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Answer **all** the questions.

- 1 Write the following numbers in order of size, starting with the **smallest**. [1]
 $-\frac{4}{7}, -\frac{4}{5}, -0.8^2, -0.\dot{8}$

- 2 During a children's day celebration, a charity organization distributed 825 files, 495 pens and 660 pencils equally among the children in a children's home. Each child received the same number of files, pens and pencils.
 - (a) Find the largest possible number of children. [2]

 - (b) Hence, find the number of files, pens and pencils each child received. [1]

- 3 It is given that $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.
 - (a) Find f when $u = 1.2$ and $v = 0.4$. [1]

 - (b) Express u in terms of f and v . [2]

- 4 A restaurant charges \$27.80 per person for a buffet lunch. On a particular day, 114 people dined in the restaurant. By approximating both the charge and the number of diners to 2 significant figures, **estimate** the total amount received by the restaurant on that particular day.
 Show your working and give your answer to a reasonable degree of accuracy. [2]

- 5 A piece of metal is heated to 375°C and then left to cool for 15 minutes. The temperature of the metal decreases at a rate of $18^{\circ}\text{C}/\text{min}$ for the first 5 minutes and then decreases at a rate of $7^{\circ}\text{C}/\text{min}$ for the next 10 minutes.

Find the time taken for the metal to cool to a temperature of 250°C . [2]

6 (a) Solve the inequality $1 - x \leq 4 + x < 13 - 2x$. [2]

(b) Write down all the integers which satisfy $1 - x \leq 4 + x < 13 - 2x$. [1]

- 7 The current, I amperes, passing through a circuit is inversely proportional to its resistance, R ohms. When the resistance of the circuit is 3 ohms, the current passing through it is 2 amperes.

(a) Find an equation connecting I and R .

[2]

(b) Calculate the resistance of the circuit when 1.5 amperes of current passes through it.

[1]

(c) Sketch the graph of I against R .

[1]

- 8 Two containers are geometrically similar.
The surface area of the larger container is 63 cm^2 and the surface area of the smaller container is 28 cm^2 .
The height of the smaller container is 5 cm.
Calculate the height of the larger container. [2]

- 9 Between 2014 and 2015, the number of pupils who applied for a particular school as their first choice school increased by 25%.
In 2015, the number of applicants for that school was 425.
Calculate the number of applicants in 2014. [2]
-

- 10 The probability that it will rain on any particular day is 0.3.

Calculate the probability that on two consecutive days, it will rain on only one of the days.

[2]

-
- 11 The table below shows the number of internet-connected devices in some households.

Number of devices	1	2	3	4	5	6
Number of households	2	4	x	7	5	3

- (a) If the modal number of devices is 4, state the maximum possible value of x . [1]

- (b) If the mean number of devices is 3.6, calculate the value of x . [2]

- (c) If the median number of devices is 4, write down all the possible values of x . [1]

- 12 Peter drove from Town X to Town Z , passing by Town Y along the way.
He took 40 minutes to drive from Town X to Town Y at an average speed of 72 km/h.
He rested in Town Y for 10 minutes before continuing his journey to Town Z .
The distance between Town Y and Town Z is 52 km.
His average speed for the whole journey is 60 km/h.

Calculate

(a) the distance between Town X and Town Y , [1]

(b) the average speed for the journey between Town Y and Town Z . [3]

- 13 The point $(1, 1)$ is marked on the diagram.

Sketch the graph of $y = 8 - x^3$ in the answer space below. [1]

- 14 David wants to invest \$500 for 3 years.
Company A offers 8% simple interest per year.
Company B offers 6% interest per year compounded quarterly.

In which company should David invest his money? Justify your answer. [3]

- 15 $\xi = \{x: x \text{ is an integer, } 1 \leq x \leq 100\}$

$$A = \{x: x \text{ is divisible by } 11\}$$

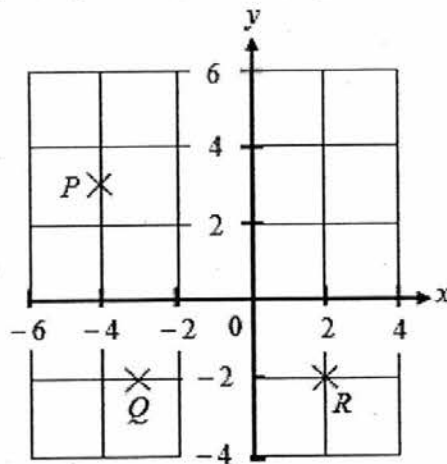
$$B = \{x: x \text{ is divisible by } 22\}$$

$$C = \{x: x \text{ is divisible by } 33\}$$

- (a) List the elements of $A \cap (B \cup C)'$. [1]

- (b) Draw, in the answer space, a clearly labelled Venn diagram to illustrate the three sets A , B and C . [2]

- 16 On the axes shown, P is $(-4, 3)$, Q is $(-3, -2)$ and R is $(2, -2)$.



Find

- (a) the gradient of PQ , [1]
- (b) $\tan \hat{PRQ}$, [1]
- (c) the equation of the line PR , [2]
- (d) the area of triangle PQR , [1]
- (e) the coordinates of two possible points S , such that the four points P , Q , R and S are the four vertices of a parallelogram. [2]

$T_1 T_2$ $T_3 T_4$

The figures $T_1, T_2, T_3, \dots$ are made up of squares.

N is the number of rows of squares in each shape.

S is the number of squares in each shape.

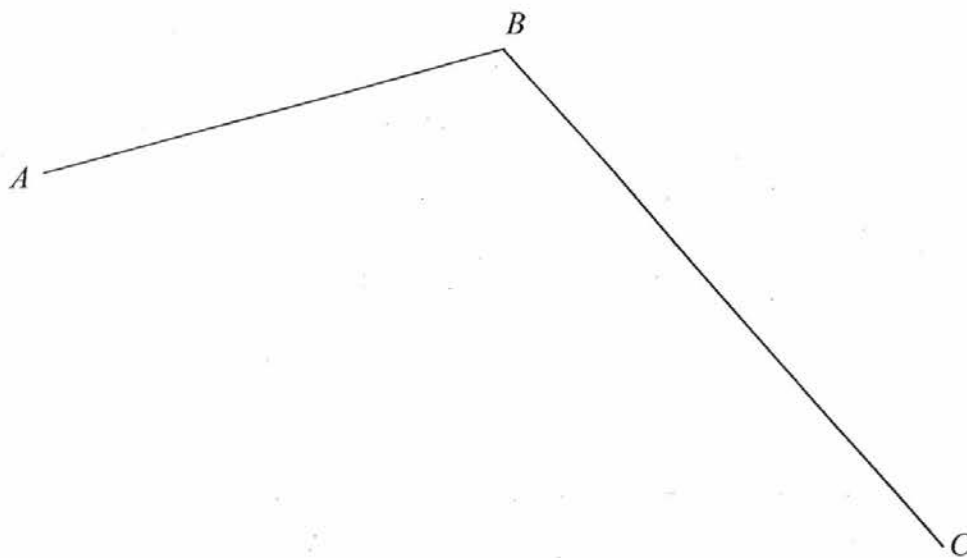
D is the number of dots in each shape.

The values of N, S and D in T_1, T_2, T_3 and T_4 are recorded in the table below.

Figure	T_1	T_2	T_3	T_4
N	1	2	3	4
S	1	4	p	16
D	4	10	q	28
$D - N^2$	3	6	r	s

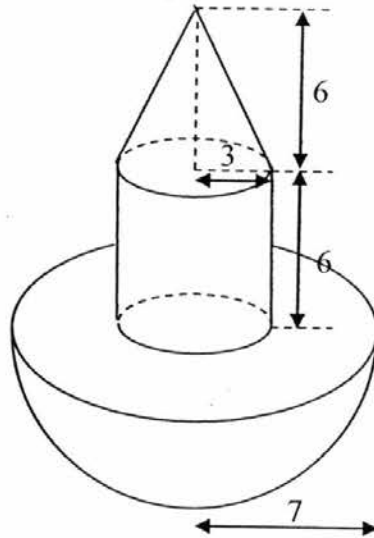
- (a) Find the values of p, q, r and s . [2]
- (b) Express S in terms of N . [1]
- (c) Express D in terms of N . [1]
- (d) Explain why the number of dots cannot be 42. [1]
-

18 Three points A , B and C are shown below.



- (a) Construct the perpendicular bisector of BC . [1]
 - (b) Construct the bisector of angle ABC . [1]
 - (c) Mark clearly a possible point, X , which is equidistant from the lines AB and BC , and equidistant from the points B than C . [1]
 - (d) The point D is such that $ABCD$ is a parallelogram. Find the position of D . [1]
-

- 19 A gold solid is formed by joining the plane faces of a cone, a cylinder and a hemisphere.
The cone and cylinder have a base radius of 3 cm and height 6 cm.
The hemisphere has a radius of 7 cm.



Calculate

- (a) the length of the slant height of the cone, [2]
(b) the surface area of the gold solid, [3]
(c) the surface area of the gold solid. [2]

The density of gold is 19.32 g/cm^3 .

A gold bar has length 25 cm, width 7 cm and height 3.5 cm.

Five gold bars were melted down and all the gold was used to make a large number of these gold solids.

- (d) Calculate the mass of gold that remains after the gold solids are made, giving your answer correct to two significant figures. [4]

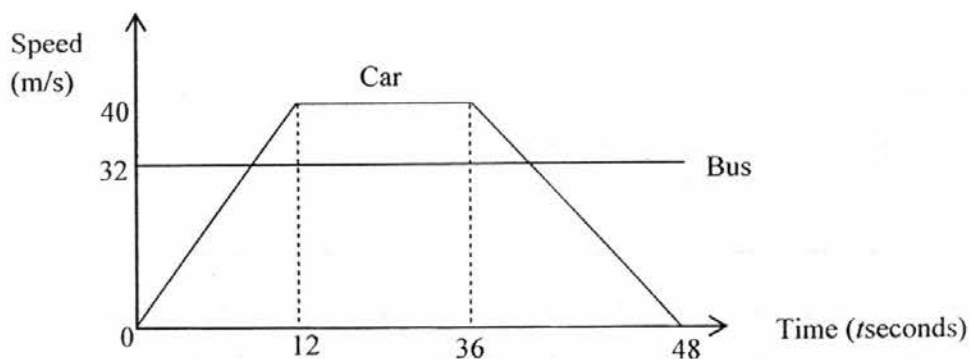
20 O is the origin. A is the point $(3, p)$. B is the point $(-4, 5)$. $\vec{BC} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$.

(a) If $\vec{BC}$ is parallel to $\vec{OA}$, find the value of p . [2]

(b) Find the ratio $OA : BC$. [1]

(c) Find the position vector of M such that $OAMB$ is a parallelogram. [2]

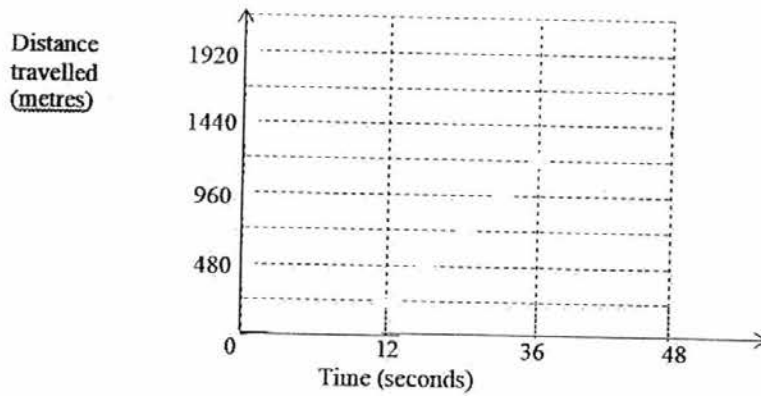
21 The diagram, not drawn to scale, shows the speed-time graph of a car and a bus during a period of 48 seconds. The car and the bus start from the same point, at the same time and travel in the same direction.



(a) Calculate the value(s) of t when the car and bus have the same speed. [3]

(b) Find the value of t when the car overtakes the bus. [3]

(c) Use the grid below to sketch the distance-time graph of the car for the same journey. [3]



Answer **all** the questions.

- 1 Write the following numbers in order of size, starting with the **smallest**.

$$-\frac{4}{7}, -\frac{4}{5}, -0.8^2, -0.\dot{8}$$

$$-0.\dot{8}, -\frac{4}{5}, -0.8^2, -\frac{4}{7}$$

- 2 During a children's day celebration, a charity organization distributed 825 files, 495 pens and 660 pencils equally among the children in a children's home. Each child received the same number of files, pens and pencils.

- (a) Find the largest possible number of children.

3	825	495	660
5	275	165	220
11	55	33	44
	5	3	4

OR

$$825 = 3 \times 5^2 \times 11$$

$$495 = 3^2 \times 5 \times 11$$

$$660 = 2^2 \times 3 \times 5 \times 11$$

$$\text{HCF} = 3 \times 5 \times 11 = 165$$

- (b) Hence, find the number of files, pens and pencils each child received.

5 files, 3 pens, 4 pencils

- 3 It is given that $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

- (a) Find f when $u = 1.2$ and $v = 0.4$.

$$\frac{1}{f} = \frac{1}{1.2} + \frac{1}{0.4}$$

$$= \frac{5}{6} + \frac{5}{2}$$

$$= \frac{20}{6}$$

$$f = 0.3$$

- (b) Express u in terms of f and v .

$$\frac{1}{u} = \frac{v-f}{fv}$$

$$u(v-f) = fv$$

$$u = \frac{fv}{v-f}$$

- 4 A restaurant charges \$27.80 per person for a buffet lunch. On a particular day, 114 people dined in the restaurant.
By approximating both the charge and the number of diners to 2 significant figures, **estimate** the total amount received by the restaurant on that particular day.

Show your working and give your answer to a reasonable degree of accuracy.

$$\begin{aligned} & 110 \times 28 \\ & = 3080 \\ & = 3100 \text{ (2 sf)} \end{aligned}$$

- 5 A piece of metal is heated to 375°C and then left to cool for 15 minutes.
The temperature of the metal decreases at a rate of $18^{\circ}\text{C}/\text{min}$ for the first 5 minutes and then decreases at a rate of $7^{\circ}\text{C}/\text{min}$ for the next 10 minutes.

Find the time taken for the metal to cool to a temperature of 250°C .

$$\begin{aligned} \text{Decrease in temp in 1}^{\text{st}} 5 \text{ min} &= 18 \times 5 \\ &= 90^{\circ}\text{C} \end{aligned}$$

$$\begin{aligned} \text{Further decr. in temp req after 1}^{\text{st}} 5 \text{ min} \\ &= 375 - 125 - 90 \\ &= 35^{\circ}\text{C} \end{aligned}$$

$$\begin{aligned} \text{Time taken} &= \frac{35}{7} + 5 \\ &= 10 \text{ min} \end{aligned}$$

- 6 (a) Solve the inequality $1 - x \leq 4 + x < 13 - 2x$.

$$\begin{aligned} 1 - x \leq 4 + x \quad \text{and} \quad 4 + x < 13 - 2x \\ -3 \leq 2x \quad \quad \quad 3x < 9 \end{aligned}$$

$$x \geq -1\frac{1}{2} \quad x < 3$$

$$\therefore -1\frac{1}{2} \leq x < 3$$

- (b) Write down all the integers which satisfy $1 - x \leq 4 + x < 13 - 2x$.

$$-1, 0, 1, 2$$

- 7 The current, I amperes, passing through a circuit is inversely proportional to its resistance, R ohms. When the resistance of the circuit is 3 ohms, the current passing through it is 2 amperes.

(a) Find an equation connecting I and R .

$$I = \frac{k}{R}$$

$$2 = \frac{k}{3}$$

$$k = 6$$

$$I = \frac{6}{R}$$

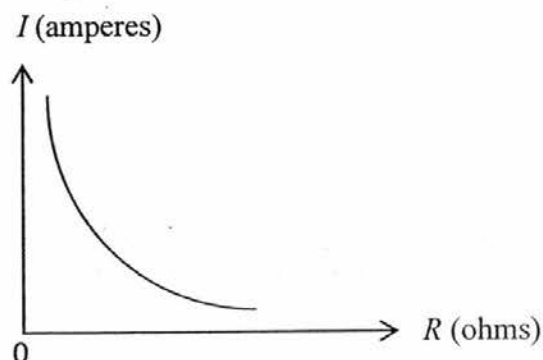
(b) Calculate the resistance of the circuit when 1.5 amperes of current passes through it.

$$1.5 = \frac{6}{R}$$

$$R = 4 \text{ ohms}$$

(c) Sketch the graph of I against R .

Answer (c)



- 8 Two containers are geometrically similar. The surface area of the larger container is 63 cm^2 and the surface area of the smaller container is 28 cm^2 . The height of the smaller container is 5 cm. Calculate the height of the larger container.

$$\left(\frac{h_l}{h_s}\right)^2 = \frac{63}{28}$$

$$\frac{h_l}{h_s} = \sqrt{\frac{63}{28}}$$

$$\frac{h_l}{5} = \frac{3}{2}$$

$$h_l = 7.5 \text{ cm}$$

- 9 Between 2014 and 2015, the number of pupils who applied for a particular school as their first choice school increased by 25%.
In 2015, the number of applicants for that school was 425.

Calculate the number of applicants in 2014.

$$\frac{100}{125} \times 425$$

$$= 340$$

- 10 The probability that it will rain on any particular day is 0.3.

Calculate the probability that on two consecutive days, it will rain on only one of the days.

$$0.3 \times 0.7 \times 2$$

$$= 0.42$$

- 11 The table below shows the number of internet-connected devices in some households.

Number of devices	1	2	3	4	5	6
Number of households	2	4	x	7	5	3

- (a) If the modal number of devices is 4, state the maximum possible value of x .

6

- (b) If the mean number of devices is 3.6, calculate the value of x .

$$\frac{1(2) + 2(4) + 3x + 4(7) + 5(5) + 6(3)}{2 + 4 + x + 7 + 5 + 3} = 3.6$$

$$\frac{81 + 3x}{21 + x} = 3.6$$

$$81 + 3x = 3.6(21 + x)$$

$$81 + 3x = 75.6 + 3.6x$$

$$5.4 = 0.6x$$

$$x = 9$$

- (c) If the median number of devices is 4, write down all the possible values of x .

0, 1, 2, 3, ..., 8 or $0 \leq x \leq 8$, x is an integer

- 12 Peter drove from Town X to Town Z , passing by Town Y along the way.
 He took 40 minutes to drive from Town X to Town Y at an average speed of 72 km/h.
 He rested in Town Y for 10 minutes before continuing his journey to Town Z .
 The distance between Town Y and Town Z is 52 km.
 His average speed for the whole journey is 60 km/h.

Calculate

- (a) the distance between Town X and Town Y ,

$$\text{Distance between } X \text{ and } Y = 72 \times \frac{2}{3} = 48 \text{ km}$$

- (b) the average speed for the journey between Town Y and Town Z .

$$\begin{aligned} \text{Total time for the whole journey} &= \frac{48 + 52}{60} \\ &= 1\frac{2}{3} \text{ h} \end{aligned}$$

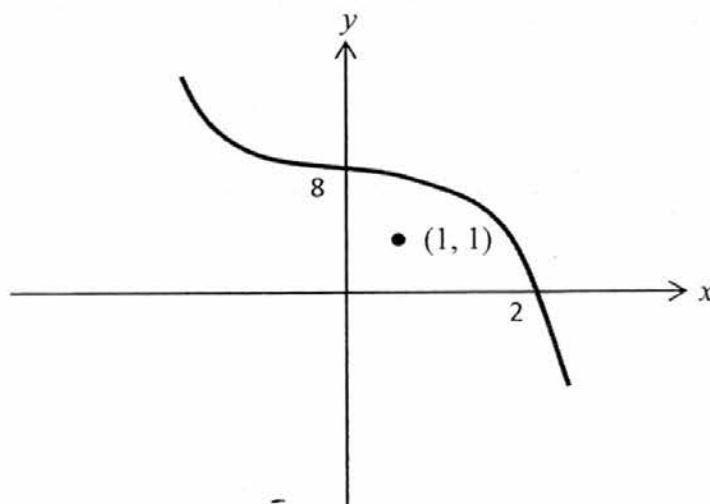
$$\begin{aligned} \text{Time taken for } Y \text{ to } Z &= 1\frac{2}{3} - \frac{2}{3} - \frac{1}{6} \\ &= \frac{5}{6} \text{ h} \end{aligned}$$

$$\begin{aligned} \text{Average speed} &= 52 \div \frac{5}{6} \\ &= 62.4 \text{ km/h} \end{aligned}$$

- 13 The point $(1, 1)$ is marked on the diagram.

Sketch the graph of $y = 8 - x^3$ in the answer space below.

Answer



- 14 David wants to invest \$500 for 3 years.
 Company A offers 8% simple interest per year.
 Company B offers 6% interest per year compounded quarterly.

In which company should David invest his money? Justify your answer.

Company A $Interest = \frac{500 \times 8 \times 3}{100}$ $= 120$		Company B $Amount = 500 \left(1 + \frac{6}{4 \times 100} \right)^{12}$ $= 597.81$	
Total = $500 + 120$ $= 620$		Interest = $597.81 - 500$ $= 97.81$	
Company A			

- 15 $\xi = \{x: x \text{ is an integer, } 1 \leq x \leq 100\}$

$$A = \{x: x \text{ is divisible by } 11\}$$

$$B = \{x: x \text{ is divisible by } 22\}$$

$$C = \{x: x \text{ is divisible by } 33\}$$

- (a) List the elements of $A \cap (B \cup C)'$.

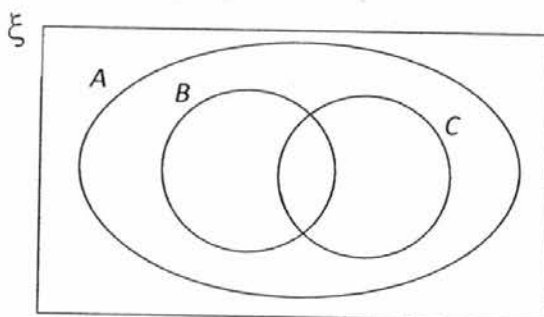
$$A = \{x: x \text{ is divisible by } 11\} = \{11, 22, 33, 44, 55, 66, 77, 88, 99\}$$

$$B = \{x: x \text{ is divisible by } 22\} = \{22, 44, 66, 88\}$$

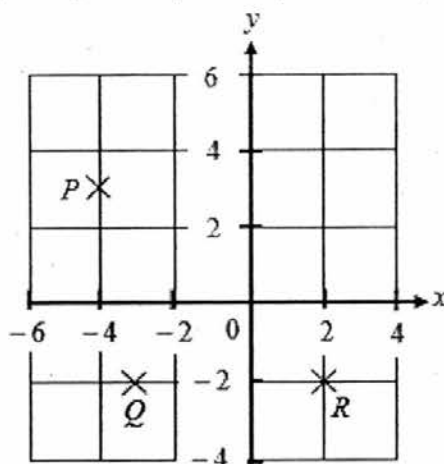
$$C = \{x: x \text{ is divisible by } 33\} = \{33, 66, 99\}$$

$$A \cap (B \cup C)' = \{11, 55, 77\}$$

- (b) Draw, in the answer space, a clearly labelled Venn diagram to illustrate the three sets A, B and C.



- 16 On the axes shown, P is $(-4, 3)$, Q is $(-3, -2)$ and R is $(2, -2)$.



Find

- (a) the gradient of PQ ,

$$\begin{aligned} \text{Gradient } PQ &= \frac{-2 - 3}{-3 - (-4)} \\ &= -5 \end{aligned}$$

- (b) $\tan \hat{PRQ}$,

$$\tan \hat{PRQ} = \frac{5}{6}$$

- (c) the equation of the line PR ,

$$\begin{aligned} \text{Gradient } PR &= -\frac{5}{6} \\ -2 &= \left(-\frac{5}{6}\right)(2) + c \\ c &= -\frac{1}{3} \\ \text{Equation: } y &= -\frac{5}{6}x - \frac{1}{3} \quad \text{OR} \\ 6y &= -5x - 2 \end{aligned}$$

- (d) the area of triangle PQR ,

$$\begin{aligned} \text{Area } PQR &= \frac{1}{2} \times 5 \times 5 \\ &= 12.5 \text{ square units} \end{aligned}$$

- (e) the coordinates of two possible points S , such that the four points P , Q , R and S are the four vertices of a parallelogram.

$$S(1,3) \text{ or } S(3,-7) \text{ or } S(-9,3)$$

$T_1 T_2$ $T_3 T_4$

The figures $T_1, T_2, T_3, \dots$ are made up of squares.

N is the number of rows of squares in each shape.

S is the number of squares in each shape.

D is the number of dots in each shape.

The values of N, S and D in T_1, T_2, T_3 and T_4 are recorded in the table below.

Figure	T_1	T_2	T_3	T_4
N	1	2	3	4
S	1	4	p	16
D	4	10	q	28
$D - N^2$	3	6	r	s

- (a) Find the values of p, q, r and s .

$$p = 9, q = 18, r = 9, s = 12$$

- (b) Express S in terms of N .

$$S = N^2$$

- (c) Express D in terms of N .

$$D = 3N + N^2$$

- (d) Explain why the number of dots cannot be 42.

$$\text{If } D = 42$$

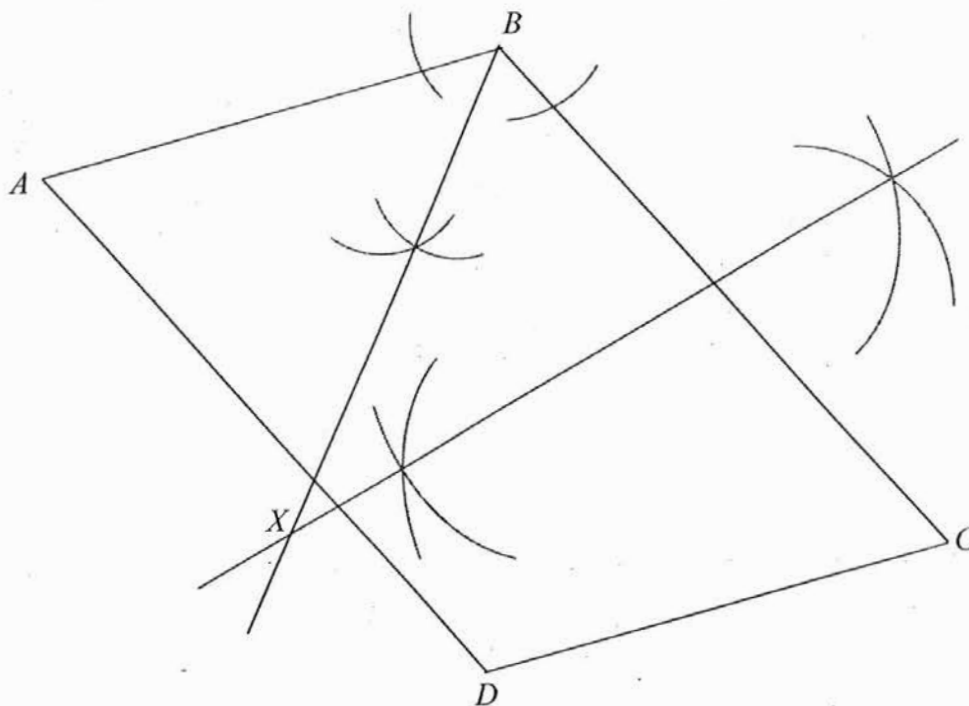
$$3N + N^2 = 42$$

$$N^2 + 3N - 42 = 0$$

$$N = \frac{-3 \pm \sqrt{9 + 168}}{2} \text{ which is not a whole number}$$

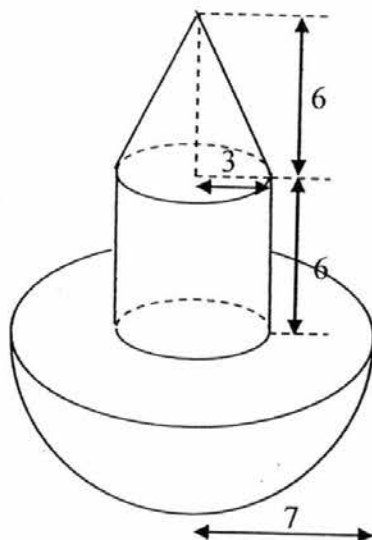
Hence D cannot be 42

18 Three points A , B and C are shown below.



- (a) Construct the perpendicular bisector of BC . [1]
- (b) Construct the bisector of angle ABC . [1]
- (c) Mark clearly a possible point, X , which is equidistant from the lines AB and BC , and equidistant from the points B than C . [1]
- (d) The point D is such that $ABCD$ is a parallelogram. Find the position of D . [1]

- 19 A gold solid is formed by joining the plane faces of a cone, a cylinder and a hemisphere.
The cone and cylinder have a base radius of 3 cm and height 6 cm.
The hemisphere has a radius of 7 cm.



Calculate

- (a) the length of the slant height of the cone,

$$\begin{aligned}\text{Slant height} &= \sqrt{3^2 + 6^2} \\ &= 6.708 \\ &= 6.71 \text{ cm}\end{aligned}$$

- (b) the surface area of the gold solid,

$$\begin{aligned}\text{Total SA} &= \pi(3)(6.708) + 2\pi(3)(6) \\ &\quad + 2\pi(7)^2 + [\pi(7^2 - 3^2)] \\ &= 609.8 \\ &\approx 610 \text{ cm}^2\end{aligned}$$

- (c) the surface area of the gold solid.

$$\begin{aligned}\text{Volume} &= \left[\frac{1}{3} \times \pi(3)^2 \times 6 \right] + [\pi(3)^2 \times 6] \\ &\quad + \left[\frac{2}{3} \times \pi(7)^3 \right] \\ &= 944.5 \\ &\approx 945 \text{ cm}^3\end{aligned}$$

The density of gold is 19.32 g/cm^3 .

A gold bar has length 25 cm, width 7 cm and height 3.5 cm.

Five gold bars were melted down and all the gold was used to make a large number of these gold solids.

- (d) Calculate the mass of gold that remains after the gold solids are made, giving your answer correct to two significant figures.

$$\begin{aligned}
 &\text{Volume five gold bars} \\
 &= 5 \times (25 \times 7 \times 3.5) \\
 &= 3062.5 \text{ cm}^3 \\
 &\text{No of figures} = 3062.5 \div 944.5 \\
 &= 3.24 \\
 &\approx 3 \text{ figures} \\
 &\text{Volume of gold that remains} \\
 &= [3062.5 - (3 \times 944.5)] \text{ or} \\
 &(3.242 - 3)(944.5) \\
 &= 229.0 \text{ cm}^3 \text{ or } 228.7 \text{ cm}^3 \\
 &\text{Mass of gold that remains} \\
 &= 229 \times 19.32 \text{ or } 228.7 \times 19.32 \\
 &= 4424.28 \text{ or } 4420.0 \\
 &\approx 4400 \text{ g}
 \end{aligned}$$

- 20 O is the origin. A is the point $(3, p)$. B is the point $(-4, 5)$. $\vec{BC} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$.

- (a) If $\vec{BC}$ is parallel to $\vec{OA}$, find the value of p .

$$\vec{BC} = m \vec{OA}$$

$$\begin{pmatrix} 6 \\ 5 \end{pmatrix} = m \begin{pmatrix} 3 \\ p \end{pmatrix}$$

$$6 = 3m$$

$$m = 2$$

$$5 = 2p$$

$$p = 2.5$$

- (b) Find the ratio $OA : BC$.

$$\vec{BC} = 2 \vec{OA} \Rightarrow OA : BC = 1 : 2$$

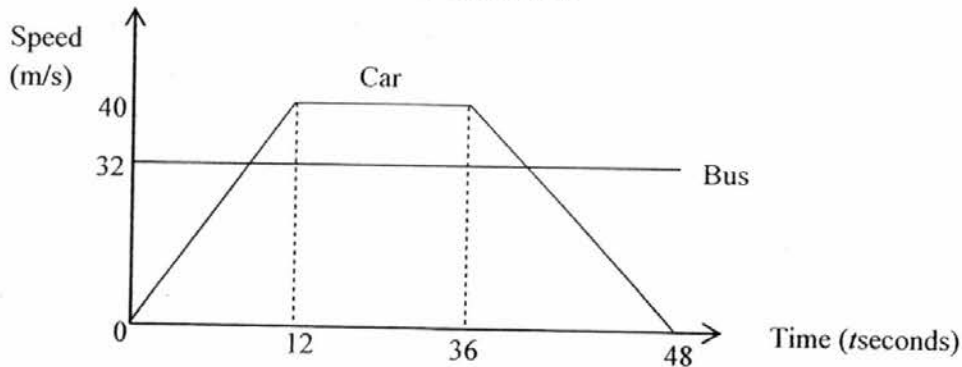
- (c) Find the position vector of M such that $OAMB$ is a parallelogram.

$$\vec{BM} = \vec{OA} \text{ or } \vec{OB} = \vec{AM}$$

$$\vec{OM} - \begin{pmatrix} -4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 2.5 \end{pmatrix} \quad \vec{OM} - \begin{pmatrix} 3 \\ 2.5 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

$$\begin{aligned}
 \vec{OM} &= \begin{pmatrix} 3 \\ 2.5 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \end{pmatrix} & \vec{OM} &= \begin{pmatrix} 3 \\ 2.5 \end{pmatrix} + \begin{pmatrix} -4 \\ 5 \end{pmatrix} \\
 &= \begin{pmatrix} -1 \\ 7.5 \end{pmatrix} & &= \begin{pmatrix} -1 \\ 7.5 \end{pmatrix}
 \end{aligned}$$

- 21 The diagram, not drawn to scale, shows the speed-time graph of a car and a bus during a period of 48 seconds. The car and the bus start from the same point, at the same time and travel in the same direction.



- (a) Calculate the value(s) of t when the car and bus have the same speed.

$$\frac{40}{12} = \frac{32}{t}$$

$$t = 32 \times \frac{12}{40}$$

$$= 9\frac{3}{5} \text{ or } 9.6$$

$$t_2 = 48 - 9\frac{3}{5}$$

$$= 38\frac{2}{5} \text{ or } 38.4$$

- (b) Find the value of t when the car overtakes the bus.

$$\frac{1}{2}(12)(40) + 40(t - 12) = 32t$$

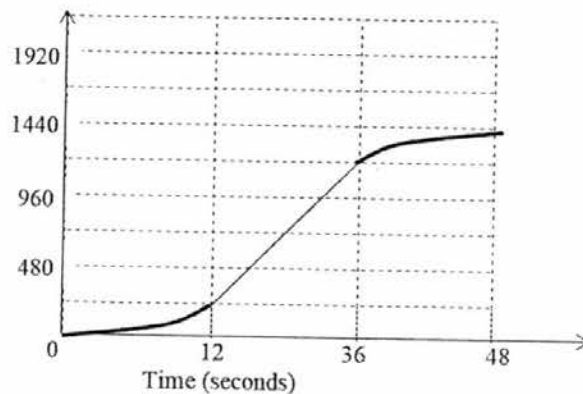
$$240 + 40t - 480 = 32t$$

$$8t = 240$$

$$t = 30$$

- (c) Use the grid below to sketch the distance-time graph of the car for the same journey.

Distance
travelled
(metres)



[3]

1 (a) Express as a single fraction in its simplest form $1 - \frac{2x}{2x-7} + \frac{7}{(2x-7)^2}$. [3]

(b) Simplify $5a^{-3}b^5 \div \frac{10}{9}a^3b^{-2}$. [2]

(c) (i) Factorise fully $11p^2 - 44pq + 4q - p$. [2]

(ii) Factorise fully $30m^2 + 14mn - 4n^2$ [2]

(d) Solve the equation $\frac{1}{x} - \frac{x-5}{2x-3} = 1$. [3]

- 2 Twenty five boys took a quiz.
The marks are shown in the stem-and-leaf diagram.

1	4 7
2	3 5 7 7 9
3	0 1 2 3 3 5 7 7 8 9 9 9
4	3 4 6 6 7
5	0

Key
1 | 4 means 14 marks

(a) Find

(i) the median mark, [1]

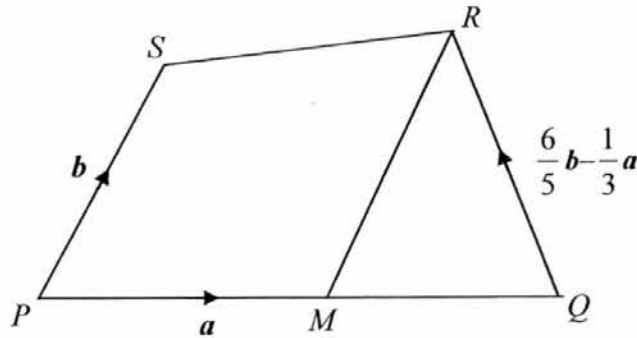
(ii) the interquartile range. [3]

Twenty five girls took the same quiz.
The median mark and interquartile range of the girls' marks are 35 and 6 respectively.

- (b) Compare and comment on the performance of the boys and girls in this quiz. [2]

- 3 $PQRS$ is a quadrilateral. M is the mid-point of PQ .

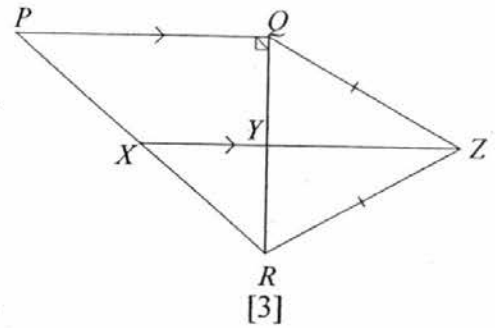
$$\vec{PQ} = \mathbf{a}, \vec{PS} = \mathbf{b} \text{ and } \vec{QR} = \frac{6}{5}\mathbf{b} - \frac{1}{3}\mathbf{a}.$$



- (a) Find $\vec{SR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

- (b) Use vectors to show that PS and MR are **not** parallel. [2]

- 4 In the diagram, PXR , QYR , and XYZ are straight lines.
 PQ is parallel to XZ , $QZ = RZ$, $\frac{YZ}{XZ} = \frac{3}{5}$ and $\angle PQR = 90^\circ$.



- (a) Show that triangles QYZ and RYZ are congruent.

- (b) Show that triangles PQR and XYR are similar.

[2]

- (c) Find

(i) $\frac{\text{area of } \triangle XYR}{\text{area of } \triangle RYZ},$

[1]

(ii) $\frac{\text{area of } \triangle XYR}{\text{area of } \triangle PQR}$

[1]

5 Jeannie bought some health drink for \$6400. She paid \$ x for each litre of the drink.

(a) Find, in terms of x , an expression for the number of litres she bought. [1]

(b) She gave away 8 litres of the drink to her friends. She sold the remainder of the drink for \$50 per litre more than she paid for it. Write down an expression, in terms of x , for the sum of money she received. [1]

(c) She made a profit of \$2960.

(i) Write down an equation in x to represent this information, and show that it reduces to $x^2 + 420x - 40\,000 = 0$. [2]

(ii) Solve the equation $x^2 + 420x - 40\,000 = 0$. [3]

(d) Find the number of litres of drink Jeannie sold. [1]

- 6 Two satay stalls sell 3 types of satay.

The number of sticks of each type of satay sold per day is given by the matrix S .

$$S = \begin{array}{ccc|c} \text{Chicken} & \text{Mutton} & \text{Beef} & \\ \hline 400 & 300 & 200 & \text{Stall A} \\ 200 & 500 & 300 & \text{Stall B} \end{array}$$

- (a) The price of each stick of chicken, mutton and beef satay is \$0.35, \$0.45 and \$0.40 respectively. Represent these prices in a 3×1 column matrix P . [1]

- (b) Evaluate the matrix $T = SP$. [1]

- (c) State what the elements of T represent. [1]

- (d) In June 2016, Stall A operated 20 days and Stall B operated 25 days. Use matrix multiplication to find the total amount of money collected by the two stalls in June 2016. [2]

- (e) In July, the number of sticks of each type of satay sold per day is increased by 10%. The information is given by the matrix Q .

$$Q = \begin{array}{ccc|c} \text{Chicken} & \text{Mutton} & \text{Beef} & \\ \hline 440 & 330 & 220 & \text{Stall A} \\ 220 & 550 & 330 & \text{Stall B} \end{array}$$

Write down the matrix R such that $Q = SR$. [1]

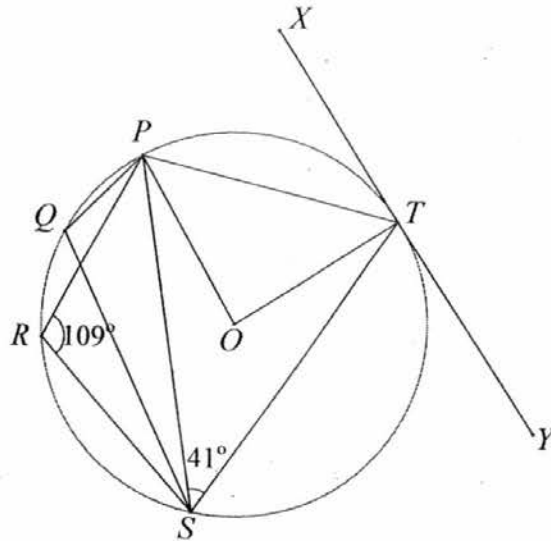
7 A box contains 5 Chocolate doughnuts, 3 Glazed doughnuts and 1 Strawberry doughnut.

(a) Two doughnuts were taken out of the box at random, without replacement. Copy and complete the tree diagram to show this information.[3]

(b) Find, as a fraction in its simplest form, the probability that
(i) the two doughnuts are the same flavour, [2]

(ii) at least one of the doughnuts is Chocolate. [3]

- 8 In the diagram, the points P, Q, R, S and T lie on a circle, centre O . XTY is a tangent to the circle. Angle $PRS = 109^\circ$ and angle $PST = 41^\circ$.



- (a) Find, giving reasons for each answer,

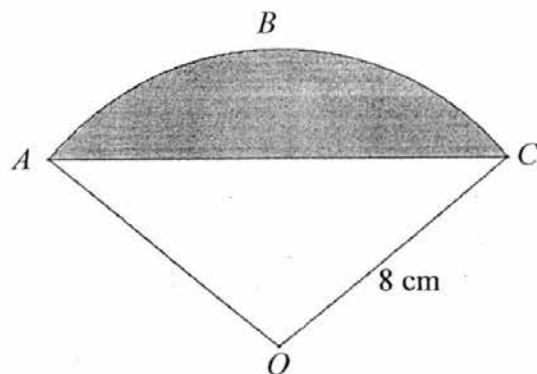
(i) $\hat{PQS}$, [1]

(ii) $\hat{PTS}$, [1]

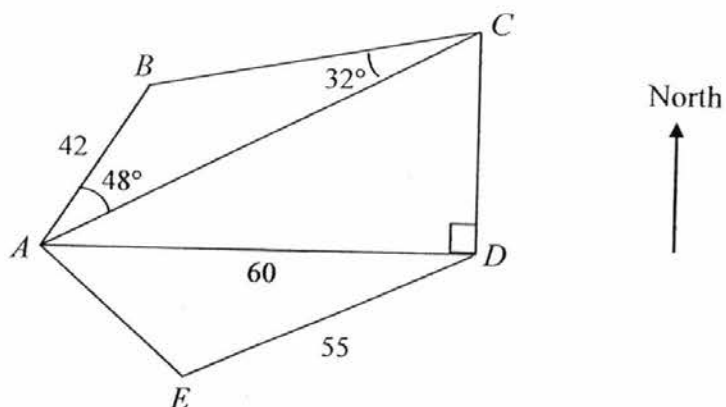
(iii) $\hat{YTS}$. [2]

(iv) $\hat{OTP}$, [2]

- (b) $OABC$ is a sector of a circle, centre O and radius 8 cm. The perimeter of the sector is 30 cm.



- (i) Show that angle $AOC = 1.75$ radians. [1]
- (ii) Calculate the area of the shaded region. [3]



The diagram shows a field, $ABCDE$, which is crossed by two paths, AC and AD . AD is perpendicular to CD . $AB = 42$ m, $AD = 60$ m, $DE = 55$ m, angle $BAC = 48^\circ$ and angle $ACB = 32^\circ$.

- (a) Show that $AC = 78.05$ m, correct to four significant figures. [2]
- (b) Calculate CD . [2]
- (c) A bird is at P , which is 8 m vertically above E . Calculate the angle of depression of D from P . [2]
- (d) Given that the area of triangle ADE is 1300 m^2 , calculate angle ADE . [2]
- (e) D is due east of A . Calculate the bearing of E from A . [3]

10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = \frac{5x^2}{4} + \frac{60}{x} - 40$.

Some corresponding values of x and y are given in the following table.

x	1	1.5	2	3	3.5	4	4.5	5	6
y	p	2.81	-5	-8.75	-7.54	-5	-1.35	3.25	15

- (a) Find the value of p . [1]
- (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $1 \leq x \leq 6$.
Using a scale of 2 cm to represent 5 units, draw a vertical y -axis for $-10 \leq y \leq 25$.
On your axes, plot the points given in the table and join them with a smooth curve. [2]
- (c) Using your graph, find the range of values of x for which $\frac{5x^2}{4} + \frac{60}{x} - 40 < 0$. [3]
- (d) By drawing a tangent, find the gradient of the curve at the point where $x = 4$. [2]
- (e) Draw the tangent to the curve which the gradient is -10 .
Write down the equation of this tangent. [2]

(f) The line l intersects the curve $y = \frac{5x^2}{4} + \frac{60}{x} - 40$ at $x = 2$ and $x = 6$.

(i) Find the equation of l . [2]

(ii) By using your answer from (f)(i), find the value of A and of B . [3]

- 11 **Diagram I** shows a pencil before it is sharpened. It is made up of a piece of cylindrical carbon encased in wood. The length of the pencil is 19 cm.
Diagram II shows the cross-sectional area of the pencil. $ABCDEF$ is a regular hexagon with side 0.45 cm. The diameter of the carbon is 0.2 cm.

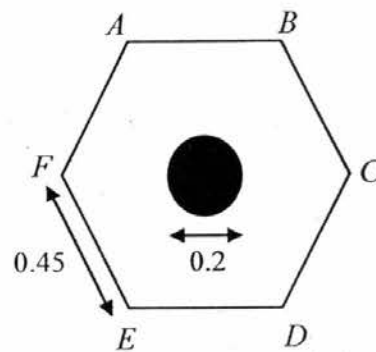
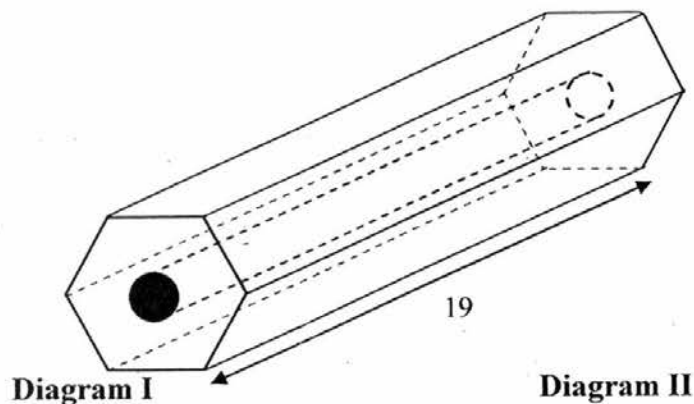


Diagram I
 (a) Find

(i) the interior angle of the regular hexagon $ABCDEF$, [2]

(ii) CF , [1]

(b) Show that $AE = 0.7794$ cm. [2]

(c) Calculate the area of the regular hexagon $ABCDEF$. [2]

(d) Calculate the volume of the carbon as a percentage of the volume of the pencil. [2]

Diagram III shows ten of these pencils which just fit into a rectangular box which is open on one side.

Diagram IV shows ten of these pencils which just fit into a box whose cross-sectional area is an equilateral triangle which is open on one side.

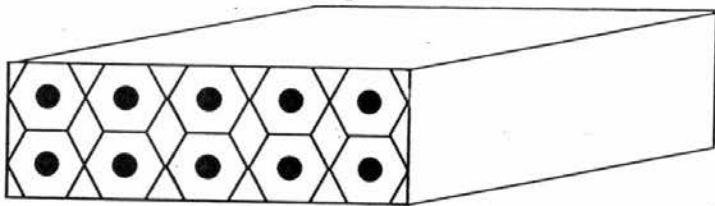


Diagram III

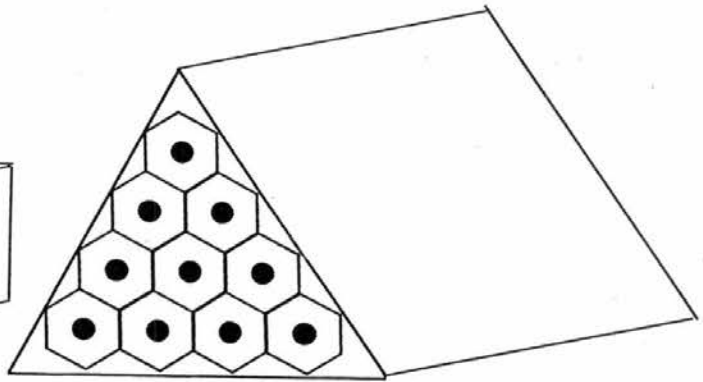


Diagram IV

(e) The boxes are made of cardboard which cost \$10 per m^2 . Determine which box will be cheaper to produce for 1000 boxes. Justify your decision with calculations.

[5]

- 1 (a) Express as a single fraction in its simplest form $1 - \frac{2x}{2x-7} + \frac{7}{(2x-7)^2}$. [3]

$$\begin{aligned} & 1 - \frac{2x}{2x-7} + \frac{7}{(2x-7)^2} \\ &= \frac{(2x-7)^2 - 2x(2x-7) + 7}{(2x-7)^2} \\ &= \frac{4x^2 - 28x + 49 - 4x^2 + 14x + 7}{(2x-7)^2} \\ &= \frac{56 - 14x}{(2x-7)^2} \text{ or } \frac{14(4-x)}{(2x-7)^2} \end{aligned}$$

- (b) Simplify $5a^{-3}b^5 \div \frac{10}{9}a^3b^{-2}$. [2]

$$\begin{aligned} & \frac{5b^5}{a^3} \div \frac{10a^3}{9b^2} \\ &= \frac{5b^5}{a^3} \times \frac{9b^2}{10a^3} \\ &= \frac{9b^7}{2a^6} \text{ or } \frac{9}{2}a^{-6}b^7 \end{aligned}$$

- (c) (i) Factorise fully $11p^2 - 44pq + 4q - p$. [2]

$$\begin{aligned} & 11p^2 - 44pq + 4q - p \\ &= (11p^2 - 44pq) - (p - 4q) \\ &= 11p(p - 4q) - (p - 4q) \\ &= (11p - 1)(p - 4q) \end{aligned}$$

- (ii) Factorise fully $30m^2 + 14mn - 4n^2$ [2]

$$\begin{aligned} & 30m^2 + 14mn - 4n^2 \\ &= 2(15m^2 + 7mn - 2n^2) \\ &= 2(3m + 2n)(5m - n) \end{aligned}$$

- (d) Solve the equation $\frac{1}{x} - \frac{x-5}{2x-3} = 1$. [3]

$$\begin{aligned} & \frac{1}{x} - \frac{x-5}{2x-3} = 1 \\ & \frac{(2x-3) - x(x-5)}{x(2x-3)} = 1 \\ & 2x - 3 - x^2 + 5x = 2x^2 - 3x \\ & 0 = 3x^2 - 10x + 3 \\ & (3x - 1)(x - 3) = 0 \\ & x = \frac{1}{3} \quad \text{or} \quad 3 \end{aligned}$$

- 2 Twenty five boys took a quiz.
The marks are shown in the stem-and-leaf diagram.

1	4 7
2	3 5 7 7 9
3	0 1 2 3 3 5 7 7 8 9 9 9
4	3 4 6 6 7
5	0

Key
1 | 4 means 14 marks

- (a) Find

- (i) the median mark,
Median = 35

[1]

- (ii) the interquartile range. [3]

$$Q_1 = \frac{27 + 29}{2} = 28 \quad Q_3 = \frac{39 + 43}{2} = 41$$

$$\text{Interquartile range} = 41 - 28 = 13$$

Twenty five girls took the same quiz.

The median mark and interquartile range of the girls' marks are 35 and 6 respectively.

- (b) Compare and comment on the performance of the boys and girls in this quiz. [2]

$Q_{2B} = Q_{2G} \Rightarrow$ The performance of the boys and girls is the same (is similar).

$IQR_G < IQR_B \Rightarrow$ The quiz marks of the girls has a narrower spread.

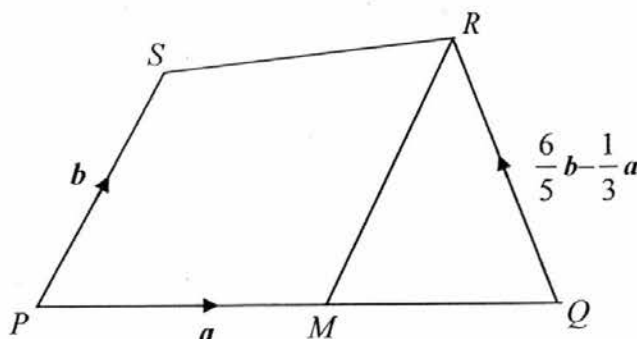
OR

The results of the girls is more consistent.

Note : Must see $IQR_G < IQR_B$ or in words

- 3 $PQRS$ is a quadrilateral. M is the mid-point of PQ .

$$\vec{PQ} = \mathbf{a}, \vec{PS} = \mathbf{b} \text{ and } \vec{QR} = \frac{6}{5}\mathbf{b} - \frac{1}{3}\mathbf{a}.$$



- (a) Find $\vec{SR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

$$\begin{aligned} \vec{SR} &= \vec{SP} + \vec{PQ} + \vec{QR} \\ &= -\mathbf{b} + \mathbf{a} + \frac{6}{5}\mathbf{b} - \frac{1}{3}\mathbf{a} \\ &= \frac{2}{3}\mathbf{a} + \frac{1}{5}\mathbf{b} \end{aligned}$$

- (b) Use vectors to show that PS and MR are **not** parallel. [2]

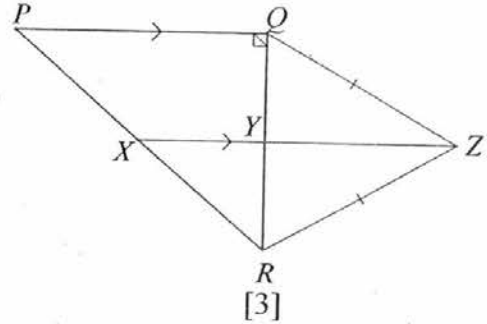
$$\begin{aligned} \vec{MR} &= \vec{MQ} + \vec{QR} \\ &= \frac{1}{2}\mathbf{a} + \frac{6}{5}\mathbf{b} - \frac{1}{3}\mathbf{a} \\ &= \frac{1}{6}\mathbf{a} + \frac{6}{5}\mathbf{b} \end{aligned}$$

$\vec{MR} \neq m\mathbf{b}$, where m is a scalar.

OR

$\therefore \vec{MR}$ is not a multiple of $\vec{PS}$, hence PS and MR are not parallel

- 4 In the diagram, PXR , QYR , and XYZ are straight lines.
 PQ is parallel to XZ , $QZ = RZ$, $\frac{YZ}{XZ} = \frac{3}{5}$ and $\angle PQR = 90^\circ$.



- (a) Show that triangles QYZ and RYZ are congruent.

$QZ = RZ$ (given)

$YZ = YZ$ (common side)

$\angle QYZ = 90^\circ$ (alt $\angle$ s, $PQ \parallel XY$)

$\angle RYZ = 180^\circ - 90^\circ$ (adj $\angle$ s on straight line)
 $= 90^\circ$

$\therefore \angle QYZ = \angle RYZ$

$\therefore \triangle QYZ \equiv \triangle RYZ$ (RHS)

Alternate Solution

$QZ = RZ$ (given)

$\angle ZQY = \angle ZRY$ (base angles, isos $\triangle$)

$\angle QYZ = 90^\circ$ (alt $\angle$ s, $PQ \parallel XY$)

$\angle RYZ = 180^\circ - 90^\circ$ (adj $\angle$ s on straight line)
 $= 90^\circ$

$\therefore \angle QYZ = \angle RYZ$

$\therefore \triangle QYZ \equiv \triangle RYZ$ (AAS)

- (b) Show that triangles PQR and XYR are similar.

$\angle PQR = \angle XYR$ (corr. $\angle$ s, $PQ \parallel XY$)

$\angle QPR = \angle YXR$ (corr. $\angle$ s, $PQ \parallel XY$)

$\angle PRQ = \angle XRY$ (common angle)

$\therefore \triangle PQR$ is similar to $\triangle XYR$.

Any 2 reasons

[2]

- (c) Find

(i) $\frac{\text{area of } \triangle XYR}{\text{area of } \triangle RYZ}$,

[1]

$$\frac{\text{area of } \triangle XYR}{\text{area of } \triangle RYZ} = \frac{2}{3}$$

(ii) $\frac{\text{area of } \triangle XYR}{\text{area of } \triangle PQR}$

[1]

$$\frac{\text{area of } \triangle XYR}{\text{area of } \triangle PQR} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

- 5 Jeannie bought some health drink for \$6400. She paid \$x for each litre of the drink.

- (a) Find, in terms of x , an expression for the number of litres she bought. [1]

$$\text{No of Litres bought} = \frac{6400}{x}$$

- (b) She gave away 8 litres of the drink to her friends. She sold the remainder of the drink for \$50 per litre more than she paid for it. Write down an expression, in terms of x , for the sum of money she received. [1]

$$\text{No of litres left} = \frac{6400}{x} - 8$$

Sum of money received

$$\begin{aligned} &= (x + 50) \left(\frac{6400}{x} - 8 \right) \\ &= 6400 - 8x + \frac{320\,000}{x} - 400 \\ &= \$ \left(\frac{320\,000}{x} - 8x + 6000 \right) \end{aligned}$$

- (c) She made a profit of \$2960.

- (i) Write down an equation in x to represent this information, and show that it reduces to $x^2 + 420x - 40\,000 = 0$. [2]

$$\left(\frac{320\,000}{x} - 8x + 6000 \right) - 6400 = 2960$$

$$8x^2 + 3360x - 320\,000 = 0$$

$$x^2 + 420x - 40\,000 = 0$$

- (ii) Solve the equation $x^2 + 420x - 40\,000 = 0$. [3]

$$(x + 500)(x - 80) = 0$$

$$x = -500 \text{ or } x = 80$$

- (d) Find the number of litres of drink Jeannie sold. [1]

$$\begin{aligned} \text{Number of litres sold} &= \frac{6400}{80} - 8 \\ &= 72 \end{aligned}$$

- 6 Two satay stalls sell 3 types of satay.

The number of sticks of each type of satay sold per day is given by the matrix S .

$$S = \begin{array}{ccc|c} \text{Chicken} & \text{Mutton} & \text{Beef} & \\ \hline 400 & 300 & 200 & \text{Stall A} \\ 200 & 500 & 300 & \text{Stall B} \end{array}$$

- (a) The price of each stick of chicken, mutton and beef satay is \$0.35, \$0.45 and \$0.40 respectively. Represent these prices in a 3×1 column matrix P . [1]

$$P = \begin{pmatrix} 0.35 \\ 0.45 \\ 0.40 \end{pmatrix}$$

- (b) Evaluate the matrix $T = SP$. [1]

$$\begin{aligned} T &= \begin{pmatrix} 400 & 300 & 200 \\ 200 & 500 & 300 \end{pmatrix} \begin{pmatrix} 0.35 \\ 0.45 \\ 0.40 \end{pmatrix} \\ &= \begin{pmatrix} 355 \\ 415 \end{pmatrix} \end{aligned}$$

- (c) State what the elements of T represent. [1]

The total amount of money collected by each stall (per day from the selling the satay)

- (d) In June 2016, Stall A operated 20 days and Stall B operated 25 days. Use matrix multiplication to find the total amount of money collected by the two stalls in June 2016. [2]

$$\begin{aligned} & \begin{pmatrix} 20 & 25 \end{pmatrix} \begin{pmatrix} 355 \\ 415 \end{pmatrix} \\ &= \begin{pmatrix} 17 & 475 \end{pmatrix} \end{aligned}$$

Total amount of money collected from the two stalls in June 2016 was \$17 475.

- (e) In July, the number of sticks of each type of satay sold per day is increased by 10%. The information is given by the matrix Q .

$$Q = \begin{array}{ccc|c} \text{Chicken} & \text{Mutton} & \text{Beef} & \\ \hline 440 & 330 & 220 & \text{Stall A} \\ 220 & 550 & 330 & \text{Stall B} \end{array}$$

Write down the matrix R such that $Q = SR$.

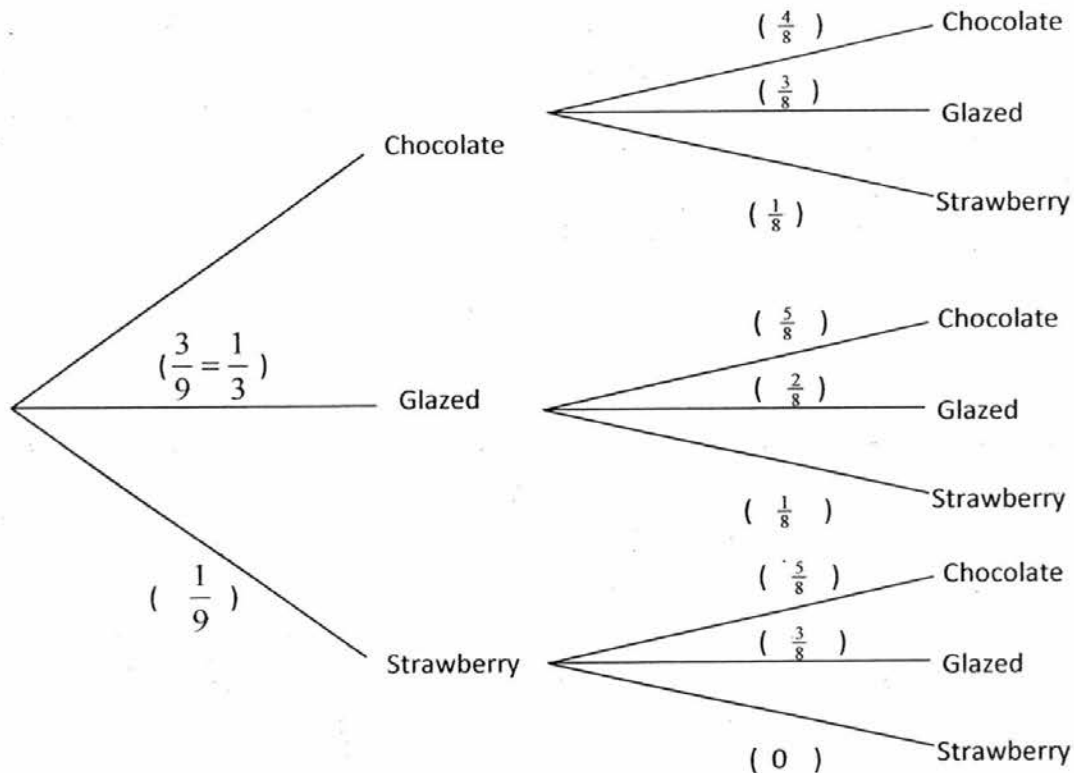
$$Q = SR$$

[1]

$$\begin{pmatrix} 440 & 330 & 220 \\ 220 & 550 & 330 \end{pmatrix} = \begin{pmatrix} 400 & 300 & 200 \\ 200 & 500 & 300 \end{pmatrix} \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}$$

$$R = \begin{pmatrix} 1.1 & 0 & 0 \\ 0 & 1.1 & 0 \\ 0 & 0 & 1.1 \end{pmatrix}$$

- 7 A box contains 5 Chocolate doughnuts, 3 Glazed doughnuts and 1 Strawberry doughnut.
- (a) Two doughnuts were taken out of the box at random, without replacement. Copy and complete the tree diagram to show this information.[3]



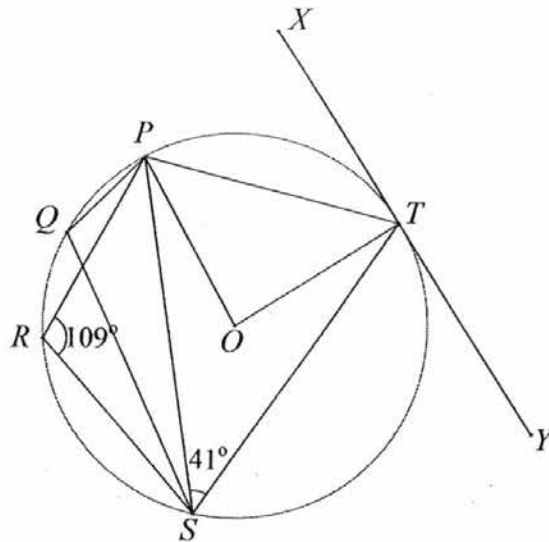
- (b) Find, as a fraction in its simplest form, the probability that
- (i) the two doughnuts are the same flavour, [2]

$$\begin{aligned}
 & \left(\frac{5}{9} \times \frac{4}{8} \right) + \left(\frac{3}{9} \times \frac{2}{8} \right) \\
 &= \frac{20}{72} + \frac{6}{72} \\
 &= \frac{13}{36}
 \end{aligned}$$

- (ii) at least one of the doughnuts is Chocolate. [3]

$$\begin{aligned}
 & \left(\frac{5}{9} \times \frac{4}{8} \right) + \left[2 \left(\frac{5}{9} \times \frac{3}{8} \right) + 2 \left(\frac{5}{9} \times \frac{1}{8} \right) \right] 1 - \left[\left(\frac{3}{9} \times \frac{2}{8} \right) + \left(\frac{1}{9} \times 0 \right) + 2 \left(\frac{3}{9} \times \frac{1}{8} \right) \right] \\
 &= \frac{20}{72} + \frac{40}{72} \qquad \qquad \qquad = 1 - \frac{12}{72} \\
 &= \frac{5}{6} \qquad \qquad \qquad = \frac{5}{6}
 \end{aligned}$$

- 8 In the diagram, the points P, Q, R, S and T lie on a circle, centre O . XTY is a tangent to the circle. Angle $PRS = 109^\circ$ and angle $PST = 41^\circ$.



- (a) Find, giving reasons for each answer,

(i) $\hat{PQS}$, [1]
 $\hat{PQS} = 109^\circ$ (angles in same segment)

(ii) $\hat{PTS}$, [1]
 $\hat{PTS} = 180^\circ - 109^\circ$ (angles in opp segment)
 $= 71^\circ$

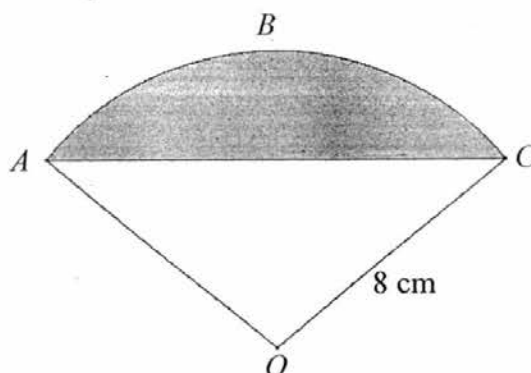
(iii) $\hat{YTS}$. [2]
 $\hat{OTS} = 71^\circ - 49^\circ$
 $= 22^\circ$
 $\hat{YTS} = 90^\circ - 22^\circ$ (rad $\perp$ tan)
 $= 68^\circ$

Alternate Solution

$\hat{SPT} = 180^\circ - 71^\circ - 41^\circ$ (angle sum of Δ)
 $\hat{YTS} = 68^\circ$ (angles in alt segment)

(iv) $\hat{OTP}$, [2]
 $\hat{POT} = 41^\circ \times 2$ ($\angle$ at center = $2\angle$ at cir)
 $= 82^\circ$
 $\hat{OTP} = \frac{180^\circ - 82^\circ}{2}$ (base angles, isos Δ)
 $= 49^\circ$

- (b) $OABC$ is a sector of a circle, centre O and radius 8 cm. The perimeter of the sector is 30 cm.



- (i) Show that angle $AOC = 1.75$ radians. [1]

$$2(8) + 8\hat{AOC} = 30$$

$$\hat{AOC} = 1.75 \text{ rad}$$

- (ii) Calculate the area of the shaded region. [3]

$$\text{area of } \triangle OAC = \frac{1}{2} \times 8^2 \times \sin 1.75$$

$$= 31.487 \text{ cm}^2$$

$$\text{Area of sector} = \frac{1}{2} \times 8^2 \times 1.75$$

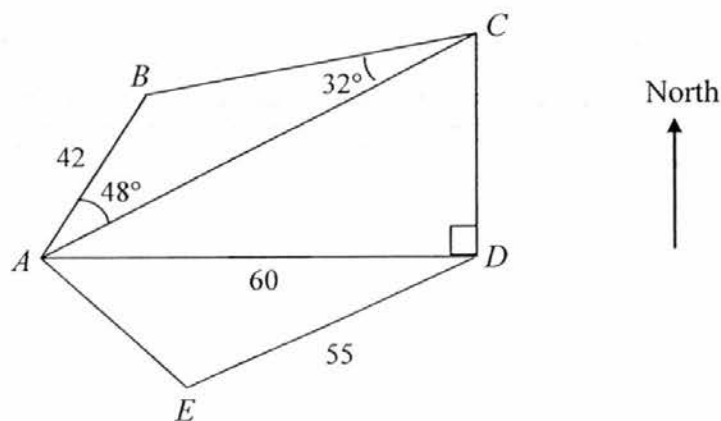
$$= 56 \text{ cm}^2$$

$$\text{Shaded area} = 56 - 31.487$$

$$= 24.513$$

$$= 24.5 \text{ cm}^2 \text{ (3 sf)}$$

9



The diagram shows a field, $ABCDE$, which is crossed by two paths, AC and AD . AD is perpendicular to CD . $AB = 42$ m, $AD = 60$ m, $DE = 55$ m, angle $BAC = 48^\circ$ and angle $ACB = 32^\circ$.

- (a) Show that $AC = 78.05$ m, correct to four significant figures. [2]

$$\angle ABC = 180 - (48 + 32) \\ = 100^\circ \text{ (sum of } \Delta)$$

$$\frac{AC}{\sin 100^\circ} = \frac{42}{\sin 32^\circ}$$

$$AC = \frac{42}{\sin 32^\circ} \times \sin 100^\circ \\ = 78.053 \\ \approx 78.05 \text{ cm}$$

- (b) Calculate CD . [2]

$$CD = \sqrt{78.053^2 - 60^2} \\ = 49.92 \\ \approx 49.9 \text{ cm}$$

- (c) A bird is at P , which is 8 m vertically above E . Calculate the angle of depression of D from P . Let angle of depression be θ [2]

$$\tan \theta = \frac{8}{55}$$

$$\theta = 8.27 \\ \approx 8.3^\circ$$

- (d) Given that the area of triangle ADE is 1300 m^2 , calculate angle ADE . [2]

$$\frac{1}{2} \times 60 \times 55 \times \sin \angle ADE = 1300$$

$$\sin \angle ADE = \frac{2 \times 1300}{60 \times 55}$$

$$\angle ADE = 51.98 \text{ or } 128.02 \\ \approx 52.0^\circ \text{ or } 128.0^\circ \text{ (NA)}$$

- (e) D is due east of A . Calculate the bearing of E from A . [3]

$$AE^2 = 60^2 + 55^2 - 2(60)(55) \cos 51.98^\circ \\ AE = \sqrt{60^2 + 55^2 - 2(60)(55) \cos 51.98^\circ} \\ = 50.59$$

$$\frac{\sin \angle EAD}{55} = \frac{\sin 51.98}{50.59}$$

$$\sin \angle EAD = \frac{\sin 51.98}{50.59} \times 55$$

$$\angle EAD = 58.92 \text{ or } 121.08 \text{ (NA)}$$

$$\text{Bearing of } E \text{ from } A = 90 + 58.92 \\ = 148.92 \\ \approx 148.9^\circ$$

10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = \frac{5x^2}{4} + \frac{60}{x} - 40$.

Some corresponding values of x and y are given in the following table.

x	1	1.5	2	3	3.5	4	4.5	5	6
y	p	2.81	-5	-8.75	-7.54	-5	-1.35	3.25	15

- (a) Find the value of p . [1]

$$y = \frac{5x^2}{4} + \frac{60}{x} - 40$$

$$p = \frac{5(1)^2}{4} + \frac{60}{1} - 40$$

$$= 21.25$$

- (b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x -axis for $1 \leq x \leq 6$.
Using a scale of 2 cm to represent 5 units, draw a vertical y -axis for $-10 \leq y \leq 25$.
On your axes, plot the points given in the table and join them with a smooth curve. [2]
Correct Scale + Label axes
7 Correct Points + Curve

- (c) Using your graph, find the range of values of x for which $\frac{5x^2}{4} + \frac{60}{x} - 40 < 0$. [3]
 $1.65 < x < 4.65$

- (d) By drawing a tangent, find the gradient of the curve at the point where $x = 4$. [2]
Draw tangent
 $m = 6.25$

- (e) Draw the tangent to the curve which the gradient is -10 . [2]
Write down the equation of this tangent.
Draw any tangent line with gradient -10

Draw tangent at/near $x = 2$ and $y = -10x + 15$

- (f) The line l intersects the curve $y = \frac{5x^2}{4} + \frac{60}{x} - 40$ at $x = 2$ and $x = 6$.

(i) Find the equation of l .

[2]

$(2, -5), (6, 15)$

$$m = \frac{15 - (-5)}{6 - 2}$$

$$= 5$$

$$y + 5 = 5(x - 2)$$

$$y = 5x - 15$$

It is given that $x = 2$ and $x = 6$ are solutions of the equation
 $5x^3 + Ax^2 + Bx + 240 = 0$.

(ii) By using your answer from (f)(i), find the value of A and of B .

[3]

$$\frac{5x^2}{4} + \frac{60}{x} - 40 = 5x - 15$$

$$\frac{5x^2}{4} - 5x - 25 + \frac{60}{x} = 0$$

$$5x^3 - 20x^2 - 100x + 240 = 0$$

$$A = -20,$$

$$B = -100$$

- 11 **Diagram I** shows a pencil before it is sharpened. It is made up of a piece of cylindrical carbon encased in wood. The length of the pencil is 19 cm.
Diagram II shows the cross-sectional area of the pencil. $ABCDEF$ is a regular hexagon with side 0.45 cm. The diameter of the carbon is 0.2 cm.

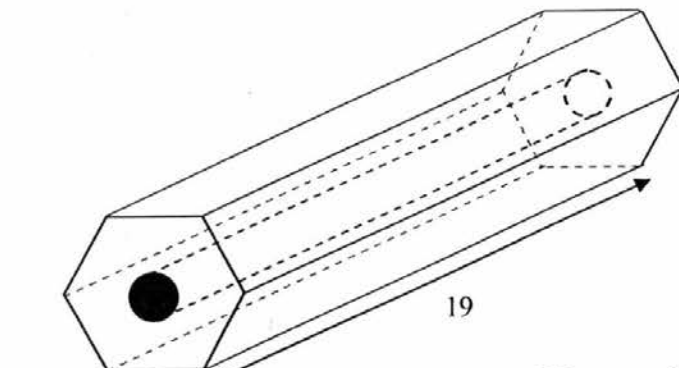


Diagram I
 (a) Find

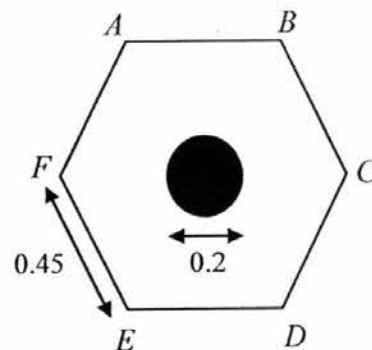


Diagram II

- (i) the interior angle of the regular hexagon $ABCDEF$, [2]

$$\text{Interior } \angle = (180 \times 4) \div 6 \text{ or } \text{Interior } \angle = 180 - \left(\frac{360}{6}\right)$$

$$= 120^\circ = 120^\circ$$

- (ii) CF , [1]

$$\angle ABF = \angle AFB$$

$$= 30^\circ \text{ (base } \angle \text{ in iso } \Delta)$$

$$\angle FBC = 90^\circ$$

$$\angle AFC = 60^\circ \text{ (int } \angle, AB \parallel CF)$$

$$\angle BFC = 60 - 30$$

$$= 30^\circ$$

$$\sin 30^\circ = \frac{0.45}{CF}$$

$$CF = \frac{0.45}{\sin 30^\circ} CF = 0.45 \times 2$$

$$= 0.9 \text{ cm} = 0.9 \text{ cm}$$

- (b) Show that $AE = 0.7794 \text{ cm}$. [2]

$$AE^2 = 0.45^2 + 0.45^2$$

$$- 2(0.45)(0.45)\cos 120^\circ$$

$$AE = \sqrt{0.6075}$$

$$AE = 0.77942$$

$$\approx 0.7794 \text{ cm}$$

- (c) Calculate the area of the regular hexagon $ABCDEF$.

[2]

$$\sin 60^\circ = \frac{h}{0.45}$$

$$h = 0.45 \sin 60^\circ$$

$$\begin{aligned} \text{Area } ABCDEF &= 2 \left[\frac{1}{2} \times (0.45 + 0.9) \times 0.45 \sin 60^\circ \right] = 2 \left[\frac{1}{2} (0.45)^2 \sin 120^\circ \right] = 6 \left[\frac{1}{2} (0.45)^2 \sin 60^\circ \right] \\ &= 0.5261 = 0.5261 = 0.5261 \\ &\approx 0.526 \text{ cm}^2 \approx 0.526 \text{ cm}^2 \approx 0.526 \text{ cm}^2 \end{aligned}$$

- (d) Calculate the volume of the carbon as a percentage of the volume of the pencil.

[2]

$$\begin{aligned} \text{Percentage} &= \frac{\pi(0.1)^2 \times 19}{0.5261 \times 19} \times 100\% \\ &= 5.971 \\ &\approx 5.97\% \end{aligned}$$

Diagram III shows ten of these pencils which just fit into a rectangular box which is open on one side.

Diagram IV shows ten of these pencils which just fit into a box whose cross-sectional area is an equilateral triangle which is open on one side.

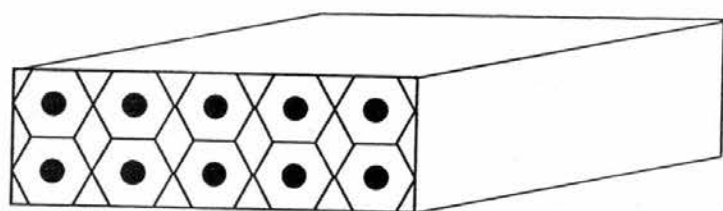


Diagram III

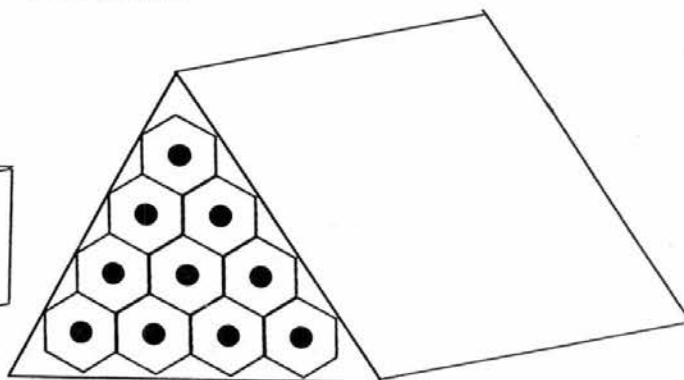


Diagram IV

- (e) The boxes are made of cardboard which cost \$10 per m^2 . Determine which box will be cheaper to produce for 1000 boxes. Justify your decision with calculations. [5]

Diagram III

$$\begin{aligned} \text{Length of III} &= 5 \times 0.9 \\ &= 4.5 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Width of III} &= 2 \times 0.7794 \\ &= 1.5588 \text{ cm} \end{aligned}$$

$$\begin{aligned} SA &= 2(4.5 \times 19) + 2(1.5588 \times 19) \\ &+ (4.5 \times 1.5588) \\ &= 237.249 \text{ cm}^2 \\ &= 2.37249 \times 10^{-2} m^2 \end{aligned}$$

$$\begin{aligned} \text{Cost} &= 2.37249 \times 10^{-2} \times 10 \times 1000 \\ &= 237.249 \\ &\approx \$237.25 \end{aligned}$$

Diagram IV

$$\begin{aligned} \text{Length of } \Delta &= 5 \times 0.7794 \\ &= 3.897 \text{ cm} \end{aligned}$$

$$SA = 3(3.897 \times 19)$$

$$\begin{aligned} &+ \frac{1}{2}(3.897)(3.897) \sin 60^\circ \\ &= 228.704 \text{ cm}^2 \\ &= 2.28704 \times 10^{-2} m^2 \end{aligned}$$

$$\begin{aligned} \text{Cost} &= 2.28704 \times 10^{-2} \times 10 \times 1000 \\ &= 228.704 \\ &\approx \$228.70 \end{aligned}$$

Design IV will be cheaper to produce for 1000 boxes