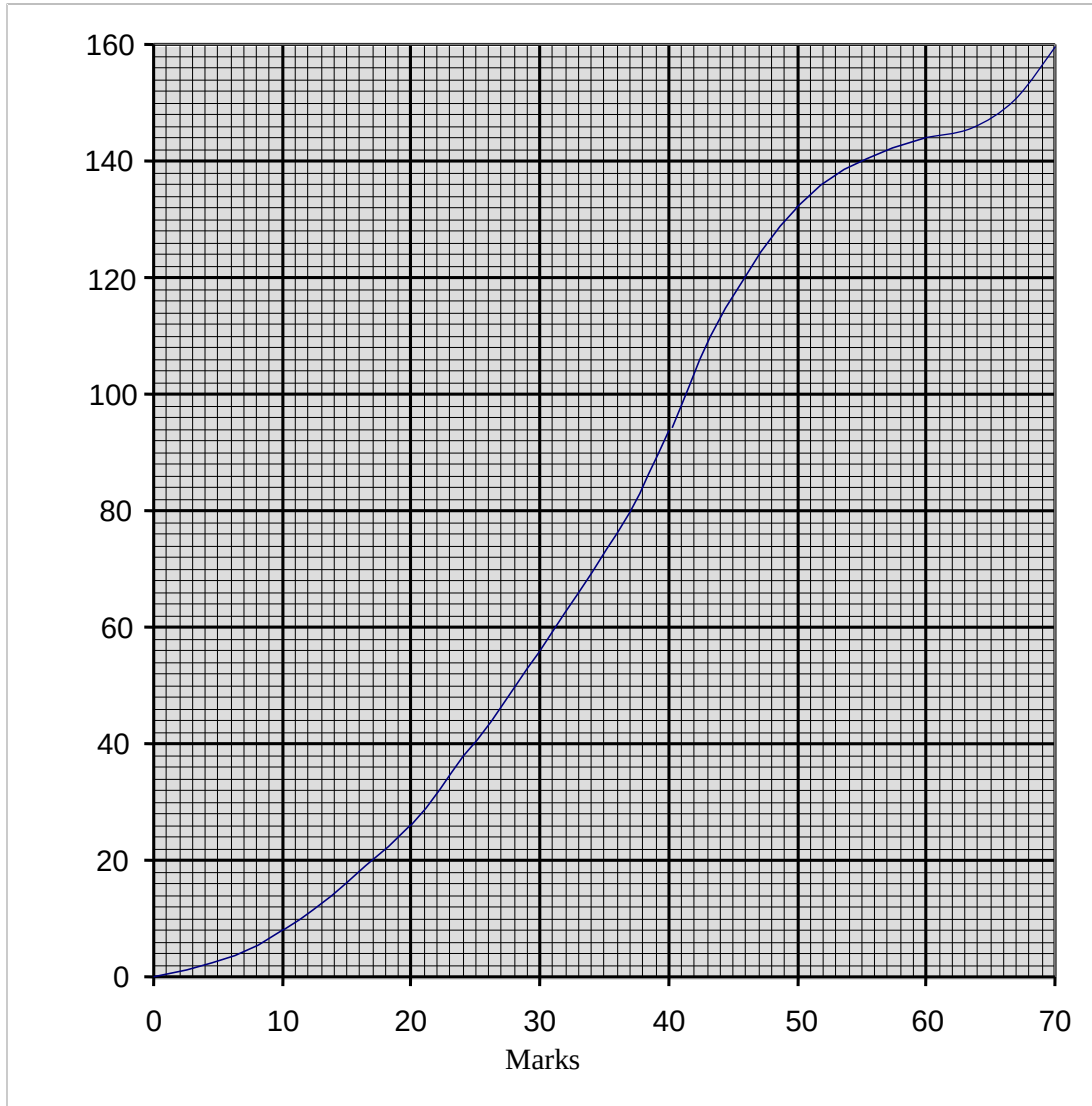
$$= \frac{3}{4}\overrightarrow{AC}$$

$$\frac{AE}{AC} = \frac{3}{4}$$

$$\frac{EC}{AE} = \frac{1}{3}$$

Answer[2]

- 8 The cumulative frequency curve below shows the distribution of the marks obtained by 160 students for Mathematics Test 1.



- (a) Estimate from the graph,
(i) the median,

Answer 37.....[1]

- (iii) the interquartile range,

46 - 25

Answer 21.....[1]

- (iii) the percentage of students who obtained more than 50 marks.

$$\frac{28}{160} \times 100\% = 17.5\%$$

Answer 17.5%.....[1]

- (b) Given that 15% of the students will be awarded a distinction grade, use the graph to find the lowest mark scored by this group of students.

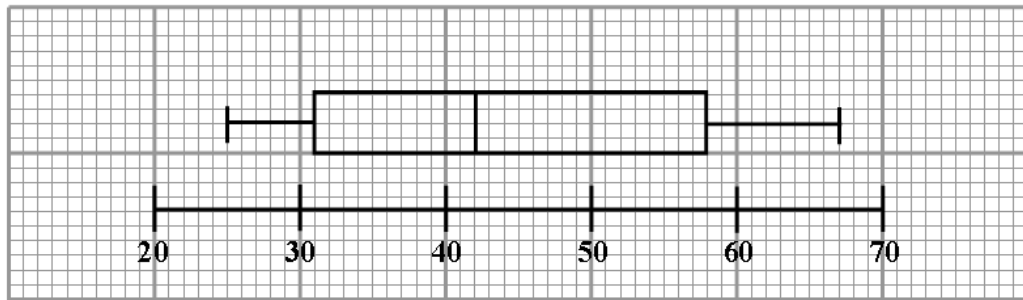
Answer 52.....[1]

- (c) Find the probability of choosing 2 students whose marks are in the 90th percentile and above.

$$\frac{16}{160} \times \frac{15}{159} = \frac{1}{106}$$

Answer[2]

- (d) The marks obtained by the same group of 160 students who sat for Mathematics Test 2 are also noted. The results are summarised by a box-and-whisker plot shown below.



- (i) State the median mark.

Answer 42..... [1]

- (ii) Find the interquartile range of the marks.

$$58 - 31 = 27$$

Answer 27[1]

- (e) Make 2 comparisons between the marks obtained by the students in the two tests.

- 1 Students did better in Test 2 as the median mark in Test 2 is higher.

..... [1]

- 2 Students' performance more consistent in Test 1 as the interquartile range is smaller which implies a smaller spread about the median mark

..... [1]

- 9 An event company is holding a 5-day food exhibition for 200 exhibitors to promote their food items. The event company collects a registration fee of \$88 and a daily booth rental of \$200 from each exhibitor.
- (a) Calculate the total amount of money collected from each exhibitor.

$$88 + 200 (5) = \$1088$$

Answer \$..... [1]

The event company needs to rent **one entire exhibition space** for the food exhibition.
The space required for the exhibition is as follows:

Size of each exhibition booth	20m ²
Space for visitors' movement and dining area	60% of the total area occupied by the exhibition booths

The layout of the 5 available exhibition halls is as shown:

Hall A	Hall B	Hall C	Hall D	Hall E
2000 m ²	3000 m ²	3500 m ²	4500 m ²	2500 m ²
\$6000 per day	\$8500 per day	\$9000 per day	\$10000 per day	\$7000 per day

Note: Adjacent exhibition halls have removable partitions to expand into a bigger exhibition hall.

- (b) Calculate the **minimum** amount payable by the event company for the daily rental of the exhibition space needed.

$$\text{Space needed for exhibitor} = 200 \times 4 \times 5 = 4000\text{m}^2$$

$$\text{Total space needed} = 4000 \times 1.6 = 6400 \text{ m}^2$$

Hall D and E cheapest

[A1]

Answer \$17 000 [2]

The event company will set up the entire exhibition one day before the actual exhibition starts. They need to include the rental of the entire exhibition space, tables and chairs for this extra day of set up.

50 long tables and 350 chairs will be set up in the dining area.
Each exhibition booth will require 4 square tables and 4 chairs.

Items available for rental for exhibition booths set-up:

Rental of square tables:	Per 100 tables at \$30 per day
Rental of long tables:	\$2 each table per day
Rental of chairs:	Per 50 chairs at \$10 per day

The event company also needs to employ 3 security guards and 2 part-time staff for 12 hours per exhibition day.

Information of other costs:

Security Guards: Company A: \$10 per hour per guard Company B: \$8 per hour for first 8 hours per guard per day and \$12 for subsequent hour per guard per day.
Part-time staff: \$9 per hour per staff

- (c) It is estimated that there will be a total of 48 000 visitors for the 5-day exhibition. The event company wishes to earn a total of \$200 000 profit from organizing the event. Using your answers in parts (a) and (b), calculate and suggest a suitable amount of entrance fee that the event company should charge for each visitor. Explain your answer.

Square Tables

Number of tables required: $4 \times 200 = 800$

Cost of rental = $800/100 \times 30 \times 6 = \$1440$

Long Tables

Cost of rental = $50 \times 2 \times 6 = \$600$

Chairs

Number of chairs required = $800 + 350 = 1150$

Cost of Rental = $1150/50 \times 10 \times 6 = \$1380$

Venue

Cost of rental = $6 \times 17000 = \$102\,000$

Labour

Wages = $[8 \times 8 + 12 \times 4] \times 3 \times 5 + 9 \times 12 \times 2 \times 5 = \$2760$

Total cost = $1440 + 600 + 1380 + 102\,000 + 2760$
= \$108 180

Collection from exhibitors = $1088 \times 200 = \$217\,600$

Entrance Fee = $\frac{108180 + 200000 - 217600}{48000}$
= \$1.887

Answer \$2 because this amount can cover all costs and profits needed and there is also an excess in case there are less than 48 000 visitors. [8]

End of Paper

