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Name : _____

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BENDEMEER SECONDARY SCHOOL

2017 PRELIMINARY TWO EXAMINATION

SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

Elementary Mathematics Paper 1

4048/01

DATE : 22 August 2017
DURATION : 2 hours
TOTAL : 80 Marks

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on the work you hand in.
 Write in dark blue or black pen on both sides of the paper.
 You may use a 2B pencil for any diagrams or graphs.
 Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.

Write your answers in the spaces provided on the question paper.

All the diagrams in this paper are **not** drawn to scale.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

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80

This document consists of **19** printed pages including this cover page.

[Turn over

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

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- 1 (a) By rounding each number to its nearest ten, calculate $\frac{216.1 + 1083.7}{14.99}$.
 (b) Write your answer to part (a) correct to 1 significant figure.

Answer (a)[1]

(b)[1]

- 2 If the length of a rectangle is 340mm and width is 200mm, both are corrected to the nearest 10mm, calculate the
 (a) maximum possible area of this rectangle in cm^2 ,
 (b) lowest possible value of the ratio $\frac{\text{width}}{\text{length}}$.

Answer (a) [2]

(b)[1]

- 3 James was 82kg and 15% above his ideal weight. He exercised and lost 6% of his initial weight. How many percent of his current weight must James lose in order to reach his ideal weight?

Answer..... [3]

- 4 (a) Solve $4a(a - 3) = 2 - (20 - 6a)$.
 (b) Factorise $x^2y^2 + 36 - 4x^2 - 9y^2$ completely.

Answer (a) [2]

(b) [3]

- 5 A flight leaving Singapore to London takes about 13 hours and 15 minutes. If the departure time on a Tuesday from Singapore is 1310 hours and Singapore is 7 hours ahead of London, what day and time, in 24 hour format, does the flight reach London?

Answer hours on [2]

- 6 In $\triangle DEF$, $DF = 10\text{cm}$, $EF = 12\text{cm}$ and $\angle EDF = 39^\circ$.
 (a) Find $\angle DEF$.
 (b) Which is the acceptable answer to part (a)? Explain why the other answer is not applicable.

Answer (a) $\angle DEF = \dots\dots\dots^\circ$, $\dots\dots\dots^\circ$ [2]

(b)
 [2]

- 7 Given that $\frac{a^2}{c^2} - \frac{b^2}{d^2} = 1$, make b the subject.

Answer [2]

- 8 (a) Evaluate $(2^{-1} - 5^{-2})$ without using a calculator. Show your working clearly.
 (b) Simplify $\frac{\sqrt[3]{b^2} \times b^6}{b^{\frac{2}{3}} \times b}$, giving your answer in the form of b^n .

Answer (a) [2]

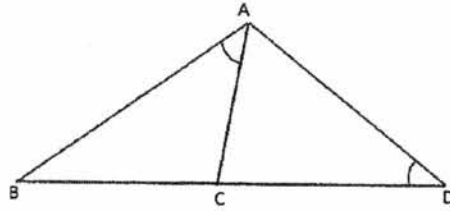
(b) [2]

- 9 Siew Teng is x years old and her brother Victor is 2 years older. Their mother is 6 times older than Victor.
- (a) Write down the ratio of Siew Teng's age: Victor's age: Mother's age in terms of x .
- (b) Ten years from now, their total ages will be 76. How old was Siew Teng's mother five years ago?

Answer (a) [1]

(b) [2]

- 10 In the diagram, given that $\angle BAC = \angle BDA$ and C lies on a straight line BD. It is given that $AB = 6$ cm and $BC = 4$ cm.



- (a) Show that $\triangle ABC$ and $\triangle DBA$ are similar.

.....

 [2]

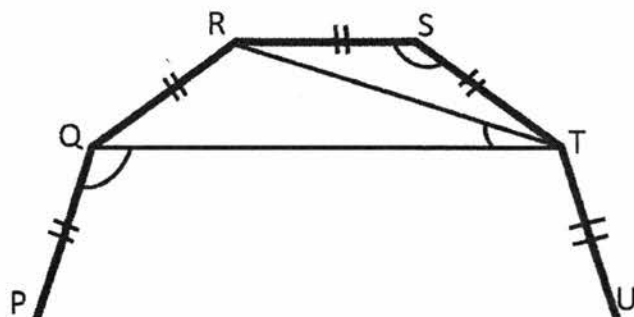
- (b) Find BD.

- (c) Given the area of $\triangle ABD$ is 42 cm^2 , find the shortest distance from D to AB.

Answer (b) cm [1]

(c)..... cm^2 [2]

11 The below diagram is part of a regular decagon.



Find

- (a) $\angle RST$
- (b) $\angle RTQ$
- (c) $\angle PQT$

Answer (a) $^{\circ}$ [1]

(b) $^{\circ}$ [1]

(c) $^{\circ}$ [2]

12 Two fair six-sided dice are thrown.

Find the probability that

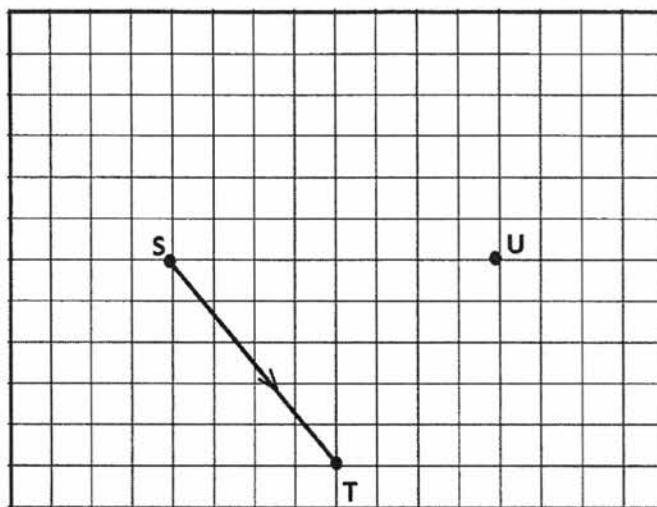
- (a) both dice show different numbers,
- (b) the sum of the two numbers shown is 12,
- (c) the sum of the two numbers shown is a prime number.

Answer (a)..... [1]

(b)..... [1]

(c)..... [2]

- 13 The figure below shows the positions of the points S, T and U.



- (a) Express $\overrightarrow{ST}$ as a column vector.
- (b) V is a point such that STUV is a parallelogram. Draw the parallelogram on the diagram above.
- (c) Find the magnitude of $|\overrightarrow{ST}|$ and $|\overrightarrow{TU}|$.
- (d) Hence, from your answer in part (c), is $|\overrightarrow{ST}| = |\overrightarrow{TU}|$? What is the specific name of the parallelogram?

Answer (a) [1]

(b) See above..... [1]

(c) , ... [2]

(d) [2]

- 14 (a) Hasan invested part of \$8000 at 2.4% per annum simple interest and the remaining at 1.8% per annum simple interest. He received a total interest of \$348 after two years. How much did he invest at 2.4% per annum simple interest?
- (b) Amin bought a car at \$70000 and the car depreciated by 25 % at end of first year, 20% at end of second year and 15% at end of third year. What was Amin's car value after 3 years?

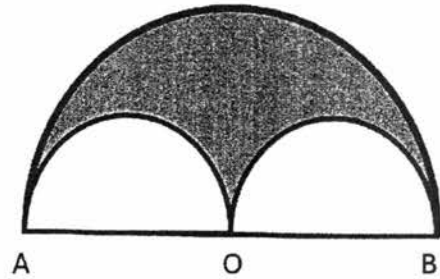
Answer (a) \$..... [2]

(b) \$..... [2]

- 15 The diagram shows 2 small semicircles inside a big semicircle. Given that AB is the diameter of the big semicircle with center O and area of each small semicircle is $\frac{9}{2}\pi \text{ cm}^2$.

Find

- the radius of the small semicircle,
- the perimeter of the shaded area in terms of π ,
- the area of the shaded region in terms of π .

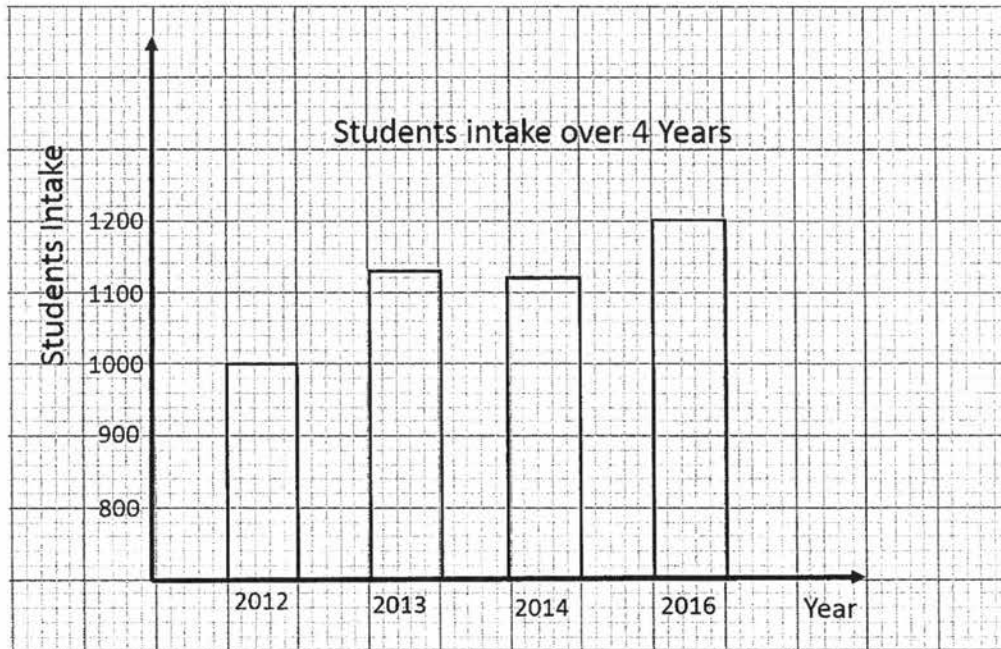


Answer (a) cm [1]

(b) cm [1]

(c) cm² [1]

- 16 The graph shows the students intake of ABC Secondary school over 4 years.



- Express the ratio of the height of the bar representing the students intake in 2012 to that in 2016.
- Express the ratio of the student intake in 2012 to the student intake in 2016.
- Should both answers you obtain in (a) and (b) be the same?
- Explain the similarity or difference in your answers of (a) and (b).

Answer (a) [1]

(b) [1]

(c) [1]

(d)

..... [1]

17 Given the equation of line L_1 is $\frac{1}{2}x - 3y = 9$, find

- (a) the coordinates when it cuts the x-axis,
- (b) the gradient of the line,
- (c) the value of k if the point $(-6, k)$ lies on the line,
- (d) the equation of line L_2 that cuts y-axis at 5 and is parallel to L_1 .

Answer (a)..... [1]

(b)[1]

(c).....[1]

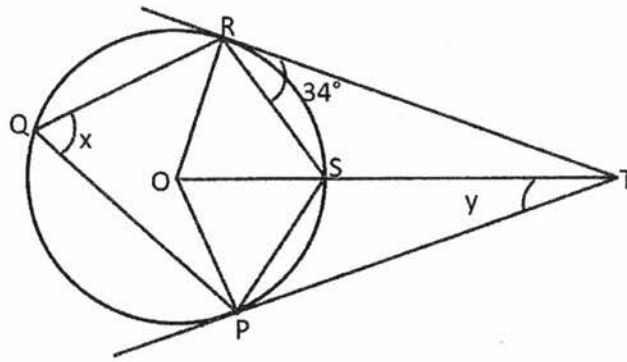
(d).....[1]

- 18 In the diagram, O is the center of the circle and RT and PT are tangents to the circle at R and P respectively. Find the angles,

(a) x and

(b) y .

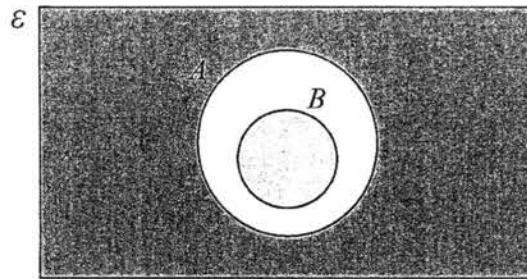
State your reasons clearly.



Answer (a) $x = \dots\dots\dots$ [3]

(b) $y = \dots\dots\dots$ [1]

- 19 (a) Use set notation to describe the shaded area in the following Venn diagram.



- (b) $E = \{\text{numbers from 1 to 10}\}$
 $A = \{\text{even numbers}\}$
 $B = \{\text{prime numbers}\}$
 $C = \{\text{multiples of 2 greater than 6}\}$
- (i) List the elements in $A \cap B'$.
 (ii) State the relationship between set A and C.

Answer (a) [1]

(b)(i) [1]

(b)(ii) [1]

- 20 The scale drawing in the answer space below shows the position of towns A and B. Town B is 36 km due South of A. The map scale is given as 1:600 000.

Construct the map of ABCD using the information given below:

- Town C which is 54 km from B with a bearing of 085° from B.
- Town D is located 18 km from C and on the perpendicular bisector of A and B.
- Measure the bearing of Town D from Town A.

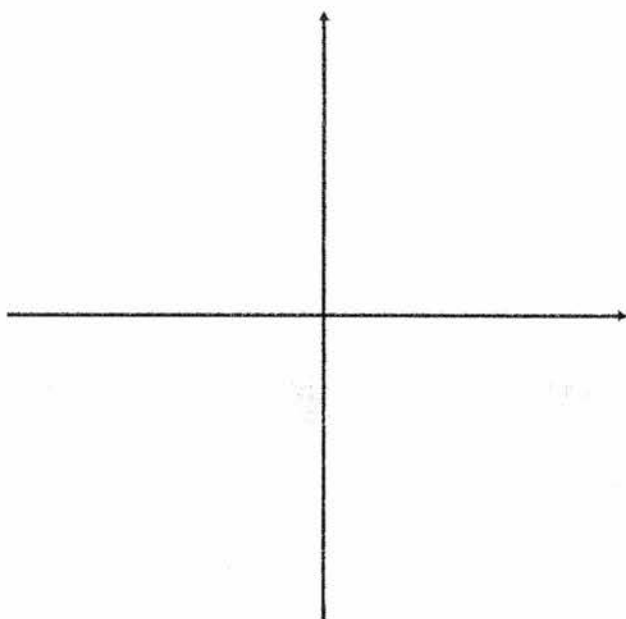


Answer (a) See above..... [2]

(b) See above [2]

(c)..... [1]

- 21 (a) Express the function $y = -x^2 + 8x - 5$ in the form $y = -(x - h)^2 + k$.
 (b) Sketch the graph of the function $y = -x^2 + 8x - 5$. Label the y-intercept and turning point.
 (c) Hence, or otherwise, solve the equation $-x^2 + 8x - 5 = -10$



Answer (a) [2]

(b) See above [2]

(c) $x =$, ... [2]

--- End of Paper ---

Name : _____

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BENDEMEER SECONDARY SCHOOL
2017 PRELIMINARY TWO EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
Elementary Mathematics Paper 2
4048/02

DATE : 23 August 2017
DURATION : 2 hours 30 minutes
TOTAL : 100 marks

Additional Materials: Cover page
Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer all questions.

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For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

FOR EXAMINER'S USE
100

This document consists of 11 printed pages including this cover page.

[Turn over

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

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Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

1 (a) Solve the inequality $\frac{p-2}{4} \leq \frac{1}{2} - \frac{15-2p}{5}$. [3]

(b) (i) Factorise $2q - 18q^3$ completely. [2]

(ii) Hence simplify $\frac{2q - 18q^3}{(4q^2 - 2q)(3q + 1)}$. [2]

(c) (i) In January, Joseph's best time to swim 200 metres was 2 minutes 30 seconds.
Calculate his speed in kilometres per hour. [2]

(ii) In December, Joseph's best time is 10% less than his best time in January.
Calculate, in minutes and seconds, his best time in December. [2]

2 The first four terms in a sequence of numbers are given below.

$$T_1 = 3 + 2^0 = 4$$

$$T_2 = 5 + 2^1 = 7$$

$$T_3 = 7 + 2^2 = 11$$

$$T_4 = 9 + 2^3 = 17$$

(a) Find T_5 . [1]

(b) Find the n th term of the sequence, T_n . [1]

(c) Hence or otherwise, find T_{20} . [1]

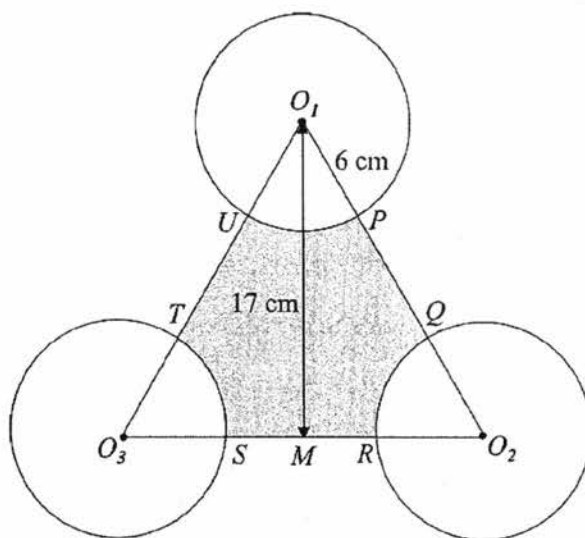
(d) Explain why the value of T_n is always odd for all values of n . [1]

(e) T_m and T_{m+1} are consecutive terms in the sequence.

Show that $T_{m+1} - T_m = 2 + 2^{m-1}$. [3]

- 3 A factory produces bottles in both the small and the large size.
- (a) It is found that x large bottles can be produced in a minute.
Write down an expression in terms of x , the time taken to produce 1 large bottle, in seconds. [1]
- (b) 4 more small bottles can be produced in a minute, compared to the large bottles.
Write down an expression in terms of x , the time taken to produce 1 small bottle, in seconds. [1]
- (c) Given that it takes 2.5 seconds longer to produce a large bottle than a small bottle, form an equation in x and show that it reduces to $x^2 + 4x - 96 = 0$. [3]
- (d) Solve the equation $x^2 + 4x - 96 = 0$. [2]
- (e) Hence find the time taken to produce 4000 small bottles, in hours and minutes. [2]
- (f) It is known that the factory sells each small bottle at \$0.30 and each large bottle at \$0.50.
Is it more profitable for the factory to produce small or large bottles?
Explain your answer. [3]

- 4 The figure below shows the outline of a spinner toy, which is made up of an equilateral triangle and 3 identical circles with centre O_1 , O_2 and O_3 respectively. It is given that the radii of the circles are 6 cm and $O_1M = 17$ cm, where M is the midpoint of SR .



- Find
- (a) PQ , [2]
- (b) the perimeter of the shaded region $PQIRSTU$ and [3]
- (c) the area of the shaded region $PQIRSTU$. [3]

- 5 (a) The stem and leaf diagram below shows the marks attained by 15 students in a Mathematics test.

1	3	7				
2	3	6	6			
3	0	4	4	5	7	9
4	1	2	5			
5	0					

Key : 1 | 0 means 10 marks

- (i) Using the data given, find the
- (a) median mark, [1]
 - (b) interquartile range and [2]
 - (c) standard deviation of the marks. [2]
- (ii) It was later found that there was a mistake in the marking for the test. As such, every student should get an additional 2 marks.

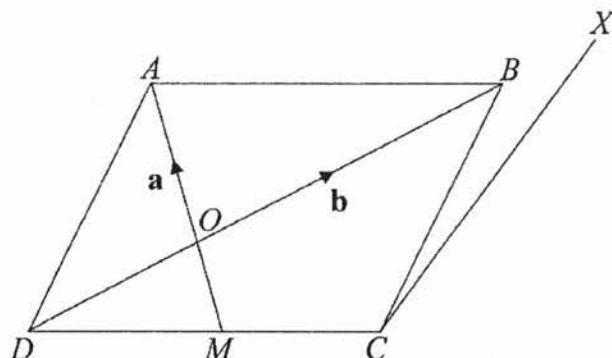
Describe how the change in marks will affect the median mark and interquartile range. [2]

- (b) It is given that a box contains 15 apples and 9 oranges.

Two fruits are then selected from the box at random. If an apple is selected, it is replaced. If an orange is selected, it is not replaced.

- (i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]
- (ii) Find, as a fraction in its simplest form, the probability that
- (a) both fruits selected are the same, [2]
 - (b) at least one of the fruit is an apple. [2]

- 6 In the following diagram, $ABCD$ is a parallelogram where M is the midpoint of CD and $OD = \frac{1}{3}BD$.



Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$,

- (a) express as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,
- $\overrightarrow{BD}$, [1]
 - $\overrightarrow{AB}$, [1]
 - $\overrightarrow{BC}$, [2]
 - $\overrightarrow{OM}$. [2]
- (b) Given that $\overrightarrow{CX} = \mathbf{a} + \frac{3}{4}\mathbf{b}$, prove that B, D and X are collinear points. [2]
- (c) Find the exact value of
- $\frac{\text{area of } \triangle ODM}{\text{area of } \triangle OAB}$, [2]
 - $\frac{\text{area of } \triangle ODM}{\text{area of } ABCD}$. [1]

- 7 Petrol stations A and B sell two grades of petrol, R92 and P98.

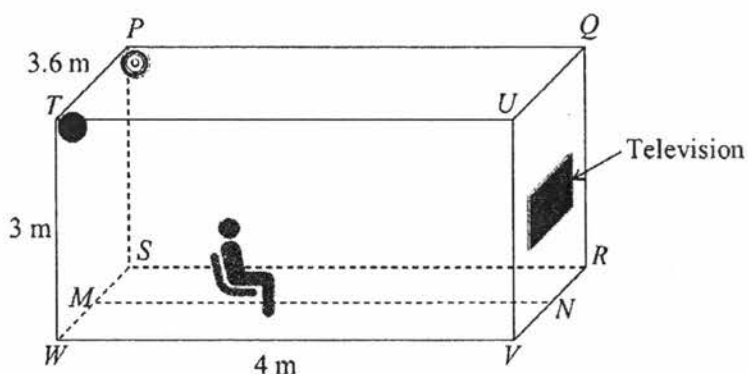
The matrix **L** shows the average amount of petrol sold at the two stations on a day in Week 1.

$$\mathbf{L} = \begin{matrix} & \begin{matrix} \text{R92} & \text{P98} \end{matrix} \\ \begin{pmatrix} 250 & 180 \\ 280 & 180 \end{pmatrix} & \begin{matrix} \text{Station A} \\ \text{Station B} \end{matrix} \end{matrix}$$

- (a) Evaluate the matrix $\mathbf{Q} = 7\mathbf{L}$. [1]
- (b) It is given that the petrol price (per litre) of grade R92 and P98 are \$2.00 and \$2.40 respectively.
- Represent the petrol prices as a column matrix **P**. [1]
- (c) Evaluate the matrix $\mathbf{S} = \mathbf{QP}$. [1]
- (d) State what the elements of **S** represent. [1]
- (e) In Week 2, the average amount of all petrol sold at both petrol stations dropped by 5%. At the same time, the prices of all grades of petrol increased by 5%.
- Calculate the earnings made by Station A and Station B respectively in Week 2. [3]
- (f) Write down a matrix **X** such that the total earnings of both petrol stations in Week 2 can be calculated using matrix multiplication.
- Hence find the total earnings of both petrol stations in Week 2. [2]
-

- 8 Figure 1 shows the three-dimensional layout of Roy's living room. The room is shaped like a cuboid with dimensions 4 m by 3.6 m by 3 m, where path MN lies across the centre of the room.
- A television is fixed on the wall $QRVU$ such that Y , the centre of the television, is 1.6 m above N .
 - Two speakers are fixed at corners P and T respectively.

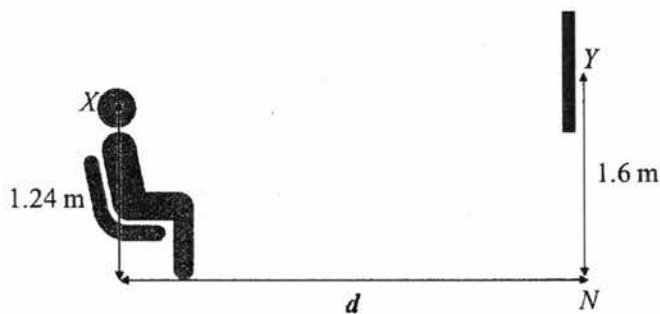
Figure 1



Roy is deciding on the best position to place his armchair along MN . The best position will allow him to have an optimal view of the television when seated in the armchair.

Figure 2 shows Roy's eye level at X , which is 1.24 m when seated at distance d from the television. It is given that $1.8 \text{ m} \leq d \leq 3.8 \text{ m}$ for Roy to have an optimal view of the television.

Figure 2



For this question, the dimensions of the television and speakers are negligible.

- If Roy chose to place the armchair at the furthest possible optimal distance, find
 - TX , [3]
 - $\angle PXT$, [2]
 - the angle of elevation of Y from X . [2]
- When the angle of elevation of Y from X is 12° , will Roy still have an optimal view of the television? Justify your answer. [2]

9 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

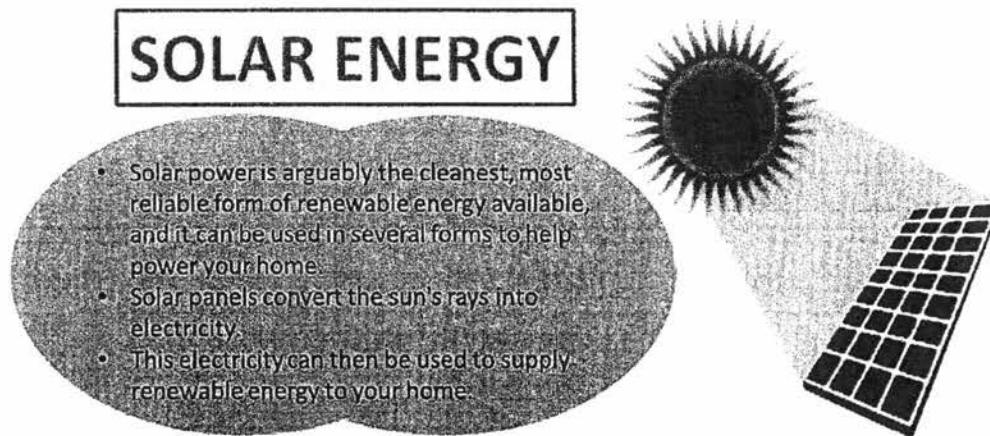
$$y = 5 + \frac{2}{x} - \frac{1}{4}x^2$$

Some corresponding values of x and y are given in the table below.

x	-6	-5	-4	-3	-2	-1.5	-1	-0.5	-0.3
y	-4.33	-1.65	p	2.08	3	3.10	2.75	0.94	-1.69

- (a) Find the value of p . [1]
- (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $-6 \leq x \leq 0$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y -axis for $-5 \leq y \leq 4$.
On your axes, plot the points given above and join them with a smooth curve. [3]
- (c) By drawing a tangent, find the gradient of the curve at $(-1, 2.75)$. [2]
- (d) (i) On the same axes, draw the line L with gradient 0.5 and passes through the point $(-4, -3)$. [1]
(ii) Write down the equation of the line L . [1]
(iii) The x -coordinate of the point(s) where the line L intersects the curve are the solution(s) to the equation $x^3 + Ax^2 - Bx - 8 = 0$.
Find the values of A and B . [2]
- (e) Using the graph, show that $\frac{2}{x} - \frac{1}{4}x^2 + 1 = 0$ has no solution for $x < 0$. [2]

- 10 Mrs Lim is currently staying at a bungalow with her family. After learning about solar energy from the brochure below, she is thinking of installing solar panels at the bungalow to help reduce the family's electricity bills.



Brochure on Solar Energy

Information that Mrs Lim needs to consider in order to make a decision on the installation can be found under Annex A on the next page.

- (a) For the first half of 2017,
- calculate the average amount of electricity (in kWh) used by Mrs Lim's family in a month, and [2]
 - calculate the average amount (in dollars) paid for electricity usage in a month. [2]
- (b) Considering all the information given, should Mrs Lim go ahead with the installation of solar panels for the bungalow?

Justify your answer. [4]

ANNEX A

Table 1: Records of electricity usage by Mrs Lim's family

Electricity Usage for 2017 (in kWh)					
January	February	March	April	May	June
1107.8	1066.3	1123.6	1259	1249.5	1281.6

Table 2: Charges for electricity usage

Electricity tariff: **21.39 cents per kWh**
(Charges subjected to 7% Goods & Services Tax)

Table 3: Details on installing solar panels for Mrs Lim's bungalow

Dimensions of roof area for solar panel installation	9 m by 4 m
Dimensions of 1 solar panel	1.65 m by 1 m
Cost of installing every 10 solar panels	\$6,250



Table 4: More about the solar panels

Average amount of electricity produced by 1 solar panel: 19 kWh per month	Lifespan of solar panels: 20 years
--	---

~END OF PAPER~

Answers:

1a) $p \geq 13\frac{1}{3}$

1b)(i) $2q(1-3q)(1+3q)$ 1b)(ii) $\frac{1-3q}{2q-1}$

1c)(i) 4.8 km/h 1c)(ii) 2 min 15 sec

2a) 27

2b) $2n+1+2^{n-1}$

2c) 524329

3a) $\frac{60}{x}$ s

3b) $\frac{60}{x+4}$ s

3d) $x = -12, 8$

3e) 5 h $33\frac{1}{3}$ min or 5 h 34 min

3f) It is more profitable for the factory to produce **large** bottles.

4a) PQ = 7.63 cm

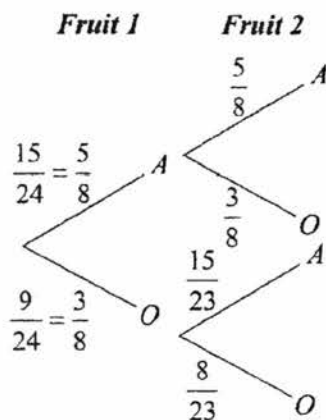
4b) Perimeter = 41.7 cm

4c) Area = 110 cm²

5a)(i)(a) Median = 34 marks 5a)(i)(b) IQR = 15 marks 5a)(i)(c) SD = 9.99 marks

5a)(ii) The median will increase by 2 and the interquartile range will remain the same.

5b)(i)



5b)(ii)(a) $P(\text{both are the same}) = \frac{767}{1472}$

5b)(ii)(b) $P(\text{at least 1 apple}) = \frac{20}{23}$

6a)(i) $\overrightarrow{BD} = -\frac{3}{2}\mathbf{b}$

6a)(ii) $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$

6a)(iii) $\overrightarrow{BC} = -\mathbf{a} - \frac{1}{2}\mathbf{b}$

6a)(iv) $\overrightarrow{OM} = -\frac{1}{2}\mathbf{a}$

$$6c)(i) \quad \frac{\text{area of } \triangle ODM}{\text{area of } \triangle OAB} = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad 6c)(ii) \quad \frac{\text{area of } \triangle ODM}{\text{area of } ABCD} = \frac{1}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{12}$$

$$7a) \quad Q = \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix} \quad 7b) \quad P = \begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix} \quad 7c) \quad S = \begin{pmatrix} 6524 \\ 6944 \end{pmatrix}$$

7d) The earnings of Station A (\$6,524) and Station B (\$6,944) respectively for Week 1.

7e) The earnings of Station A (\$6,507.69) and Station B (\$6,926.64) respectively for Week 2.

7f) $X = \begin{pmatrix} 1 & 1 \end{pmatrix}$

$$\begin{aligned} \text{Total earnings} &= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 6507.69 \\ 6926.64 \end{pmatrix} \\ &= (13434.33) \end{aligned}$$

Total earnings of both stations (Week 2) = \$13,434.33

8a)(i) $TX = 2.53 \text{ m}$

8a)(ii) $\angle PXT \approx 90.9^\circ$

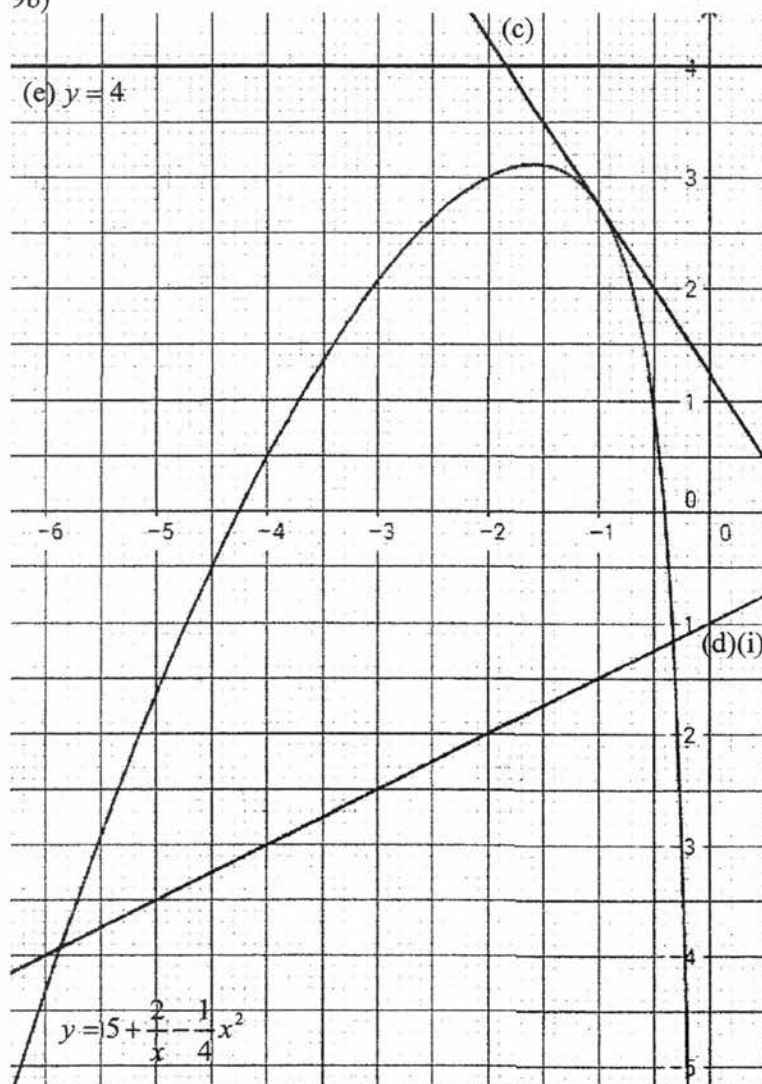
8a)(iii) Angle of elevation = 5.4°

8b) $\tan 12^\circ = \frac{0.36}{d} \quad \rightarrow d \approx 1.69 \text{ m}$

Since 1.69 m is less than the minimum optimal distance 1.8 m, Roy will not have an optimal view of the TV in this case.

9a) $p = 0.5$

9b)



9c) Gradient = -1.5 (± 0.2)

9d)(ii) $y = \frac{1}{2}x - 1$

9d)(iii) $A = 2$ and $B = 24$

10a)(i) 1181.3 kWh

10a)(ii) \$270.37

10b) Since the average amount paid by Mrs Lim per month will be lesser than what she is currently paying for electricity usage, she should go ahead with the installation.

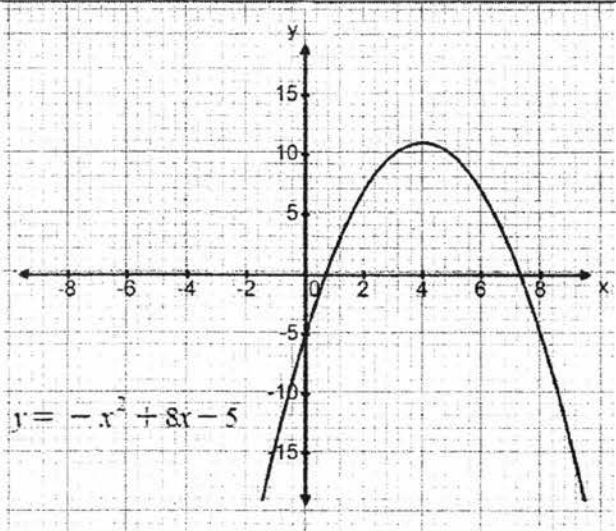
2017 Sec 4E/5NA Preliminary One Mathematics Marking Scheme

Qn	Answer	Marks
1(a)	130	B1
1(b)	100	B1
2(a)	$344 \times 204 = 70176 \text{ mm}^2$ $1 \text{ mm}^2 = 0.1^2 \text{ cm}^2$ $71196 \text{ mm}^2 = 0.1 \times 0.1 \times 70176 \text{ cm}^2$ $= 701.76 \text{ cm}^2 = 702 \text{ cm}$	M1 A1 (accept exact value)
2(b)	$\frac{195}{344}$	B1
3	Ideal weight = $\frac{82}{115} \times 100 = 71.30 \text{ kg}$ Current weight = $\frac{94}{100} \times 82 = 77.08 \text{ kg}$ Per cent = $\frac{77.08 - 71.30}{77.08} \times 100 = 7.4987 = 7.50\%$	M1 M1, A1
4(a)	$4a^2 - 12a = 2 - 20 + 6a$ $4a^2 - 18a + 18 = 0$ $2a^2 - 9a + 9 = 0$ $(2a - 3)(a - 3) = 0$ $a = \frac{3}{2}, 3$	M1 A1
4(b)	$x^2y^2 + 36 - 4x^2 - 9y^2$ $= x^2y^2 - 4x^2 - (9y^2 - 36)$ $= x^2(y^2 - 4) - 9(y^2 - 4)$ $= (x^2 - 9)(y^2 - 4)$ $= (x + 3)(x - 3)(y + 2)(y - 2)$	M1 M1 A1
5	Singapore Tuesday 1310 => London Tuesday 0610 Flight 13 hours and 15 minutes => Arrival Tuesday 1925 Or Flight 13 hours and 15 minutes => Arrival 0225 Wednesday Singapore time Singapore 0225 Wednesday => London Tuesday 1925	M1 B1 Or M1 B1 (If no working,

6(a)	$\frac{\sin 39}{12} = \frac{\sin \angle DEF}{10}$ $\angle DEF = \sin^{-1} \frac{10 \sin 39}{12}$ $\angle DEF = 31.63, 180 - 31.63$ $= 31.6, 148.4$	M1 A1
6(b)	Acceptable answer $\Rightarrow 31.6^\circ$. Reject 148.4° because $(148.4 + 39) > 180$ which is more than angle sum of a triangle.	B1 B1
7	$a^2 d^2 - b^2 c^2 = c^2 d^2$ $b^2 c^2 = d^2 (a^2 - c^2)$ $b = \pm \frac{d}{c} \sqrt{a^2 - c^2}$	M1 A1 No mark if no $\pm$.
8(a)	$\frac{1}{2} - \frac{1}{5^2} = \frac{25-2}{50} = \frac{23}{50}$	M1, A1
8(b)	$b^{\frac{2}{3}+6-\frac{2}{3}-1} = b^5$	M1, A1
9(a)	$x : x + 2 : 6(x + 2)$	B1
9(b)	$(x + 10) + (x + 12) + (6x + 22) = 76$ $x = 4$ <i>Mother's age = $6(4 + 2) - 5 = 31$ years old</i>	M1 A1
10(a)	<i>In $\triangle ABC$ and $\triangle DBA$</i> $\angle BAC = \angle BDA$ (given) $\angle ABC = \angle DBA$ (Common $\angle$) $\triangle ABC$ is similar to $\triangle DBA$ (AA Similarity)	}B1 (order of vertices must be in corresponding order B1 (statement and reason) No reason no mark
10(b)	$\frac{BC}{BA} = \frac{BA}{BD}$ $\frac{4}{6} = \frac{6}{BD}$	

13(c)	$ \vec{ST} = \sqrt{4^2 + (-5)^2} = 6.40 \text{ unit}$ $ \vec{TU} = \sqrt{4^2 + 5^2} = 6.40 \text{ unit}$	B2
13(d)	Yes. $ST=TV$. Parallelogram is a rhombus.	B1 B1
14(a)	Let p be the amount invested at 2.4% p.a. $\frac{2.4 \times 2}{100} \times p + \frac{1.8 \times 2}{100} \times (8000 - p) = 348$ $4.8p + 3.6p = 34800 - 28800$ $p = \$5000$	M1 A1
14(b)	$70000 \times 0.75 \times 0.8 \times 0.85 = \$35700$	M1, A1
15(a)	3cm	B1
15(b)	$2(\text{Arc length of small semicircle}) = 2(\pi \times 3) = 6\pi \text{ cm}$ Radius of big semicircle = 6cm Arc length of big semicircle = $\pi \times 6 = 6\pi \text{ cm}$ Perimeter = $6\pi + 6\pi = 12\pi \text{ cm}$	B1
15(c)	$\text{Area} = \frac{1}{2}\pi(6^2) - 2(\frac{1}{2}\pi 3^2) = 9\pi \text{ cm}^2$	B1
16(a)	$\frac{3}{5}$	B1
16(b)	$\frac{5}{6}$	B1
16(c)	Yes. Both answers are supposed to be the same.	B1
16(d)	There is a difference in answer because the scale of the vertical	B1

	axis does not start from zero	
17(a)	(18,0)	B1
17(b)	$\frac{1}{6}$	B1
17(c)	$k = -4$	B1
17(d)	$y = \frac{1}{6}x + 5$	B1
18(a)	$\angle ORS = 90 - 34 = 56$ (radius perpendicular to tangent) $\angle ROS = 180 - 2(56) = 68$ (angle sum of issos. triangle) $\angle ROP = 2(68) = 136$ $\angle ROP = 2x$ (angle at center = 2 angles at circumference) $x = 68^\circ$	M1 M1 A1 (If more than 2 reasons not given, deduct 1m overall)
18(b)	$y = 180 - 90 - 68 = 22^\circ$ (angle sum of triangle)	B1
19(a)	$A' \cup B$	B1
19(b)(i)	{4, 6, 8, 10}	B1
19(b)(ii)	$C \subset A$	B1
20(a) / 20(b)		C1 – Correct angle measurement C1- Correct scale conversion C1 – Perpendicular bisector C1 – Label of Town D (accept either D ₁ or D ₂)
20(b)	$104^\circ \pm 1^\circ, 114^\circ \pm 1^\circ$	B1
21(a)	$y = -(x^2 - 8x + 5)$ $y = -[(x - 4)^2 + 5 - 4^2]$ $y = -(x - 4)^2 + 11$	M1 A1
21(b)		P1- correct shape P1 – correct intercepts and

	 $y = -x^2 + 8x - 5$	turning point.
21(c)	8.58, -0.583	B2

Name : _____

Register No.	Class



BENDEMEER SECONDARY SCHOOL
2017 PRELIMINARY TWO EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
Elementary Mathematics
4048 / 02

DATE : 23 August 2017
DURATION : 2 hours 30 minutes
TOTAL : 100 marks

MARK SCHEME

[Turn over

MARK SCHEME

Qn.	Solutions	Remarks
1(a)	$\frac{p-2}{4} \leq \frac{1}{2} - \frac{15-2p}{5}$ $\frac{p-2}{4} \leq \frac{-25+4p}{10}$ $10(p-2) \leq 4(-25+4p)$ $-6p \leq -80$ $\therefore p \geq 13\frac{1}{3}$	[B1] [B1] [B1]
1(b)	(i) $2q-18q^3 = 2q(1-9q^2)$ [B1] $= 2q(1-3q)(1+3q)$ [B1] (ii) $\frac{2q-18q^3}{(4q^2-2q)(3q+1)} = \frac{2q(1-3q)(1+3q)}{(4q^2-2q)(3q+1)}$ $= \frac{2q(1-3q)}{2q(2q-1)}$ [B1] $= \frac{1-3q}{2q-1}$ [B1]	
1(c)	(i) $200 \text{ m} \rightarrow 0.2 \text{ km}, \quad 2 \text{ min } 30 \text{ s} \rightarrow \frac{1}{24} \text{ h}$ $\text{Speed} = 0.2 / \frac{1}{24}$ [M1] $= 4.8 \text{ km/h}$ [A1] (ii) $\text{Best time (Dec)} = 0.9 \times \frac{1}{24}$ [B1] $= \frac{3}{80} \text{ h}$ $= 2 \text{ min } 15 \text{ seconds}$ [B1]	
Total Marks: 11		

MARK SCHEME

2(a)	$T_5 = 11 + 2^4 = 27$	[B1]	
2(b)	$n^{\text{th}} \text{ term} = 2n + 1 + 2^{n-1}$	[B1]	
2(c)	$T_{20} = 2(20) + 1 + 2^{20-1} = 524\,329$	[B1]	
2(d)	Since $2n$ and 2^{n-1} are even, then $T_n = 2n + 1 + 2^{n-1} = \text{even} + 1 + \text{even}$ $= \text{odd}$	[B1]	
2(e)	$T_{m+1} - T_m = 2(m+1) + 1 + 2^{m+1-1} - (2m + 1 + 2^{m-1})$ $= 2m + 2 + 1 + 2^m - 2m - 1 - 2^{m-1}$ $= 2 + 2^m - 2^{m-1}$ $= 2 + 2^m - \frac{1}{2}(2^m)$ $= 2 + \frac{1}{2}(2^m)$ $= 2 + 2^{m-1} \text{ (shown)}$	[B1] [B1] [B1]	
Total Marks: 7			
3(a)	Time taken to produce 1 large bottle $= \frac{60}{x} \text{ s}$	[B1]	
3(b)	Time taken to produce 1 small bottle $= \frac{60}{x+4} \text{ s}$	[B1]	
3(c)	$\frac{60}{x} - \frac{60}{x+4} = 2.5$ $60(x+4) - 60x = 2.5x(x+4)$ $240 = 2.5x^2 + 10x$ $x^2 + 4x - 96 = 0 \quad \text{(shown)}$	[B1] [B1] [B1]	
3(d)	$x^2 + 4x - 96 = 0$ $(x-8)(x+12) = 0$ $\therefore x = -12 \text{ (N.A.)}, \quad 8$	[M1] [A1]	
3(e)	Time taken to produce 4000 small bottles $= 4000 \times \frac{60}{8+4}$ $= 5 \text{ h } 33 \frac{1}{3} \text{ min}$	[B1] [B1]	Accept: $\approx 5 \text{ h } 33 \text{ min}$
3(f)	In the same duration of time y seconds, Amount earned for selling large bottles $= \$0.50 \times (y/7.5)$ $\approx \$0.067y$ Amount earned for selling small bottles $= \$0.30 \times (y/5)$ $= \$0.06y$ $\therefore$ It is more profitable for the factory to produce large bottles.	[B1] [B1] [B1]	

MARK SCHEME

	or Amount earned in 1 min (Large) = 8(\$0.50) = \$4.00 [B1] Amount earned in 1 min (Small) = 12(\$0.30) = \$3.60 [B1] ∴ It is more profitable for the factory to produce large bottles. [B1]	
Total Marks: 12		
4(a)	Since M is the midpoint, then O_1M is perpendicular to $O_2 O_3$. So, $\sin 60^\circ = \frac{17}{6 + PQ + 6}$ [M1] ∴ $PQ = \frac{34}{\sqrt{3}} - 12 \approx 7.629909152$ ≈ 7.63 cm [A1] or Let $O_1 O_2$ be $2x$. $(2x)^2 = x^2 + 17^2 \rightarrow 3x^2 = 17^2$ $\rightarrow x = \sqrt{96\frac{1}{3}}$ [B1] ∴ $PQ = 2 \times \sqrt{96\frac{1}{3}} - 2(6) \approx 7.63$ cm [B1]	
4(b)	Arc length $PU = 6\left(\frac{\pi}{3}\right)$ or $\pi \times 2(6) \times \left(\frac{60^\circ}{360^\circ}\right)$ ≈ 6.283185307 cm [B1] Perimeter of shaded region $PQRSTU = (6.283185307 \times 3) + (7.629909152 \times 3)$ [B1] ≈ 41.7 cm [B1] or Perimeter of shaded region $PQRSTU$ $= 7.629909152 + (3 \times \frac{\pi}{3} \times 6)$ [B2] ≈ 41.7 cm [B1]	

MARK SCHEME

4(c)	Area of $\triangle O_1O_2O_3 = \frac{1}{2} \times 17 \times (6 + 6 + \frac{34}{\sqrt{3}} - 12)$ $\approx 166.8542278 \text{ cm}^2$ [B1] Area of sector $O_1PU = \frac{1}{2} \times 6^2 \times \frac{\pi}{3}$ or or $\pi \times 6^2 \times \left(\frac{60^\circ}{360^\circ}\right)$ $\approx 18.84955592 \text{ cm}^2$ [B1] Area of shaded region $PQRSTU = 166.8542278 - 3(18.84955592)$ $\approx 110 \text{ cm}^2$ [B1] or Area of shaded region $PQRSTU$ $= \frac{1}{2} \times 17 \times 2 \left(\sqrt{96\frac{1}{3}} - \frac{1}{2} \pi (6^2) \right)$ [B2] $\approx 110 \text{ cm}^2$ [B1]	
Total Marks: 8		
5(a)	(i)(a) Median = 34 marks [B1] (i)(b) IQR = $41 - 26$ [M1] = 15 marks [A1] (i)(c) Mean = $\frac{492}{15} = 32.8$ marks S.D. = $\sqrt{\frac{17636}{15} - 32.8^2}$ [M1] ≈ 9.99 marks [A1] (ii) The median will increase by 2 and become 36 marks. [B1] The interquartile range will remain the same at 15 marks. [B1]	

MARK SCHEME

5(b)	(i) <table border="0" style="width: 100%;"> <thead> <tr> <th style="text-align: left;">Fruit 1</th> <th style="text-align: left;">Fruit 2</th> <th></th> </tr> </thead> <tbody> <tr> <td>$\frac{15}{24} = \frac{5}{8}$</td> <td>$\frac{5}{8}$</td> <td>A</td> </tr> <tr> <td></td> <td>$\frac{3}{8}$</td> <td>O</td> </tr> <tr> <td>$\frac{9}{24} = \frac{3}{8}$</td> <td>$\frac{15}{23}$</td> <td>A</td> </tr> <tr> <td></td> <td>$\frac{8}{23}$</td> <td>O</td> </tr> </tbody> </table> [B1] Correct branches [B1] Correct probabilities (ii)(a) $P(\text{both are the same}) = \left(\frac{5}{8} \times \frac{5}{8}\right) + \left(\frac{3}{8} \times \frac{8}{23}\right)$ [B1] $= \frac{767}{1472}$ [B1] (ii)(b) $P(\text{at least 1 apple}) = 1 - \left(\frac{3}{8} \times \frac{8}{23}\right)$ [B1] $= \frac{20}{23}$ [B1]	Fruit 1	Fruit 2		$\frac{15}{24} = \frac{5}{8}$	$\frac{5}{8}$	A		$\frac{3}{8}$	O	$\frac{9}{24} = \frac{3}{8}$	$\frac{15}{23}$	A		$\frac{8}{23}$	O	
Fruit 1	Fruit 2																
$\frac{15}{24} = \frac{5}{8}$	$\frac{5}{8}$	A															
	$\frac{3}{8}$	O															
$\frac{9}{24} = \frac{3}{8}$	$\frac{15}{23}$	A															
	$\frac{8}{23}$	O															
Total Marks: 13																	
6(a)	(i) $\overrightarrow{BD} = -\frac{3}{2}\mathbf{b}$ [B1] (ii) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$ [B1] (ii) $\overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{DC} = -\frac{3}{2}\mathbf{b} + (-\mathbf{a} + \mathbf{b})$ [B1] $= -\mathbf{a} - \frac{1}{2}\mathbf{b}$ [B1] (iv) $\overrightarrow{OM} = \overrightarrow{OD} + \overrightarrow{DM} = -\frac{1}{2}\mathbf{b} + \frac{1}{2}(-\mathbf{a} + \mathbf{b})$ [B1] $= -\frac{1}{2}\mathbf{a}$ [B1]																
6(b)	$\overrightarrow{XB} = \overrightarrow{XC} + \overrightarrow{CB} = -\mathbf{a} - \frac{3}{4}\mathbf{b} + \mathbf{a} + \frac{1}{2}\mathbf{b} = -\frac{1}{4}\mathbf{b}$ [B1] Since $\overrightarrow{BD} = 6\overrightarrow{XB} \rightarrow BD \parallel XB$ and B is a common point, [B1] then B, D and X must be collinear points.																
6(c)	(i) $\frac{\text{area of } \triangle ODM}{\text{area of } \triangle OAB} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ [B1, B1] (ii) $\frac{\text{area of } \triangle ODM}{\text{area of } ABCD} = \frac{1}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{12}$ [B1]																
Total Marks: 11																	

MARK SCHEME

7(a)	$Q = \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}$	[B1]	
7(b)	$P = \begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix}$	[B1]	
7(c)	$S = \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix} \begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix} = \begin{pmatrix} 6524 \\ 6944 \end{pmatrix}$	[B1]	
7(d)	The earnings of Station A (\$6,524) and Station B (\$6,944) respectively for Week 1.	[B1]	
7(e)	Amount of petrol sold (Week 2) = $0.95 \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}$ $= \begin{pmatrix} 1662.5 & 1197 \\ 1862 & 1197 \end{pmatrix}$ [B1] Prices of petrol (Week 2) = $1.05 \begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix}$ $= \begin{pmatrix} 2.10 \\ 2.52 \end{pmatrix}$ [B1] Earnings (Week 2) = $\begin{pmatrix} 6507.69 \\ 6926.64 \end{pmatrix}$ The earnings of Station A (\$6,507.69) and Station B (\$6,926.64) respectively for Week 2.	[B1]	
7(f)	$X = \begin{pmatrix} 1 & 1 \end{pmatrix}$ Total earnings = $\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 6507.69 \\ 6926.64 \end{pmatrix}$ $= \begin{pmatrix} 13434.33 \end{pmatrix}$ Total earnings of both stations (Week 2) = \$13,434.33	[B1]	
			Total Marks: 9

MARK SCHEME

8(a)	(i) At furthest possible optimal distance, $d = 3.8$ m, $\rightarrow M$ to foot of $X = 4 - 3.8 = 0.2$ m [B1] By Pythagoras' Theorem, W to foot of $X = \sqrt{0.2^2 + (3.6 \div 2)^2} = \sqrt{3.28}$ [B1] So, $TX = \sqrt{3.28 + (3 - 1.24)^2} = \sqrt{6.3776}$ ≈ 2.53 m [B1] (ii) By Cosine Rule, $3.6^2 = 6.3776 + 6.3776 - 2(6.3776)\cos \angle PXT$ [M1] $\angle PXT \approx 90.9^\circ$ [A1] (iii) Let the angle of elevation here be θ. $\tan \theta = \frac{1.6 - 1.24}{3.8}$ [M1] $\theta \approx 5.4^\circ$ [A1]	
8(b)	$\tan 12^\circ = \frac{0.36}{d} \rightarrow d \approx 1.69$ m [B1] Since 1.69 m is less than the minimum optimal distance 1.8 m, Roy will not have an optimal view of the TV in this case. [B1]	
Total Marks: 9		

MARK SCHEME

9(a)	$p = 0.5$	[B1]	
9(b)	Correct scale + labeling [B1] Correct plotting of points [B1] Smooth curve [B1]		
9(c)	Drawing of suitable tangent at $x = -1$ [B1] Gradient $= \frac{3.5 - 2.75}{-1.5 - (-1)} = -1.5 (\pm 0.2)$ [B1]		M1 is given if tangent not accurate but correct formula used to find gradient
9(d)	(i) Drawing of correct straight line [B1] (ii) $y = \frac{1}{2}x - 1$ [B1]		

MARK SCHEME

	(iii) For $5 + \frac{2}{x} - \frac{1}{4}x^2 = \frac{1}{2}x - 1$ [B1] $x^3 + 2x^2 - 24x - 8 = 0$ So, $A = 2$ and $B = 24$ [B1]	
9(e)	For $\frac{2}{x} - \frac{1}{4}x^2 + 1 = 0 \rightarrow \frac{2}{x} - \frac{1}{4}x^2 + 5 = 4$ [B1] For $x < 0$, No point of intersection with $y = 4$. $\rightarrow$ No solution (shown) [B1]	
Total Marks: 12		
10(a)	(i) Ave. amount of electricity used per month $= (1107.8 + 1066.3 + 1123.6 + 1259 + 1249.5 + 1281.6)/6$ [M1] $= 1181.3$ kWh [A1] (ii) Ave. amount paid per month $= 1181.3 \times \\$0.2139 \times 1.07$ [B1] $\approx \\$270.37$ [B1]	
10(b)	Max. no. of solar panels that can be installed = 20 [B1] (Based on calculations $(9 \div 1.65) \approx 5$ [length] and $(4 \div 1) = 4$ [width]) <i>After installation,</i> Ave. amount of electricity saved per month = 19×20 $= 380$ kWh Ave. amount paid per month = $(1181.3 - 380) \times \\$0.2139 \times 1.07$ $\approx \\$183.40$ [B1] Ave. cost of solar panels per month = $(2 \times \\$6250) \div (20 \times 12)$ $\approx \\$52.08$ [B1] Total ave. amount paid per month = $\\$183.40 + \\$52.08$ $= \\$235.48 (< \\$270.37)$ Since the average amount paid by Mrs Lim per month will be lesser than what she is currently paying for electricity usage, she should go ahead with the installation. [B1]	
Total Marks: 8		

