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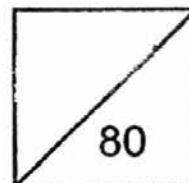
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INDEX NO: _____

Anglo-Chinese School (Independent)



PRELIMINARY EXAMINATION 2016 YEAR FOUR EXPRESS MATHEMATICS PAPER 1

4048/01

FRIDAY

29 JULY 2016

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The number of the marks for this paper is 80.

This document consists of 18 printed pages.

[Turn over

*Mathematical Formulae**Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer ALL the questions.

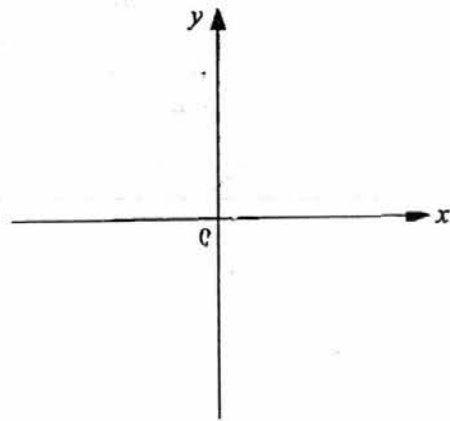
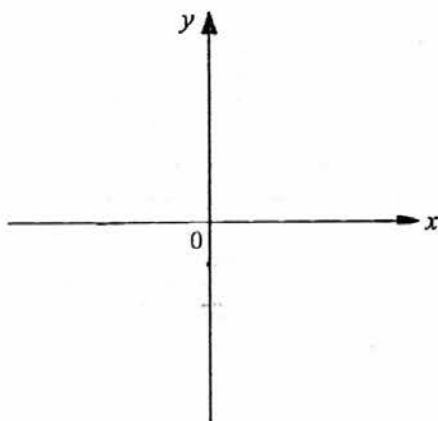
- 1 Solve the inequality $\frac{1}{3}(2-4x) < \frac{1}{2}(9x+1)$.

Answer: [2]

- 2 On the axes given, sketch the following graphs, indicating the x and y intercepts where relevant.

(i) $y = -2 - x^2$

(ii) $y = 3^x$ [2]



[Turn over

- 3 At the recent Great Singapore Sale, a store assistant accidentally raised the price of an electric cooker by 10% instead of lowering it by 15%. As a result, a customer who bought this electric cooker was overcharged by \$ 140. Find the original price of the electric cooker.

Answer:..... [2]

- 4 In a library, the ratio of the number of books to magazines is 3 : 2. After a donation drive to help an overseas primary school, many books and magazines were given away. If the number of books is reduced by 40% and the number of magazines is reduced by 20%, find the new ratio of the number of books to the number of magazines.

Answer: [2]

- 5 Factorize completely $3a^{n+1} - 14a^n - 5a^{n-1}$.

Answer: [2]

- 6 The kinetic energy of a particle, y Kilojoules (KJ) is directly proportional to the square of its speed x (m/s). When the particle moves at a certain speed, the kinetic energy is 20 KJ. If the speed is tripled, find the kinetic energy, in KJ.

Answer: KJ [2]

[Turn over

7 Solve the equation $\frac{2x}{4-3x} + 4 = \frac{1}{3x-4}$.

Answer: $x = \dots\dots\dots$ [3]

8 Simplify $\left(\frac{ab-ac}{a^2-b^2} \div \frac{c-b}{a+b}\right) + \frac{3a}{a-b}$.

Answer: $\dots\dots\dots$ [3]

- 9 Simplify $\left(\frac{x^9}{125}\right)^{\frac{-1}{3}} \div (256x^8)^{\frac{1}{4}}$, leaving your answer in positive index.

Answer: [3]

- 10 A rectangular football field measuring 156 m by 96 m is to be enclosed by pieces of fences of equal length.

- (i) Find the largest possible length of each piece of fence so that the number of pieces of fences used for each side of the field is a whole number.

Answer: (i)..... [2]

- (ii) Calculate the total pieces of fences used.

Answer: (ii)..... [1]

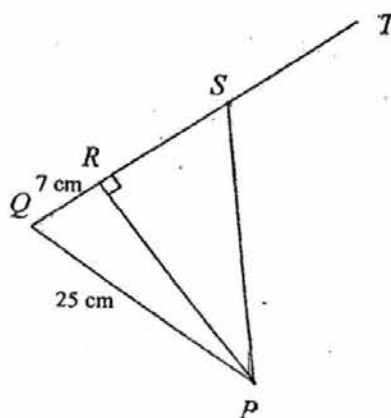
[Turn over

- 11 Express $\frac{3}{4 - \frac{3}{\left(\frac{x+1}{x}\right)}} - \frac{5}{x+4}$ as a single fraction.

Answer: [3]

- 12 In the diagram, $PQ = 25$ cm, $QR = 7$ cm and $\angle SRP = 90^\circ$. Given that $\tan \angle TSP = -\frac{4}{3}$, calculate

(i) SR ,



Answer: (i) [2]

- (ii) $\cos \angle TSP$, leaving your answer in fraction in its simplest form.

Answer: (ii) [1]

13

The first four terms in a sequence of numbers $T_1, T_2, T_3, T_4, \dots$, are given as follows:

$$T_1 = \frac{2}{1} - \frac{3}{2}$$

$$T_2 = \frac{3}{2} - \frac{4}{2^2}$$

$$T_3 = \frac{4}{2^2} - \frac{5}{2^3}$$

$$T_4 = \frac{5}{2^3} - \frac{6}{2^4}$$

$$T_n =$$

- (a) Write down the n^{th} line and show that it can be expressed as $T_n = \frac{n}{2^n}$

Answer: (a) [2]

- (b) Hence, evaluate the following sum and leave your answer as a fraction.

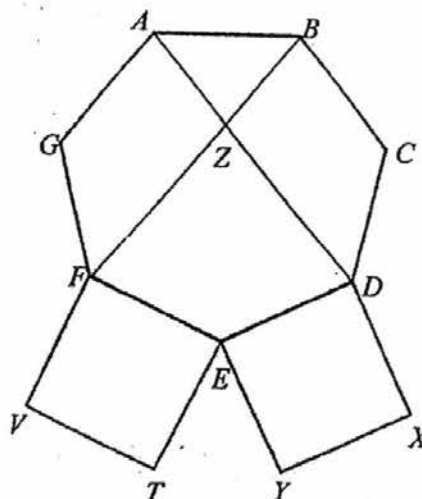
$$T_1 + T_2 + T_3 + \dots + T_{11}.$$

Answer: (b) [2]

[Turn over

- 14 (a) The figure $ABCDEFG$ shown below is a regular heptagon.

Calculate angle BCD .



Answer: (a) $\angle BCD = \dots\dots\dots [1]$

- (b) It is given that $FETV$ and $EDXY$ are squares. Line AD meets line BF at Z .

Find

- (i) angle YET ,

Answer : (b) (i) $\angle YET = \dots\dots\dots [1]$

- (ii) angle BZD .

Answer : (b) (ii) $\angle BZD = \dots\dots\dots [2]$

- 15 In a secondary school, E represents all the students, $P = \{\text{students who listen to Pop music}\}$, $R = \{\text{students who listen to Rap music}\}$, $n(E) = 1500$, $n(P) = 1000$ and $n(R) = 700$.

(a) Find the largest possible value of $n(P \cup R)'$.

Answer: (a) [1]

(b) Describe $R \cap P' \neq \emptyset$.

Answer: (b) [1]

(c) Find $n(P \cap R')$ if $n(R \cap P') = 200$.

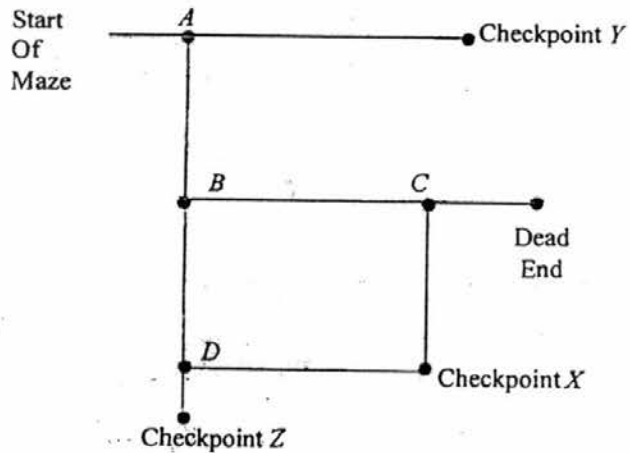
Answer: (c) [2]

(d) Express in set notation "All students who listens to Rap music also listen to Pop music".

Answer: (d) [1]

[Turn over

- 16 Sean is participating in the game 'Maze Runner' during his school's carnival. Participants have to find Checkpoint X in the shortest time to win a prize. There are no prizes awarded at Checkpoint Y and Z . The diagram shows four junctions A , B , C and D in the maze. Once Sean runs pass a junction, he is not able to make a turnaround. The probability that Sean goes straight at every junction is $\frac{2}{7}$.



- (a) Find the probability that Sean hits the dead end.

Answer: (a) [2]

- (b) Find the probability that Sean wins a prize.

Answer: (b) [3]

- 17 The following diagram shows a right pyramid which is cut horizontally into three portions A , B and C , such that the lengths of the slant edges are 1 cm, 2 cm and 3 cm respectively. Find the ratio volume of A : volume of B : volume of C .

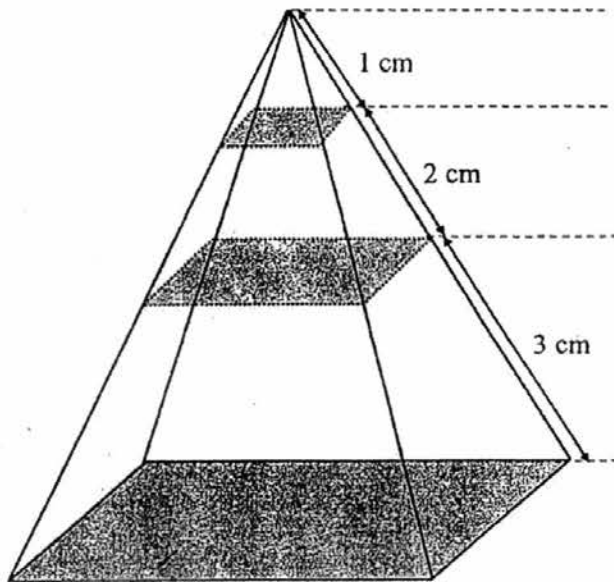


Diagram is not drawn to scale

Answer:[5]

[Turn over

- 18 The scale drawing in the answer space below shows three towns, A , B and C where C is due north of B .

(a) Measure and write down the bearing of B from A .

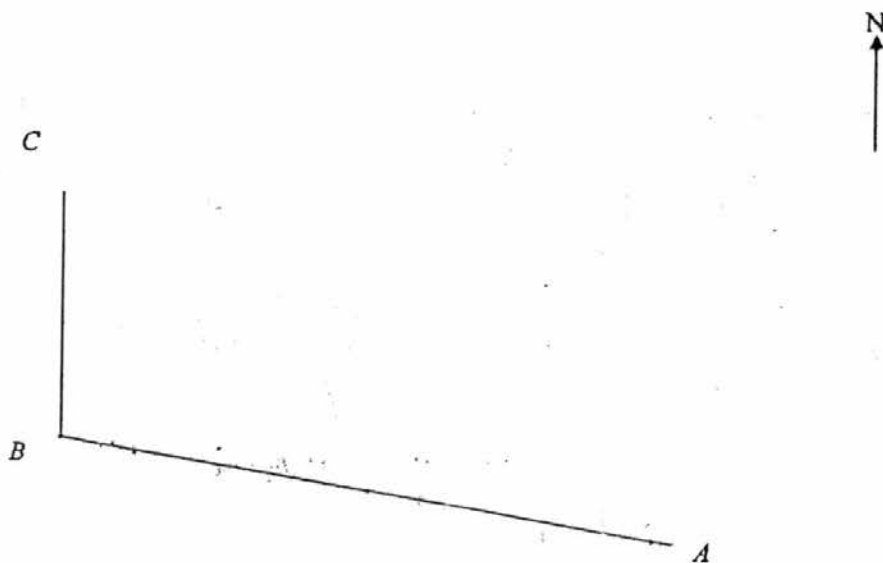
Answer: (a) [1]

(b) Construct

(i) the perpendicular bisector of AB , [1]

(ii) the bisector of angle ABC . [1]

Answer (b)



- (c) John wants to set up a bakery shop P such that P is equidistant from CB and AB .

The bakery shop P is also equidistant from Town A and Town B .

Mark and label " P " in the diagram above. Hence, write down the actual distance in meters of P from B if the scale is 1: 10000.

Answer : (c) $PB = \dots\dots\dots m$ [1]

- (d) A satellite station, S , is to be positioned in such a way that it is equidistant from the 3 towns. Mark out S on the diagram and find the actual distance of S from A . Show any relevant construction clearly.

Answer : (d) $AS = \dots\dots\dots m$ [2]

- 19 $OPQR$ is a parallelogram such that $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and P is the point $(3, 2)$

Express $\overrightarrow{RP}$ as a column vector.

Answer: [2]

The point J lies on RP produced such that $\overrightarrow{PJ} = m\overrightarrow{RP}$.

- (i) Show that $\overrightarrow{OJ} = \begin{pmatrix} 3+m \\ 2-2m \end{pmatrix}$

Answer: (i) Show on the space provided [2]

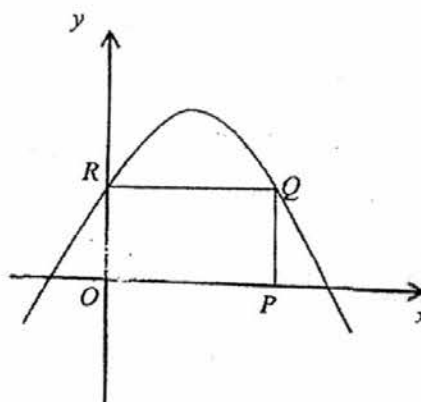
- (ii) If $|\overrightarrow{OJ}| = \sqrt{52}$, find the value(s) of m .

Answer: (ii) $m = \dots\dots\dots$ [3]

[Turn over

20 The figure shows the graph of $y = -2x^2 + bx + c$.

- (a) Find the area of the rectangle $OPQR$, leaving your answer in terms of b and c .



Answer: (a) [3]

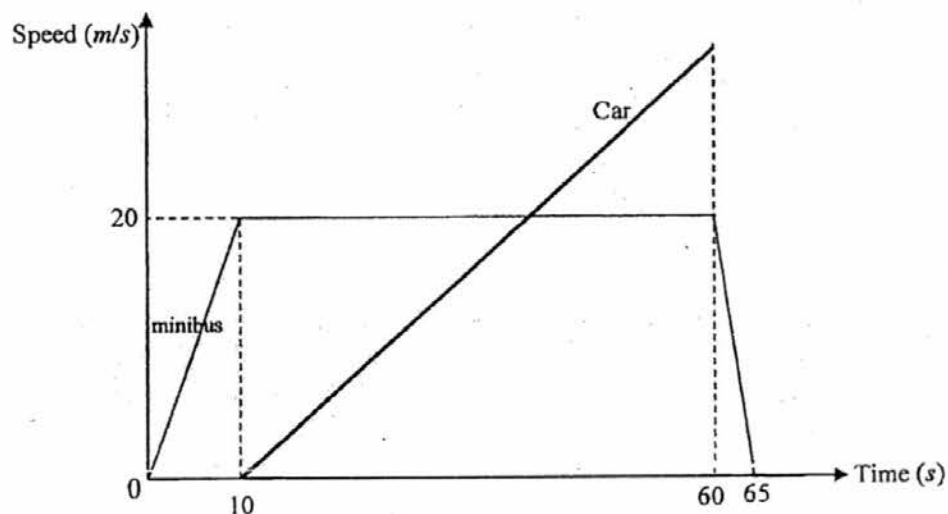
- (b) If the curve passes through $(-1, 2)$ and $(2, 5)$, find the value of b and of c .

Answer: (b) $b = \dots\dots\dots$, $c = \dots\dots\dots$ [4]

- 21 The diagram below shows the speed-time graph of a minibus and a car.

The minibus starts from rest and accelerates uniformly for 10 seconds until it reaches a speed of 20 m/s. It then travels uniformly at this speed for the next 50 seconds. It finally decelerates uniformly until it comes to rest after travelling a further 5 seconds.

The car started its journey 10 seconds later from the same starting point as the minibus. It accelerated uniformly until it overtakes the minibus at $t = 60$ s.



- (a) Find the acceleration of the minibus during the first 10 seconds.

Answer (a)..... m/s² [1]

- (b) Find the speed of the minibus at $t = 63$ s.

Answer (b)..... m/s [2]

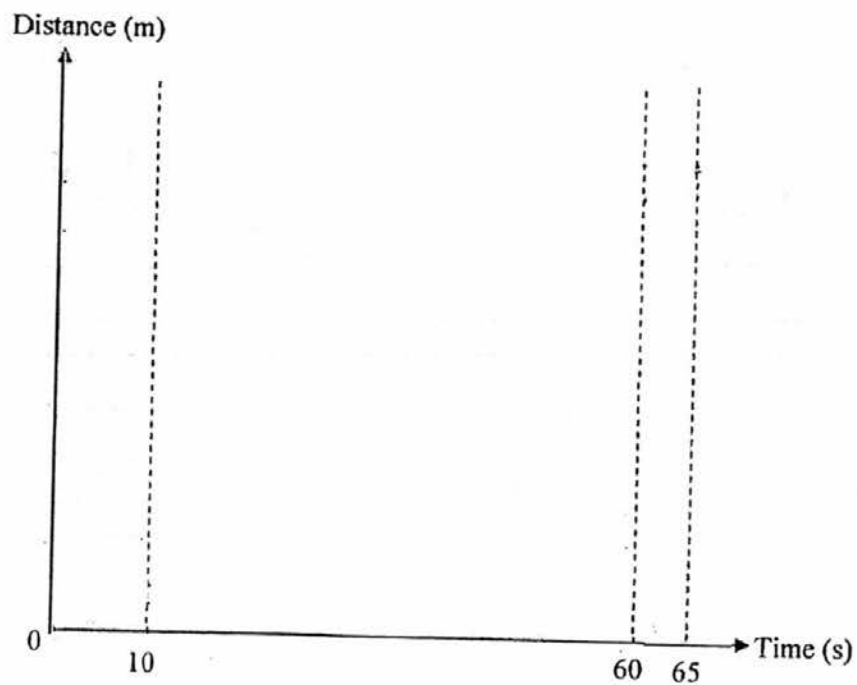
[Turn over

Q21 Continue

- (e) Calculate the speed of the car at $t = 60\text{s}$.

Answer (c) m/s [2]

- (d) On the graph below, sketch the distance – time graph for the minibus.



[2]

End of Paper

E Maths P1

1

Multiply by 6... $2(2 - 4x) < 3(9x + 1)$

$$4 - 8x < 27x + 3$$

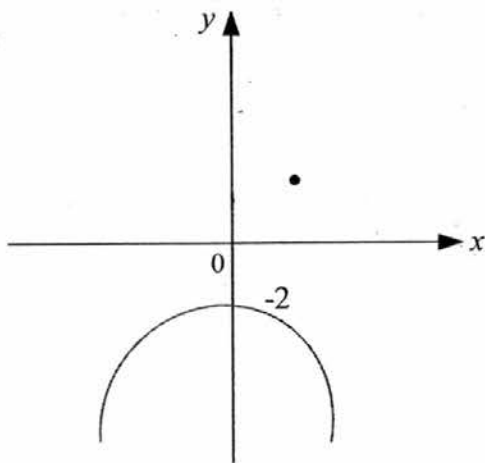
$$1 < 35x$$

$$\frac{1}{35} < x$$

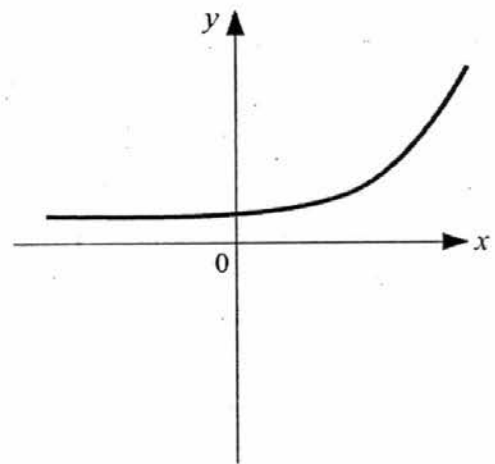
$$x > \frac{1}{35}$$

2

(i) $y = -2 - x^2$



(ii) $y = 3^x$



[Turn over

3

Let original price be \$x

$$1.10x - 0.85x = 140$$

$$0.25x = 140$$

$$x = 560$$

- 4 Let original no. of books be x. Original no. of magazine be y

$$\frac{x}{y} = \frac{3}{2}$$

$$\text{New ratio } \frac{0.6x}{0.8y} = \frac{6}{8} \times \frac{x}{y}$$

$$= \frac{6}{8} \times \frac{3}{2}$$

$$= \frac{9}{8}$$

$$= 9:8$$

5

$$3a^{n+1} - 14a^n - 5a^{n-1}$$

$$= a^{n-1}(3a^2 - 14a - 5)$$

$$= a^{n-1}(a-5)(3a+1)$$

6

$$y = kx^2$$

$$20 = kx^2$$

$$k = \frac{20}{x^2}$$

$$\text{So } y_1 = kx_1^2$$

$$= \frac{20}{x^2}(3x)^2$$

$$= 180$$

7

$$\frac{2x}{4-3x} + 4 = \frac{1}{3x-4}$$

$$\frac{2x}{4-3x} - \frac{1}{3x-4} = -4$$

$$\frac{2x}{4-3x} + \frac{1}{4-3x} = -4$$

$$\frac{2x+1}{4-3x} = -4$$

$$2x+1 = -4(4-3x)$$

$$2x+1 = -16+12x$$

$$17 = 10x$$

$$x = 1\frac{7}{10}$$

8

$$\begin{aligned}
 & \left(\frac{ab-ac}{a^2-b^2} \div \frac{c-b}{a+b} \right) + \frac{3a}{a-b} \\
 &= \left(\frac{a(b-c)}{(a+b)(a-b)} \times \frac{a+b}{c-b} \right) + \frac{3a}{a-b} \\
 &= \frac{-a}{a-b} + \frac{3a}{a-b} \\
 &= \frac{2a}{a-b}
 \end{aligned}$$

9

$$\begin{aligned}
 & \left(\frac{x^9}{125} \right)^{-\frac{1}{3}} \div (256x^8)^{\frac{1}{4}} \\
 &= \left(\frac{x^9}{5^3} \right)^{-\frac{1}{3}} \div (2x)^{8 \times \frac{1}{4}} \\
 &= \frac{x^{-3}}{5^{-1}} \div (2x)^2 \\
 &= \frac{5}{x^3} \times \frac{1}{4x^2} \\
 &= \frac{5}{4x^5}
 \end{aligned}$$

10

(i)

2	156,96
2	78,48
3	39,24
	13,8

Largest Possible Length HCF = $2 \times 2 \times 3 = 12$ m

(ii)

$$2\left(\frac{156}{12}\right) + 2\left(\frac{96}{12}\right) = 42$$

[Turn over]

11

$$\begin{aligned}
 & \frac{3}{4 - \frac{3}{\left(\frac{x+1}{x}\right)}} - \frac{5}{x+4} \\
 &= \frac{3}{4 - \frac{3x}{x+1}} - \frac{5}{x+4} \\
 &= \frac{3}{\frac{4x+4-3x}{x+1}} - \frac{5}{x+4} \\
 &= \frac{3x+3}{x+4} - \frac{5}{x+4} \\
 &= \frac{3x-2}{x+4}
 \end{aligned}$$

12

(i) SR ,

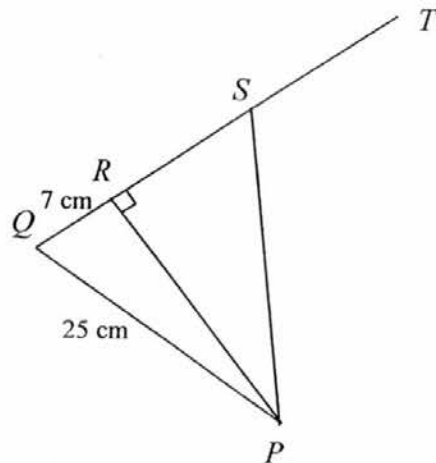
$$\begin{aligned}
 RP &= \sqrt{25^2 - 7^2} \\
 &= 24\text{cm}
 \end{aligned}$$

$$\tan RSP = \frac{4}{3}$$

$$\frac{RP}{SR} = \frac{4}{3}$$

$$\frac{24}{SR} = \frac{4}{3}$$

$$SR = 18\text{cm}$$

(i) $\cos \angle TSP$, leaving your answer in fraction in its simplest form.

$$\begin{aligned}
 SP &= \sqrt{18^2 + 24^2} \\
 &= 30\text{cm}
 \end{aligned}$$

$$\therefore \cos TSP = -\cos RSP$$

$$= \frac{-18}{30} = -\frac{3}{5}$$

13

(a)

$$\begin{aligned}
 T_n &= \frac{n+1}{2^{n-1}} - \frac{n+2}{2^n} \\
 &= \frac{n+1}{2^n} - \frac{n+2}{2^n} \\
 &= \frac{2n+2}{2^n} - \left(\frac{n+2}{2^n}\right) \\
 &= \frac{2n+2-n-2}{2^n} \\
 &= \frac{n}{2^n}
 \end{aligned}$$

(b)

$$\begin{aligned}
 &\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \dots + \frac{11}{2^{11}} \\
 \frac{2}{1} - \frac{3}{2} + \frac{3}{2} - \frac{4}{2^2} + \frac{4}{2^2} - \frac{5}{2^3} + \frac{5}{2^3} - \frac{6}{2^4} + \dots - \frac{12}{2^{10}} + \frac{13}{2^{11}} \\
 &= 2 - \frac{13}{2^{11}} \\
 &= \frac{4083}{2048}
 \end{aligned}$$

14

(a)

$$n = 7$$

$$\begin{aligned}
 BCD &= \frac{(7-2) \times 180}{7} \\
 &= 128.6^\circ
 \end{aligned}$$

(b) It is given that $FETV$ and $EDXY$ are squares. Line AD meets line BF at Z .

Find

(i) angle YET ,

$$\begin{aligned}
 YET &= 360 - 128.57 - 2(90) \\
 &= 51.4^\circ
 \end{aligned}$$

(ii) angle BZD .

Let angle $BAZ = CDZ = x$

$$2x + 2(128.57) = 360 \text{-----} M1$$

$$x = 51.43$$

$$\text{angle } ZDE = 128.57 - 51.43 = 77.14$$

$$\text{angle } FZD = 360 - 128.57 - 2(77.14) = 77.15$$

$$\text{angle } BZD = 180 - 77.15 = 102.85 \text{-----} A1$$

15 (a)

$$\text{Largest value} = 1500 - 1000 = 500$$

(b) Describe $R \cap P' \neq \emptyset$.

There are students who listen to Rap music but don't listen to Pop music

(c)

$$\text{Let } n(P \cap R) = x$$

$$700 - x = 200$$

$$x = 500$$

$$\text{Hence } n(P \cap R') = 1000 - x$$

$$= 1000 - 500$$

$$= 500$$

(d)

$$R \subset P$$

$$R \subseteq P$$

$$R \cap P' = \emptyset$$

16

(a)

$$= \frac{5}{7} \times \frac{5}{7} \times \frac{2}{7}$$

$$= \frac{50}{343}$$

(b)

$$= \left(\frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \right) + \left(\frac{5}{7} \times \frac{2}{7} \times \frac{5}{7} \right)$$

$$= \frac{175}{343} \text{ or } \frac{25}{49}$$

17

$$\frac{V_A}{V_A + V_B} = \left(\frac{1}{3}\right)^3$$

$$\frac{V_A}{V_A + V_B} = \frac{1}{27}$$

$$\frac{V_A}{V_B} = \frac{1}{26}$$

$$\frac{V_A}{V_A + V_B + V_C} = \left(\frac{1}{6}\right)^3$$

$$\frac{V_A}{27V_A + V_C} = \frac{1}{216}$$

$$216V_A = 27V_A + V_C$$

$$189V_A = V_C$$

$$\frac{V_A}{V_C} = \frac{1}{189}$$

$$V_A : V_B : V_C = 1 : 26 : 189$$

- 18 The scale drawing in the answer space below shows three towns, A , B and C where C is due north of B .

(a) Measure and write down the bearing of B from A .

280 Degrees

(b)

(c)

$$PB = 670 \text{ m} \quad [1]$$

(d) $AS = 510 \text{ m}$ [2]

19

$$\begin{aligned}\vec{RP} &= \vec{RO} + \vec{OP} \\ &= \vec{QP} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -2 \\ -4 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ -2 \end{pmatrix}\end{aligned}$$

(i)

$$\vec{PJ} = m \vec{RP}$$

$$OJ - OP = m \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$OJ = \begin{pmatrix} m \\ -2m \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$OJ = \begin{pmatrix} 3+m \\ 2-2m \end{pmatrix}$$

$$\sqrt{(3+m)^2 + (2-2m)^2} = \sqrt{52}$$

$$9 + 6m + m^2 + 4 - 8m + 4m^2 = 52$$

(ii) $15m^2 - 2m - 39 = 0$

$$(5m+13)(m-3) = 0$$

$$m = \frac{-13}{5}, m = 3$$

At y-axis, $x=0$ so $y=c$

$R(0,c)$

At Q, $y=c$

$$c = -2x^2 + bx + c$$

$$0 = x(-2x + b)$$

$$x = 0 \text{ or } -2x + b = 0$$

$$x = 0 \text{ or } x = \frac{b}{2}$$

$$Q\left(\frac{b}{2}, c\right)$$

$$\text{Area} = (OR) \times (RQ)$$

$$= c \times \frac{b}{2}$$

$$= \frac{bc}{2}$$

(b)

Use $(-1, 2)$

$$2 = -2(-1)^2 + b(-1) + c$$

$$4 = -b + c$$

Use $(2, 5)$

$$5 = -2(2)^2 + b(2) + c$$

$$13 = 2b + c$$

Solve simult eqn

$$13 = 2b + (4 + b)$$

$$b = 3$$

$$c = 7$$

21

- (a) Find the acceleration of the minibus during the first 10 seconds.

$$acc = \frac{20}{10} = 2 \text{ m/s}^2$$

- (b) Find the speed of the minibus at $t = 63\text{s}$.

$$\text{equate gradient } \frac{20-0}{60-65} = \frac{v-0}{63-65}$$

$$\frac{20}{-5} = \frac{v}{-2}$$

$$v = 8 \text{ m/s}$$

Q21 Continue

- (c) Calculate the speed of the car at $t = 60\text{s}$.

$$\begin{aligned} \text{dist of minibus} &= \frac{1}{2}(50+60) \times 20 \\ &= 1100 \text{ m} \end{aligned}$$

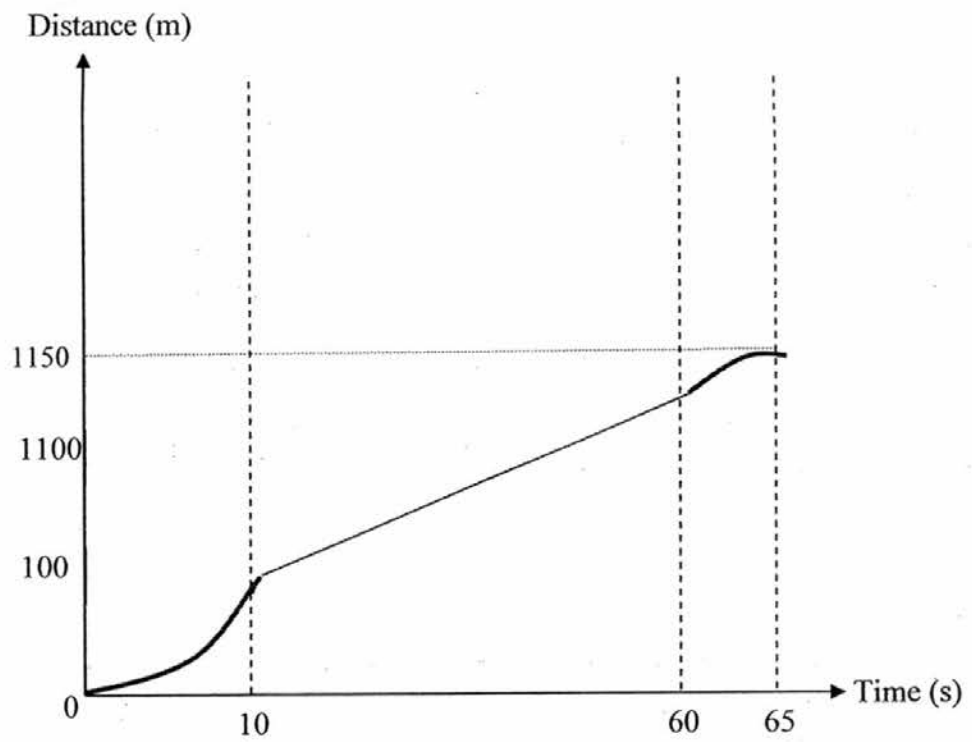
$$\begin{aligned} \text{dist of car} &= \frac{1}{2}(50)v \\ &= 25v \end{aligned}$$

$$25v = 1100$$

$$v = 44 \text{ m/s}$$

Answer (c) m/s [2]

- (d) On the graph below, sketch the distance – time graph for the minibus.



End of Paper

- 18 The scale drawing in the answer space below shows three towns, A , B and C where C is due north of B .

(a) Measure and write down the bearing of B from A .

Answer: (a) 280 [1]

(b) Construct

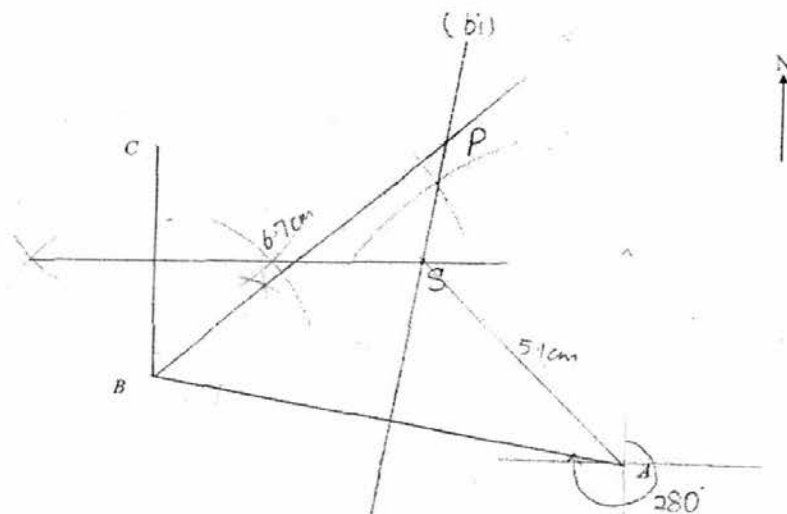
(i) the perpendicular bisector of AB ,

[1]

(ii) the bisector of angle ABC .

[1]

Answer (b)



- (c) John wants to set up a bakery shop P such that P is equidistant from CB and AB . The bakery shop P is also equidistant from Town A and Town B . Mark and label " P " in the diagram above. Hence, write down the actual distance in meters of P from B if the scale is 1:10000. 670 m

- (d) A satellite station, S , is to be positioned in such a way that it is equidistant from the 2 towns. Mark out S on the diagram and find the actual distance of S from A . Show any relevant construction clearly. 510 m

Answer: (c) $PB = \underline{670}$ m [1]
(d) $AS = \underline{510}$ m [2]

Anglo-Chinese School (Independent)



PRELIMINARY EXAMINATION 2016 YEAR FOUR EXPRESS MATHEMATICS PAPER 2

4048/02

Wednesday

3 August 2016

2 hours 30 minutes

Additional Materials:

Answer Paper (10 sheets)

Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your index number on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of 11 printed pages and 1 blank page.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

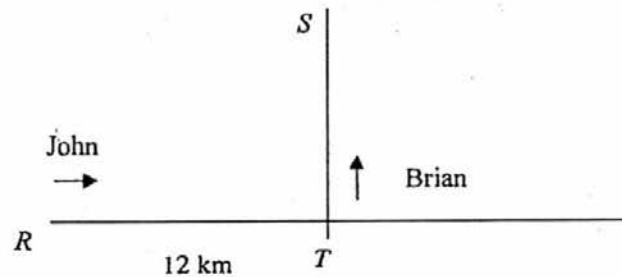
$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

- 1
- (a) Evaluate $25.2^2 - 3\sqrt{\frac{232.54 - 15.25^2}{7}}$, giving your answer correct to 2 significant figures. [1]
- (b) Factorise completely $4x^2 - 4x - 9y^2 + 1$. [2]
- (c) Solve the equation $\left(\frac{5^x}{2}\right)^{-3} = \frac{200}{\sqrt{5^x}}$. [3]
- (d) What must be added to $4x^2 - 3xy$ to make it a perfect square? [2]
- (e) Given that $3a - 2b = 3b + 5a$, find the value of $\sqrt{\frac{13b^2}{a^2 - 3b^2}}$. [3]

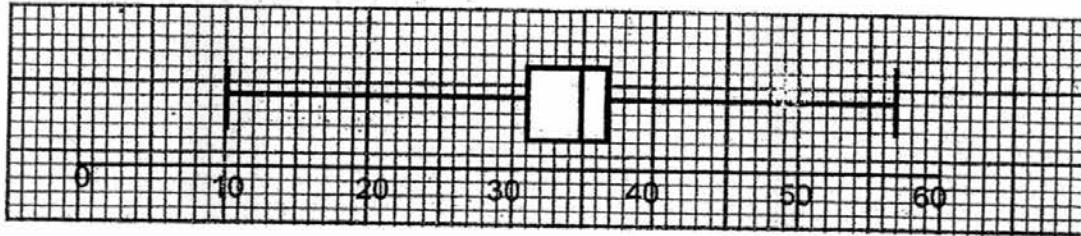
2



The diagram above shows a road junction at T where T is 12 km due east of R . During a morning walk, John starts from his home at R and walks towards T at a constant speed of 4 km/h. At the same time, his friend Brian starts from T and cycles due north towards S at a constant speed of 8 km/h.

- (a) Write down, in terms of t (where $t < 3$),
- (i) John's distance from T after t hours, Brian's [1]
- (ii) distance from T after t hours. [1]
- (b) Given that after t hours, John and Brian are x km apart, show that
- $$x^2 = 80t^2 - 96t + 144. \quad [2]$$
- (c) Find the time taken in hours for them to be $\sqrt{272}$ km apart. [3]
- Hence find John's distance from T at this time.

- 3 (a) A group of 240 students from School A took part in a Science quiz. The diagram below is the box-and-whiskers plot for the times taken by them to complete the quiz.



- (i) 25% of the students took more than p minutes. State the value of p . [1]

240 students from School B also took part in the same quiz. Their median time was 32 minutes while their interquartile range was 10 minutes. Below are two statements comparing students from School A and School B.

Statement	Agree/Disagree	Reason
The students in School A were faster.		
The performance of students in School B was more consistent.		

- (ii) Copy the table and complete the blanks. For each statement, indicate whether you agree or disagree and give a reason for your answer. [2]

- (b) Anil wanted to buy a pair of sports shoes. He looked at 11 different types and compared their prices. The prices are shown in the stem- and- leaf diagram below.

12	5 6
13	3 4 5 5 9
14	0 6 7 8

Key 13 | 4 means \$134

- (i) Find the median. [1]

- (ii) Find the standard deviation of the prices. [2]

- (iii) The salesman attending to Anil told him that if he bought any pair of shoes the next day, the price of each pair will go up by \$8.

Explain what effect, if any, will there be on the median price and the standard deviation of the prices. [2]

- 4 A confectionary shop sells two different gift boxes, *Deluxe* and *Tasty Treats*, each comprising chocolate bars, cupcakes and packets of biscuits. The contents of each box are as shown below.

Gift Box	Number of chocolate bars	Number of cupcakes	Number of packets of biscuits
<i>Deluxe</i>	3	5	2
<i>Tasty Treats</i>	5	4	3

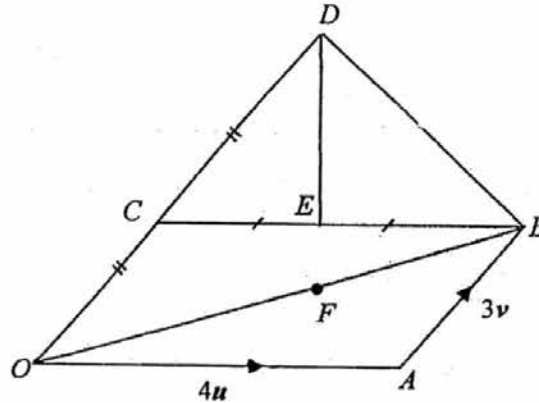
The above information can be represented by the matrix $P = \begin{pmatrix} 3 & 5 & 2 \\ 5 & 4 & 3 \end{pmatrix}$

The cost price and the selling price of each item are as shown.

	Cost price	Selling price
Chocolate bar	\$3	\$4
Cupcake	\$2.80	\$4.20
Packet of biscuits	\$2	\$3.80

- (a) (i) Madam Marina bought 12 boxes of *Deluxe* and 20 boxes of *Tasty Treats*. Given that $R = \begin{pmatrix} 12 & 20 \end{pmatrix}$, find the matrix S if $S=RP$. [2]
- (ii) Describe what is represented by the elements of S . [1]
- (b) (i) State the matrix Q such that the elements of SQ give the total cost price and the total selling price of the items used in making the gift boxes. [1]
- (ii) Evaluate SQ . [2]
- (c) Given that $SQ \begin{pmatrix} a \\ b \end{pmatrix}$ gives the total profit made, state the value of a and b and hence find the total profit. [2]

- 5 $OABC$ is a parallelogram and E is the mid-point of BC ; OC is produced to D such that $OC = CD$. F is a point on OB such that $OB : FB = 3 : 1$.



- (a) Given that $\vec{OA} = 4\mathbf{u}$ and $\vec{AB} = 3\mathbf{v}$, express, in terms of $\mathbf{u}$ and/or $\mathbf{v}$, (i) $\vec{OF}$, $\frac{8}{3}\mathbf{u} + 2\mathbf{v}$ [1]
 (ii) $\vec{BD}$, [1]
 (iii) $\vec{DE}$. [1]
- (b) Show that D , E and F are collinear points. [2]
- (c) Find the value of
 (i) $\frac{\text{Area of } \triangle DEB}{\text{Area of } \triangle DFB}$, [1]
 (ii) $\frac{\text{Area of } \triangle DEB}{\text{Area of parallelogram } OABC}$ [2]

- 6 (a) Mr David received his annual income tax form for the Year of Assessment 2015 and the rates for the year are as follows.

Chargeable Income	Rate(%)	Gross Tax (Payable)
First \$20 000	0	0
Next \$10 000	3.50	350
First \$30 000	-	350
Next \$10 000	5.50	550
First \$40 000	-	900
Next \$40 000	8.50	3 400
First \$80 000	-	4 300
Next \$80 000	14	11 200
First \$160 000	-	15 500
Next \$160 000	17	27 200
First \$320 000	-	42 700
Above \$320 000	20	

He was entitled to a personal relief of \$15000 and child relief of \$2500 for each of his two children. His CPF contributions of \$14800 were not taxed. If his income tax was \$12 700, calculate

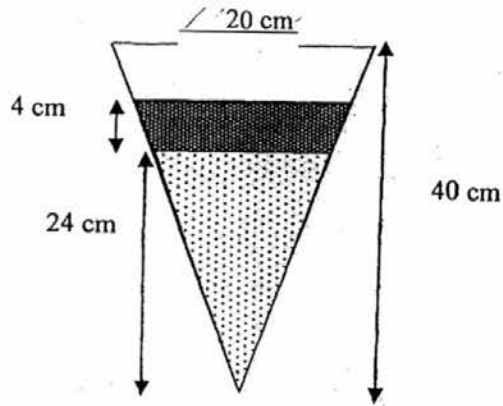
- (i) his chargeable income for the Year of Assessment 2015, [3]
- (ii) his annual income in 2015. [2]
- (b) Mr David changed his savings of S\$300 000 into Australian dollars and saved the money in an Australian bank. The bank offered an interest rate of 2.5% per annum compounded quarterly if the money was invested for 3 years.

—

Calculate, to the nearest dollar,

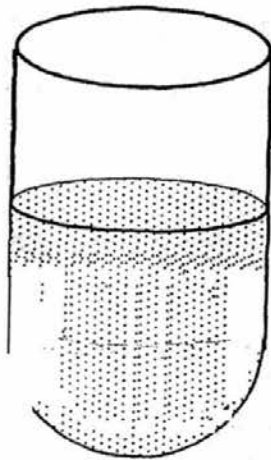
- (i) the principal amount of his savings in Australian dollars (AUD) if the exchange rate was AUD 1 = S\$1.325, [1]
- (ii) the amount of money he would receive after 3 years in Singapore currency if the currency exchange rate was S\$1 = AUD 0.745. [3]

- 7 (a) Container A is a cone with diameter 20 cm and height 40 cm. It contains water to a height of 24 cm from the vertex of the container. A layer of oil of height 4 cm floats on the water surface. The water is drained from a small opening at the vertex of the cone. The oil remains in the cone.



Container A

- (i) Given that the water was drained at the rate of $1.25 \text{ cm}^3/\text{s}$, find the time taken to drain all the water in container A . Give your answer to the nearest minute. [3]
- (ii) Find the height of the oil left behind in container A . [5]
- (b) The water that was drained from container A was poured into container B shown below. It is made up of a cylinder and a hemispherical bowl of radius 3 cm.

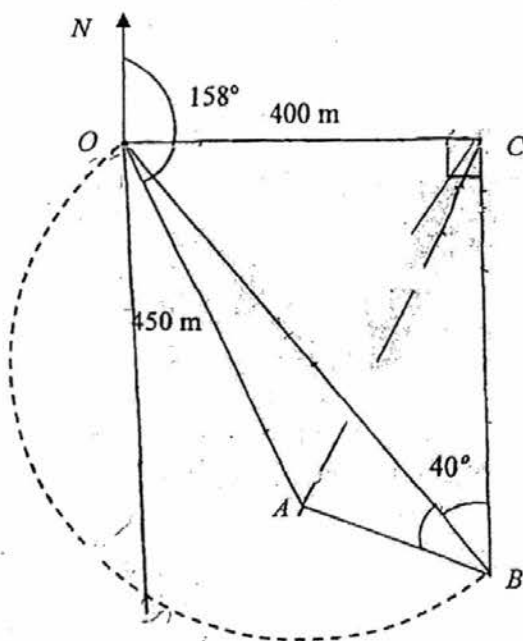


Container B

Find the height of the water in container B .

[3]

- 8 O, A, B and C are points on a level ground as shown in the figure below. A is on a bearing of 158° from O , C is due east of O and due north of B . It is given that $OC = 400$ m, $OA = 450$ m, $\angle OBC = 40^\circ$ and $\angle OBA = 30^\circ$.



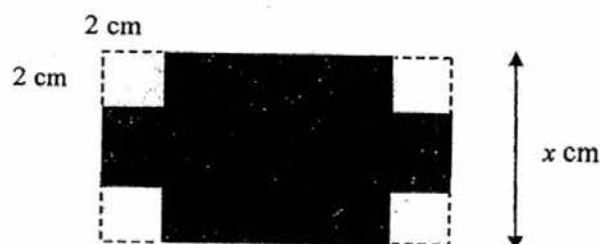
- (a) Calculate
- (i) the bearing of B from O , [1]
 - (ii) the distance between A and C , [2]
 - (iii) the bearing of A from C . [3]

- (b) A pole of height 15 m is set up at A . The angle of elevation of the top of the pole from a point X on the line OB is the greatest. Find this angle of elevation. [3]

- (c) Another pole is put up at C . The rays of the sun cast a shadow of the pole on the ground. The tip of the shadow touches a point P on the semicircle with OB as diameter. From P , the angle of elevation of the top of the pole is the least. Find the distance OP . [2]

- 9 Answer the whole of this question on a sheet of graph paper.

The breadth of a rectangular sheet of paper is x cm and its area is 48 cm^2 . Four squares with sides 2 cm are cut at the four corners of the paper and the paper is then folded to make a rectangular box.



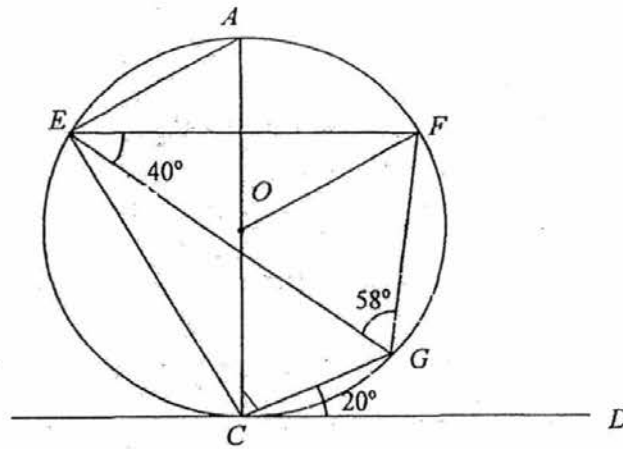
- (a) Show that the volume of the box, $V \text{ cm}^3$, is $V = 8(x-4)\left(\frac{12-x}{x}\right)$. [2]
- (b) Some corresponding values of x and V are given in the table below.

x	4	5	6	8	9	10	12
V	0	11.2	16	16	13.3	p	0

- Calculate the value of p . [1]
- (c) Using a scale of 1 cm to represent 1 unit on each axis, draw the graph of $V = 8(x-4)\left(\frac{12-x}{x}\right)$ for $0 \leq x \leq 12$. [2]
- (d) Find the range of values of x for which the volume of the box is less than 6 cm^3 . [1]
- (e) Find the area of the base of the rectangular box which has the largest volume. [2]
- (f) A cylinder with a fixed base area of 8 cm^2 has a height of $\frac{x}{7} + 1$. Insert a suitable graph on the graph in (c) to represent the volume of this cylinder. [2]
- (g) Hence find the values of x that satisfy the equation $\frac{4}{7}x^2 + 4x = 4(4-x)(x-12)$. [2]

- 10 (a) In the diagram below, the points A, E, C, G and F lie on the circle with centre O .

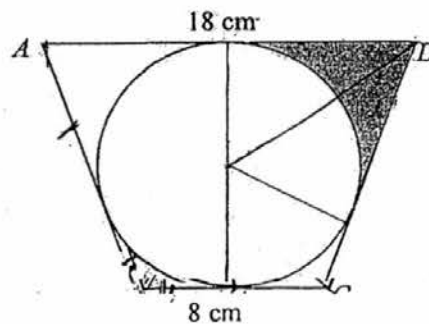
BCD is a tangent to the circle at C , AC is the diameter, $\angle GEF = 40^\circ$, $\angle EGF = 58^\circ$ and $\angle DCG = 20^\circ$



Find, stating your reasons clearly,

- | | | |
|-------|----------------|-----|
| (i) | $\angle ACG$, | [1] |
| (ii) | $\angle FOA$, | [2] |
| (iii) | $\angle GCE$, | [2] |
| (iv) | $\angle EAO$. | [2] |

- (b) $ABCD$ is a trapezium. $AD = 18$ cm, $BC = 8$ cm, $AB = DC$ and the circle touches all the sides of the trapezium.



- | | | |
|-------|--|-----|
| (i) | Find the length of AB . | [1] |
| (ii) | Calculate the radius of the circle. | [3] |
| (iii) | Calculate the area of the shaded region. | [4] |

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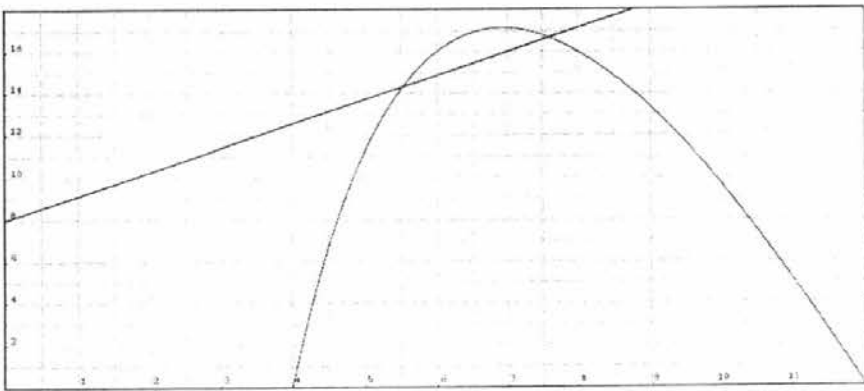
E Maths P2

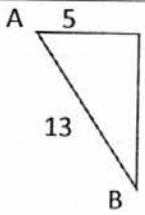
Q		METHOD
1(a)		$25.2^2 - 3\sqrt{\frac{232.54 - 15.25^2}{7}}$ $= 640$
1(b)		$4x^2 - 4x - 9y^2 + 1$ $= 4x^2 - 4x + 1 - 9y^2$ $= (2x - 1)^2 - 9y^2$ $= (2x - 1 - 3y)(2x - 1 + 3y)$
(c)		$\left(\frac{5^x}{2}\right)^{-3} = \frac{200}{\sqrt{5^x}}$ $\left(\frac{2}{5^x}\right)^3 = \frac{200}{5^{\frac{x}{2}}}$ $\frac{8}{200} = 5^{\frac{5x}{2}}$ $5^{\frac{5}{2}x} = 5^{-2}$ $x = -\frac{4}{5}$
(d)		$4x^2 - 3xy + p = \left(2x - \frac{3}{4}y\right)^2$ $p = \frac{9}{16}y^2$
(e)		$3a - 2b = 3b + 5a$ $2a = -5b$ $a = -\frac{5b}{2}$ $\sqrt{\frac{13b^2}{a^2 - 3b^2}} = \sqrt{\frac{13b^2}{\frac{13b^2}{4}}}$ $= 2$
2 (a)	(i)	12 - 4t
	(ii)	8t

(b)		$x^2 = (12 - 4t)^2 + (8t)^2$ $= 144 - 96t + 16t^2 + 64t^2$ $= 80t^2 - 96t + 144$			
(c)		$272 = 80t^2 - 96t + 144$ $80t^2 - 96t - 128 = 0$ $t = 2$ John's distance is $12 - 4(2) = 4\text{km}$			
3(a)	(i)	p=37			
	(ii)	Statement	Agree/Disagree	Reason	
		The students in School A did better.	Disagree	Median time is higher	
		The performance of students in the School B was more consistent.	Disagree	Their interquartile range is higher	
(b)	(i)	\$135			
	(ii)	$\sqrt{\frac{\sum x^2}{11} - (\bar{x})^2} = \sqrt{\frac{207346}{11} - \left(\frac{1508}{11}\right)^2}$ $= 7.46$			
	(iii)	The median price will go up by \$8 but stad deviation will remain the same			
4 (a)	(i)	$RP = \begin{pmatrix} 12 & 20 \end{pmatrix} \begin{pmatrix} 3 & 5 & 2 \\ 5 & 4 & 3 \end{pmatrix}$ $= \begin{pmatrix} 136 & 140 & 84 \end{pmatrix}$			
	(ii)	Each element represents the number of chocolate bars, cupcakes and packets of biscuits Madam Marina got respectively			
(b)	(i)	$Q = \begin{pmatrix} 3 & 4 \\ 2.8 & 4.2 \\ 2 & 3.8 \end{pmatrix}$			
	(ii)	$SQ = \begin{pmatrix} 136 & 140 & 84 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 2.8 & 4.2 \\ 2 & 3.8 \end{pmatrix}$ $= \begin{pmatrix} 968 & 1451.2 \end{pmatrix}$			
(c)		$a = -1 \text{ and } b = 1$ $\begin{pmatrix} 968 & 1451.2 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \$483.2$ Profit is \$483.2			

5(a)	(i)	$\overrightarrow{OF} = \frac{2}{3}\overrightarrow{OB} = \frac{8}{3}u + 2v$
5	(ii)	$\overrightarrow{BD} = 3v - 4u$
	(iii)	$\overrightarrow{DE} = 2u - 3v$
(b)		$\begin{aligned} \overrightarrow{DF} &= \overrightarrow{DO} + \overrightarrow{OF} \\ &= -6v + \frac{8}{3}u + 2v \\ &= \frac{8}{3}u - 4v \\ &= \frac{4}{3}(2u - 3v) \\ &= \frac{4}{3}\overrightarrow{DE} \end{aligned}$
(c)	(i)	$\frac{3}{4}$
	(ii)	$\frac{\text{Area of } \triangle DEB}{\text{Area of parallelogram } OABC} = \frac{\frac{1}{2}DBC}{2\triangle OBC}$ $= \frac{1}{4}$
6 (a)	(i)	$\begin{aligned} \$12700 &= \$4300 + \$8400 \\ \frac{x \times 14}{100} &= 8400 \\ x &= 60000 \\ \text{Chargeable income is } \$80000 + \$60000 &= \$140000 \end{aligned}$
	(ii)	$\begin{aligned} \text{Annual income} &= \$140000 + \$15000 + \$5000 + \$14800 \\ &= \$174\,800 \end{aligned}$
(b)	(i)	Amount is AUD 226415
	(ii)	$\begin{aligned} \text{Amount} &= 226415 \left(1 + \frac{2.5}{400}\right)^{12} \\ &= \text{AUD } 243992.1849 \\ \text{Amount in Singapore currency} &= \frac{243992.1849}{0.745} \\ &= \$327506 \end{aligned}$

7 (a)	(i)	$\frac{r}{24} = \frac{10}{40}$ $r = 6 \text{ cm}$ $\text{Volume of water} = \frac{1}{3} \pi \times 36 \times 24$ $= 288\pi$ $\text{Time taken} = \frac{288\pi}{1.25}$ $= 12 \text{ min}$
	(ii)	$\frac{r}{28} = \frac{10}{40}$ $\text{radius of oil} = 7 \text{ cm}$ $\text{Volume of oil} = \frac{1}{3} \pi \times 7^2 \times 28 - 288\pi$ $= 169\frac{1}{3}\pi$ $\text{Let height of oil be } h \text{ cm}$ $\frac{r}{h} = \frac{1}{4}$ $r = \frac{1}{4}h$ $\frac{1}{3} \pi \left(\frac{1}{4}h\right)^2 h = 169\frac{1}{3}\pi$ $h = 20.1 \text{ cm}$ $\text{ALTERNATIVE METHOD}$ $\frac{V_{oil} + V_{water}}{V_{water}} = \left(\frac{28}{24}\right)^3$ $V_{oil} = \left(\frac{7}{6}\right)^3 \times 288\pi - 288\pi$ $= 169\frac{1}{3}\pi$
	(iii)	$\frac{2}{3} \pi \times 3^3 + \pi \times 3^2 h = 288\pi$ $h = 30 \text{ cm}$ $\text{Height of water} = 33 \text{ cm}$
8 (a)	(i)	$\text{Bearing is } 90 + 50 = 140^\circ$

	(ii)	$AC = \sqrt{450^2 + 400^2 - 2(450)(400)\cos 68}$ $= 477.118$ $= 477m$
	(iii)	$\frac{\sin ACO}{450} = \frac{\sin 68}{477.118}$ $\angle ACO = 60.98^\circ$ $= 61^\circ$ Therefore bearing of A from C = $180 + (90 - 60.98)$ $= 209^\circ$
	(b)	$\sin 18^\circ = \frac{AX}{450}$ $AX = 139.05764$ $\tan \theta = \frac{15}{139.05764}$ $\theta = 6.2^\circ$
	(c)	$CP = OB$ $\cos 50 = \frac{400}{OB}$ $OB = 622.2895cm$ $OP = \sqrt{622.2895^2 - 400^2}$ $= 477cm$
9	(a)	Length is x Width is $48/x$ $Vol = 2(x - 4)\left(\frac{48}{x} - 4\right)$ $= 8(x - 4)\left(\frac{12}{x} - 1\right)$
	(b)	P = 9.6
	(c)	

	(d)	$x < 4.4$ or $x > 10.8$
	(e)	Max V is 17.2 cm^3 X is about 6.92 Area is 8.6 cm^2
	(f)	$\text{Line } V = \frac{8}{7}x + 8$
	(g)	Solve simultaneous: $x = 5.5, 7.6$
10(a)	(i)	$\angle ACG = 90 - 20$ (tan perpendicular to rad) $= 70^\circ$
	(ii)	$\angle GEA = 70^\circ$ (angles in same seg) $\angle FEA = 30^\circ$ $\angle FOA = 60^\circ$
	(iii)	$\angle GFE = 180 - 40 - 58$ (Angles of sum of triangle) $= 82^\circ$ $\angle GCE = 180 - 82$ (cyclic quad) $= 98^\circ$
	(iv)	$\angle ACE = 98^\circ - 70^\circ = 28^\circ$ $\angle EAO = 180 - 28 - 90$ (Angle sum of triangle) $= 62^\circ$
(b)	(i)	$AB = 9 + 4 = 13$
	(ii)	 $Ht = \sqrt{13^2 - 5^2} = 12 \text{ cm}$ Radius = 6 cm

	(iii)	$\tan \theta = \frac{6}{9}$ $\theta = 0.588 \text{ radians}$ $\text{Area} = 2\left\{\frac{1}{2}(6)(9) - \frac{1}{2}(6)^2 0.983\right\}$ $= 18.6 \text{ cm}^2$
--	-------	---