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1 (a) Sketch the graph of $y = \ln x$. [1]

(b) Determine the equation of the straight line that would be drawn on the graph of $y = \ln x$ to obtain the graphical solution of the equation $x = e^{2 - \frac{3}{2}x}$. [2]

(c) Find the number of solutions of the equation $x = e^{2 - \frac{3}{2}x}$. [1]

- 2 (a) Without using a calculator, solve the equation $\sqrt{x+3} = 2 + \sqrt{5}$, leaving your answer in exact form. [2]

- (b) A triangle ABC is such that its area is $(8 + 3\sqrt{2}) \text{ cm}^2$, the length of AB is $(4 + \sqrt{8}) \text{ cm}$ and angle ABC is 90° .

Without using a calculator, find the length of BC , in cm, in the form $a + b\sqrt{2}$, where a and b are integers. [3]

3 (a) (i) State the values between which the principal value of $\cos^{-1} x$ must lie. [1]

(ii) Find the principal value of $\tan^{-1}(\sqrt{3})$ in radian in exact form. [1]

(b) Given that $\operatorname{cosec} \theta = -\frac{2}{\sqrt{3}}$ and that $\tan \theta$ and $\sin \theta$ have opposite signs, without evaluating θ , find the exact value of each of the following.
(i) $\cos(-\theta)$, [1]

(ii) $\tan 2\theta$. [2]

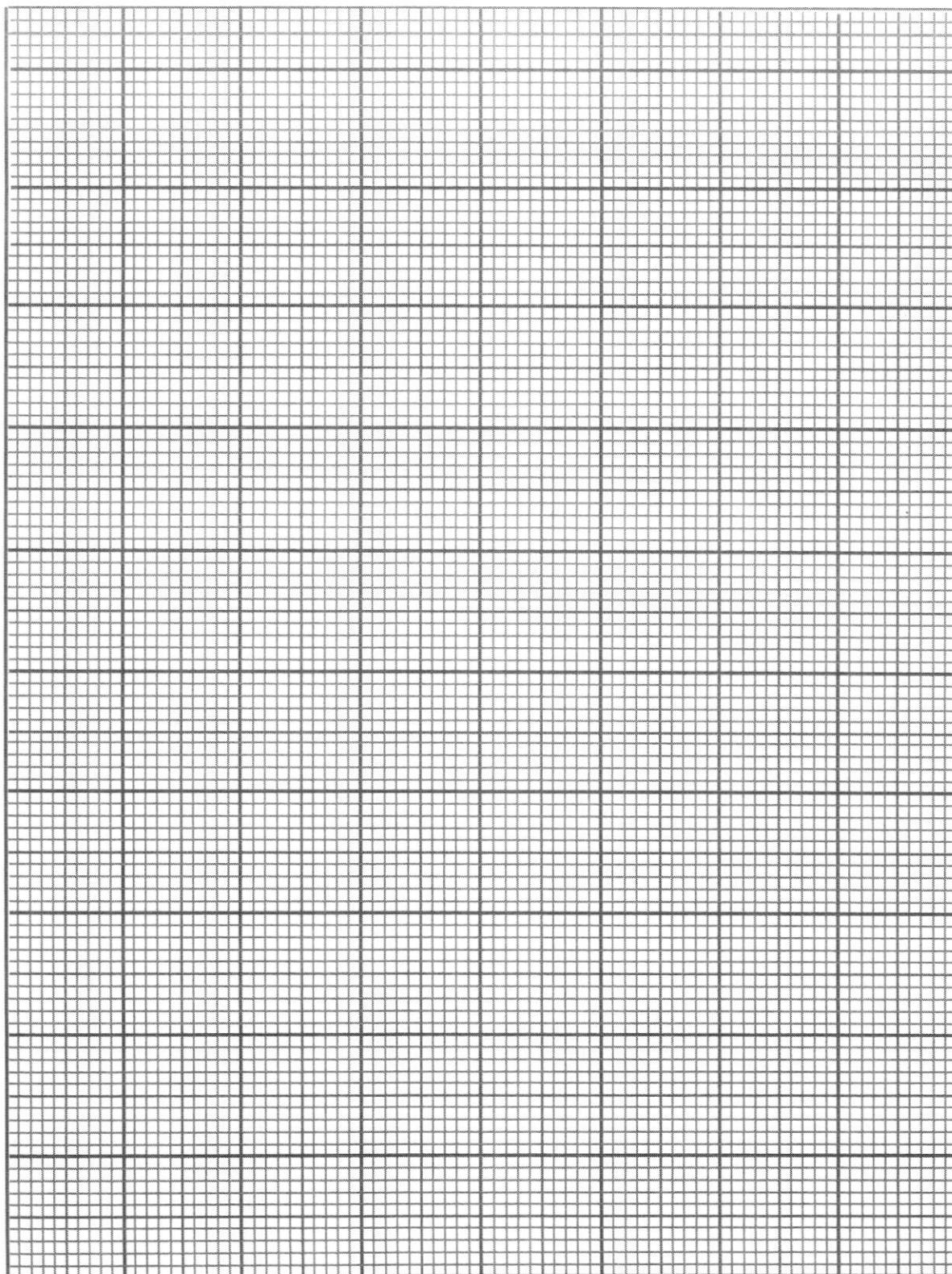
- 4 Express $\frac{x^2 - 3x + 5}{(x+1)(2x^2 - x)}$ in partial fractions.

[5]

- 5 The table below shows experimental values of two variables x and y .

x	1	1.5	2	2.5	3
y	5.00	4.90	4.95	5.06	5.20

- (a) Given that x and y are related by the equation $y = a\sqrt{x} + \frac{b}{\sqrt{x}}$, where a and b are constants, draw the straight line graph of $y\sqrt{x}$ against x . [2]



(b) Use your graph to estimate the value of a and of b . [3]

(c) Explain how the graph could be used to find the value of y when x is 0.5. [2]

- 6 A curve is such that $\frac{d^2y}{dx^2} = 3x - 2$ and the equation of the tangent to the curve at the point $(1, -5)$ is $y = -2 - 3x$. Find the equation of the curve. [6]

- 7 A stone is thrown upwards from a cliff, 40 m above the ground. It will reach a maximum vertical height and then fall back to the ground. The height of the stone from the ground at time t is h m, which is given by $h = -8t^2 + 64t + 40$, where t is the time in seconds.

(a) Find the time interval when the stone is below 160 m. [3]

(b) Express h in the form of $k(t - p)^2 + q$, where k , p and q are constants. [2]

(c) Hence, find the maximum height that the stone can reach and the time taken to reach this maximum height. [2]

- 8 Diagram I shows a spherical ornament of radius r cm.
 Diagram II shows the cross-sectional view of an open cylindrical box containing 7 such identical spherical ornaments touching one another and the side of the box. The height of the box is h cm.
 A paper label is wrapped round the box which exactly covers its vertical surface. The area of paper label is 152 cm^2 .

Diagram I

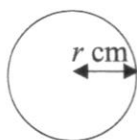
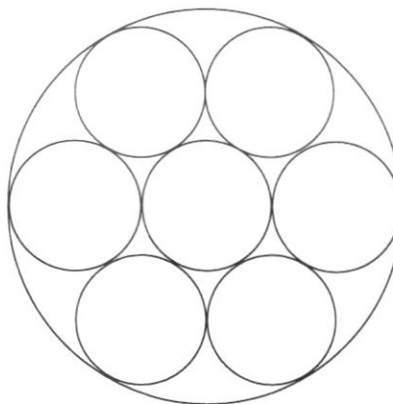


Diagram II



- (a) Express h in terms of r . [1]

- (b) Show that $V \text{ cm}^3$, the volume of the space in the box which is not occupied by the ornaments is

$$V = 228r - \frac{28}{3}\pi r^3. \quad [3]$$

- (c) Given that r can vary, find the stationary value of V . [3]

9 (a) Prove the identity $\sin 2x - \tan x = \tan x \cos 2x$. [3]

(b) Solve the equation $3 \cot^2 \theta + 10 \operatorname{cosec} \theta = 5$ for $0 \leq \theta \leq 2\pi$. [5]

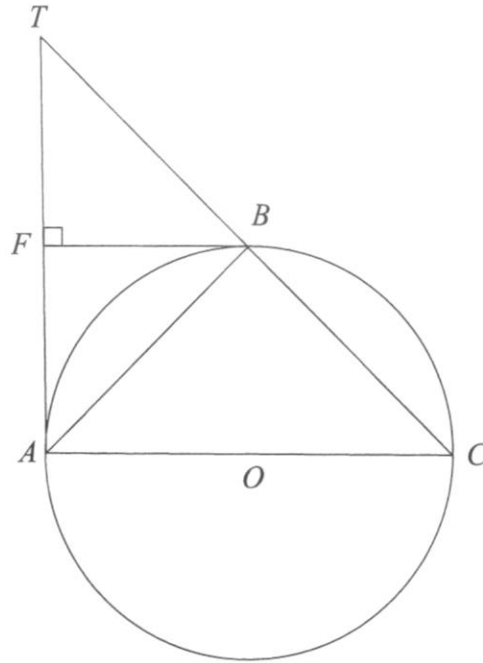
- 10** The expression $f(x) = 3x^3 + ax^2 + 2x + b$, where a and b are constants, is exactly divisible by $x - 2$ and leaves a remainder of -8 when divided by $x - 1$.

(a) Find the value of a and b . [4]

(b) Show that the equation $f(x) = 0$ has only one real root. [4]

- 11** A circle passes through the points A , B and C are $(6, 0)$, $(2, 2)$ and $(-3, -3)$.
The equation of the perpendicular bisector of the line segment BC is $y = -x - 1$.
- (a) Find the equation of the perpendicular bisector of the line segment AB . [3]
- (b) Hence, find the equation of the circle that passes through the three points A , B and C . [4]

- 12 In the diagram, a circle with centre O passes through the vertices of triangle ABC . AT is a tangent to the circle at A and BF is perpendicular to AT . CB produced and AF produced meet at T .



Prove that

- (a) triangle ABC is similar to triangle BFA ,

[2]

(b) $AB^2 = AC \times BF$, [1]

(c) $AB^2 + TB^2 = TC^2 - AC^2$. [2]

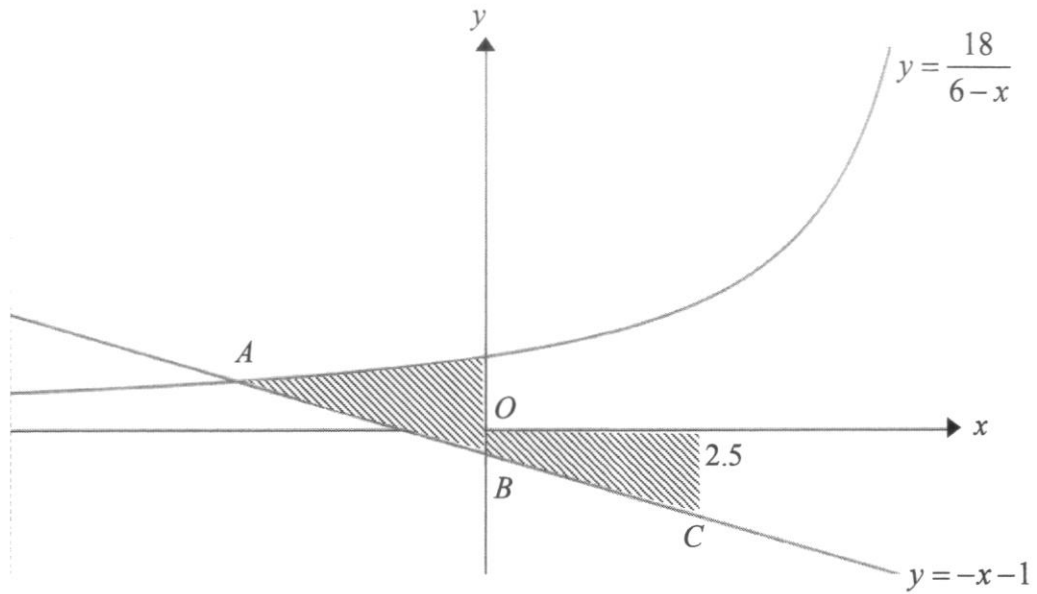
- 13 The diagram shows part of a curve $y = \frac{18}{6-x}$ and the line $y = -x - 1$.

A lies on the curve.

B and C lie on the line such that B is on the y -axis and C has a x -coordinate of 2.5.

Find the area of the shaded region.

[7]



Continuation of working space for question **13**.

- 14** A particle, A , starts at a point 2 m away from O and travels in a straight line. Its velocity, v m/s, is given by $v = 4t - t^2$, where t is the time in seconds measured from the start of motion.

Calculate

- (a) the time when the particle is instantaneously at rest, [2]

- (b) total distance travelled by particle A in the first 5 seconds. [3]

Point X is 8 m from point O . Particle B travels in the same direction along the same straight line as Particle A . It travels with an acceleration of $a = -8t + 4$, where t is the time after passing point X . The velocity of Particle B when it passes through Point X is 7 m/s.

- (c) Calculate the time when particle B collides with particle A . [4]

- End of paper -

1 It is given that $y = 7x^3 e^{-2x}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Show that $e^{2x} \left(p \frac{dy}{dx} + \frac{d^2y}{dx^2} + 4y \right) = qx$
where p and q are constants to be determined. [5]

2 (a) Find $\frac{d}{dx}(x \cos 4x)$. [2]

(b) Without using a calculator, find the value of each of the constant a and b for which

$$\int_0^{\frac{\pi}{12}} x \sin 4x \, dx = a\pi + b \quad [4]$$

- 3 (a) The coefficient of x^{13} in the binomial expansion of $\left(ax^2 - \frac{1}{x}\right)^{11}$ is -42240 .

Find the value of a .

[3]

- (b) In the expansion of $(3+7x)^n$, the coefficient of x^3 and x^4 are in the ratio $12 : 35$.

Find the value of n .

[4]

4 A curve has equation $y = \frac{4x^3 - 1}{x^2}$, $x \neq 0$.

(a) Determine the range of values of x such that y is a decreasing function. [4]

(b) Determine the coordinates of the point where the tangent of the curve at $x = -1$ cuts the x -axis. [3]

5 (a) Solve the equation $\log_2(y-4) + \log_2 y = \log_2 2y - 3$. [4]

(b) Solve the equation $\log_6 x = 3 - \log_x 36$. [4]

- 6** The equation of a curve is $y = kx^2 + 4x + 3k - 5$, where k is a constant.
The line $y = 2kx - 3$ is a tangent to the curve at the point H .

(a) Find the possible values of k .

[4]

(b) Given that the gradient of the tangent is positive, find the coordinates of H . [2]

- 7 When a piece of steak is pan-fried, the internal temperature of the steak, $T^{\circ}\text{C}$, can be modelled by the formula $T = 20 + me^{kt}$, where t is the time in minutes from the time the steak is placed in the pan to be cooked and k is a constant.

The steak must be in room temperature when it is placed in the pan.

The table below shows some information of how a piece of steak may be cooked to the preference of doneness*.

Doneness of Steak	Colour & Firmness	Internal Temperature	Estimated Time to cook a piece of steak
Medium-Rare	Red warm centre, springy firmness	52 °C	9 minutes
Medium	Hot pink centre, less springy than medium-rare	57 °C	
Medium-Well	Slight colour, cooked throughout, firm	63 °C	
Well-Done	No colour left, very firm	71 °C	14 minutes

***Doneness** is a gauge of how thoroughly cooked a piece of steak is based on its color, firmness, and internal temperature.

- (a) Using the information in the table, find the value of k and of m . [5]

- (b) Kenny prefers his steak to be done medium-well.
How long should he pan-fry his steak?

[3]

- (c) Explain why the model cannot be used for large values of t .

[1]

- 8 (a) A particle moves along the curve $y = \ln \sqrt{\frac{5x-3}{x^2+1}}$ in such a way that the y -coordinate is decreasing at a constant rate of $\frac{1}{7}$ units per second. Find the rate at which the x -coordinate of the particle is increasing at the instant when $x = 2$. [4]

(b) A curve is defined by the equation $y = 10 - \left(\frac{x}{3} - 2\right)^5$.

(i) Find the coordinates of stationary point.

[3]

(ii) By considering the sign of $\frac{dy}{dx}$, determine the nature of the stationary point. [2]

9 It is given that $y_1 = -3 \cos 2x$ and $y_2 = \tan 2x$

(a) State the amplitude and period, in degrees, of y_1 . [2]

(b) Explain why the amplitude of y_2 does not exist. [1]

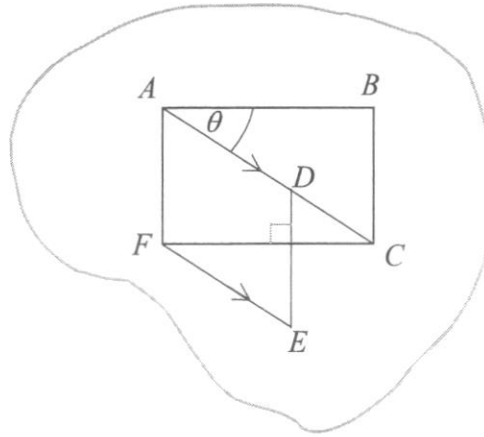
For the interval $0^\circ \leq x \leq 180^\circ$,

(c) solve the equation $y_1 = \frac{1}{y_2}$. [4]

(d) sketch on the same diagram, the graphs of y_1 and y_2 .

[4]

10



The diagram shows an enclosure in a miniature park. The lines represent the possible tracks of automated moving cars. $ABCF$ is a rectangle and AC is parallel to FE . D is a point on AC and DE is perpendicular to FC . Angle BAC is θ radians and AC is 10 metres. A car moves along the track $ABCDEFA$ to complete one cycle.

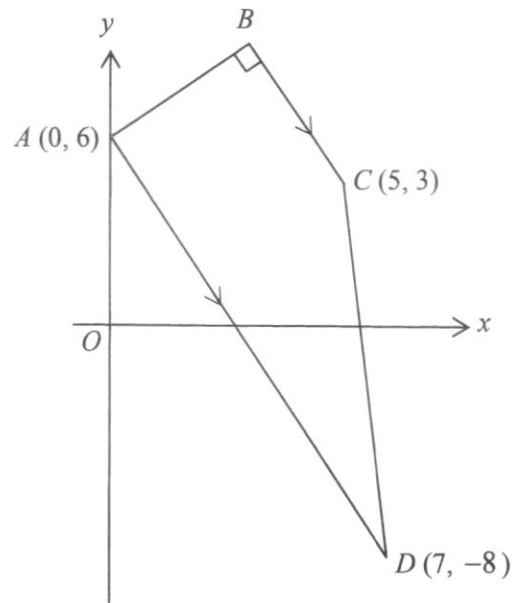
- (a) Show that the distance, S m, travelled by the car per cycle is given by

$$S = 10 \cos \theta + 30 \sin \theta + 10. \quad [3]$$

- (b) Express $10 \cos \theta + 30 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and α is acute. [3]

- (c) Find the value of θ if S is 36 m. [2]

11



The diagram shows a trapezium $ABCD$ in which AD is parallel to BC and angle $ABC = 90^\circ$. The coordinates of A , C and D are $(0, 6)$, $(5, 3)$ and $(7, -8)$ respectively.

(a) Is $AB = BC$? Justify your answer with working.

[7]

Continuation of working space for Question 11a.

- (b) The point E lies on the line AD produced such that $\frac{\text{area of } \triangle ACD}{\text{area of } \triangle ACE} = \frac{4}{5}$.

Find the coordinates of point E .

[2]

- (c) A quadrilateral $OACF$ is formed such that the area is 50 square units. Given that DF is parallel to the x -axis, find the coordinates of F .

[3]

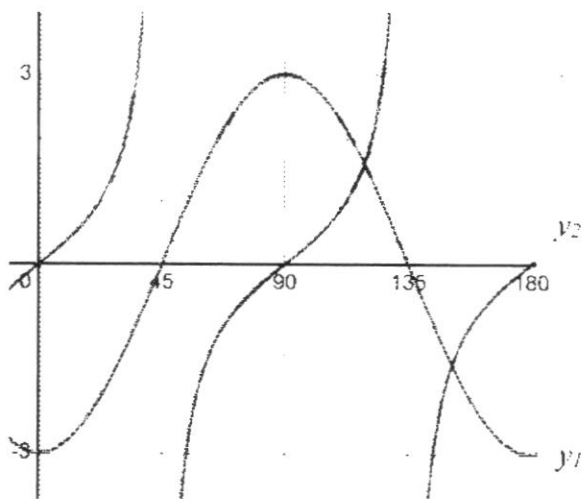
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Answer Key**2023 4E5N Prelim Additional Math Paper 1**

1a	graph	1b	$y = 2 - \frac{3}{2}x$
1c	1 solution		
2a	$x - 6 + 4\sqrt{5}$	2b	$5\sqrt{2}$
3ai	$0 \leq \cos^{-1} x \leq \pi$	3aii	$\frac{\pi}{3}$
3bi	-0.5	3bii	$-\sqrt{3}$
4	$\frac{3}{x+1} + \frac{5}{2x-1} - \frac{5}{x}$		
5a	graph	5b	$a = 2, b = 3$
5c	Draw a vertical line $X = 0.5$ From the graph, the corresponding $Y = 4$ $y = 5.66$		
6	$y = \frac{x^3}{2} - x^2 - \frac{5}{2}x + 2$		
7a	$0 \leq t < 3$ $5 < t \leq 8.58$	7b	$-8(t-4)^2 + 168$
7c	Max height = 168 m Time taken = 4 sec		
8a	$h = \frac{76}{3\pi r}$	8b	show question
8c	245 cm^3		
9a	prove question	9b	3.39 or 6.03
10a	$a = -5, b = 8$	10b	show question
11a	$y = 2x - 7$	11b	$(x-2)^2 + (y+3)^2 - 25$
12a	prove question	12b	prove question
12c	prove question		
13	11.4 units^2		
14a	$t = 0$ or 4	14b	13 m
14c	$t = 3$		

2023 4E5N AM P2 Answer Key

- 1a) $e^{-2x}(-14x^3 + 21x^2)$ b) $p = 4$ $q = 42$
- 2a) $-4x \sin 4x + \cos 4x$ b) $a = -\frac{1}{96}$ $b = \frac{\sqrt{3}}{32}$ (do not accept 3 s.f.)
- 3a) $a = \pm 2$ b) $n = 8$
- 4a) $\sqrt{\frac{1}{2}} < x < 0$ b) $\left(\frac{3}{2}, 0\right)$
- 5a) $\frac{17}{4}$ b) 6 or 36
- 6a) $k = -2$ or 1 b) $H(-1, -5)$
- 7a) $k = \frac{1}{5} \ln\left(\frac{51}{32}\right)$ o.e m = 13.8 b) 12.2 minutes
- 7c) Any logical reasons:
 For large values of t , the temperature will rise to a very high value according to the model. The steak will be burnt and will not be edible/ the steak does not need to be cooked to such high temperature/ the internal temperature of the steak will reach a certain level and will not continue to rise anymore.
- 8a) $\frac{10}{3}$ units per second b) (6, 10) c) point of inflexion
- 9a) amp = 3, period = 180°
- 9b) $y_2 = \tan 2x$ do not have maximum or minimum values c) 99.7° , 170.3°
- 9d)



10a) To be shown b) $10\sqrt{10}\cos(\theta-1.25)$ C) .644

11a) No, $B\left(\frac{14}{5}, \frac{37}{5}\right)$, $AB = \sqrt{\frac{49}{5}}$ units is not to equal to $BC = \sqrt{\frac{121}{5}}$ units

11b) $E\left(8\frac{3}{4}, -11\frac{1}{2}\right)$ c) $F(10-8)$