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Name:	Register No.:	Class:
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**CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2020
PRELIMINARY EXAMINATION**

**ADDITIONAL MATHEMATICS
Paper 1**

**4047/01
25 August 2020**

Additional Materials: NIL

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of the page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the answer spaces provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiners' Use

Question	1	2	3	4	5	6	7	8	9	10	11	12	13
Marks													

Table of Penalties		Question Number	Parent's/Guardian's Signature	80
Presentation	-1			
Units	-1			
Significant Figures	-1			

This document consists of **18** printed pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

- 1 Find the range of values of k for which the quadratic equation $(k+2)x^2 - 2kx + 1 = 0$ has real roots. [3]

- 2 A triangle ABC has an area of $(14 - 3\sqrt{2}) \text{ cm}^2$ and $AC = \left(5 - \frac{4}{\sqrt{2}}\right) \text{ cm}$. Express the perpendicular distance from B to AC in the form of $a + b\sqrt{2}$, where a and b are rational numbers. [4]

[Turn over

- 3 Given that $(1, h)$ is a solution of the simultaneous equations $\frac{1}{x} + \frac{2}{y} - 3 = 0$ and $x + ky - 6 = 0$, find the values of x and of y . [5]

- 4 Given that $y = \frac{e^{-2x}}{3x-5}$, find the range of values of x for which y is an increasing function. [5]

[Turn over

- 5 (i) Given that $\lg 2 = x$ and $\lg 3 = y$, find the value of $\log_5 24$ in terms of x and y . [4]

- (ii) Express $4^{\ln 3} + 2^{\ln 9}$ in the form 2^m , where m is a constant. [2]

6 Express $\frac{x^4}{x^4-16}$ in partial fractions.

[6]

[Turn over

- 7 Water flows out of a filled container of height 20 cm such that after t seconds, the depth of water in the container is h cm and the volume of water left in the container is given by $V = 10 + \frac{h^2(h+2)}{10}$. Given that the height of the water level is decreasing at a rate of $(4t-1)$ cm/s. find the rate of decrease of the volume of the water when $t = 2$. [6]

- 8 It is given that $\tan B = \frac{3}{4}$ where B is an acute angle. Without solving for B , find the exact value of

(i) $\cos 3B$, [4]

(ii) $\tan \frac{B}{2}$. [Hint: Use double angle formula.] [3]

[Turn over

- 9 (i) Sketch the graph of $f(x) = 3 - |2x - 3|$ for $-2 \leq x \leq 7$, indicating the coordinates of the intercepts, endpoints and vertex clearly. [3]

- (ii) State least positive value of $\frac{1}{f(x)}$. [1]

- (iii) State the range of values of m such that the line $y = mx + 2$ intersects the graph at 2 distinct points. [3]

[Turn over

10 A company wants to produce open cylindrical containers with a capacity of 6000 cm^3 each.

- (i) Find the radius of the container if the company wants to minimize the amount of material used. [4]

- (ii) For the value of the radius found in (i), show that the amount of material used is minimum. [2]

- (iii) State an assumption made in your calculations above. [1]

[Turn over

- 11** A radioactive substance decays according to the equation $A = 15e^{-bt}$, where A is the mass in grams of radioactive substance remaining and t is the time in hours after the radioactive substance starts to decay.

(i) Given that there are 10 grams of radioactive substance remaining after 30 hours, verify that the value of b is 0.013516. [2]

(ii) Find the mass of radioactive substance left at the end of 1 week. [1]

(iii) Find the number of hours for the radioactive substance to decay to half of its original mass. [3]

- (iv) Find the rate at which the substance decays at $t = 50$ hours. [2]

[Turn over

- 12 It is given that $f(x) = \frac{1}{2} - 3 \cos 2x$ for $0 \leq x \leq 2\pi$ and $g(x) = 2 \sin\left(\frac{x}{2}\right)$ for $0 \leq x \leq 2\pi$.

(i) State the period of $g(x)$. [1]

(ii) State the least and greatest values of $f(x)$. [1]

(iii) Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \leq x \leq 2\pi$. [4]

- (iv) Hence, state the number of solutions of the equation $6\cos 2x + 4\sin\left(\frac{x}{2}\right) = 1$
for $0 \leq x \leq 2\pi$. [1]

- (v) Determine the value of k for which the equation $\frac{1}{2} - 3\cos 2x = 2\sin\left(\frac{x}{2}\right) + k$
has one solution. [1]

[Turn over

- 13 (i) Find the coordinates of the foot of the perpendicular from $A(3, 11)$ to the line l , $2x + 4y = 25$. [4]

- (ii) Hence, calculate the perpendicular distance from A to the line l . [2]

- (iii) If B is the reflection of A in the line l , find the coordinates of B . [2]

END OF PAPER

Name:	Register No.:	Class:
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**CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2020
PRELIMINARY EXAMINATION**

**ADDITIONAL MATHEMATICS
Paper 2**

**4047/02
28 August 2020**

Additional Materials: NIL

2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of the page.
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You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

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The number of marks is given in brackets [] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiners' Use

Question	1	2	3	4	5	6	7	8	9	10	11
Marks											

Table of Penalties		Question Number	Parent's/Guardian's Signature	100
Presentation	–1			
Units	–1			
Significant Figures	–1			

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Mathematical Formulae

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Quadratic Equation

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

- 1 The equation of a polynomial is given by $P(x) = 2x^3 + cx^2 + 7$. The remainder when $P(x)$ is divided by $2x - 1$ is twice the remainder when $P(x)$ is divided by $x - 2$.

(i) Show that the value of c is -5 . [3]

(ii) Using the value of c in part (i), explain why the equation $P(x) = 0$ has only one real root. [4]

[Turn over]

- 2 A particle moves in a straight line so that after t seconds, its distance, s metres, from a fixed point O is given by $s = e^t - \frac{5}{2e^t} - 7$.

(i) Find the distance of the particle from O when $t = \ln 2$.

By comparing with the instant when the particle first moves off, determine if the particle is moving towards or away from O at $t = \ln 2$. [3]

(ii) Find the value of t when the velocity, $v = 5.5$ m/s.

[4]

- 3 (i) Differentiate $x \cos^2 x$ with respect to x .

[2]

- (ii) Hence show that $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x + x \sin 2x}{2} dx = \frac{\pi}{16}$.

[4]

[Turn over

- 4 (a) Find the range of values of k for which $3x^2 + 8x + k$ is always positive. [2]

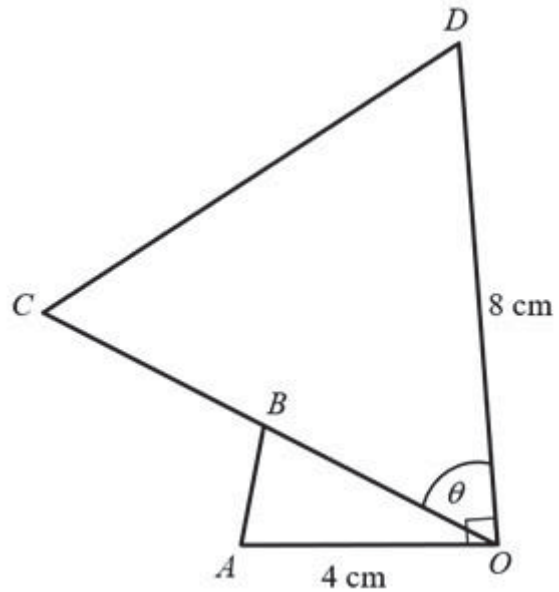
- (b) The equation $px^2 + 7qx + 9p = 0$ has equal roots. Given that p and q are positive, find the ratio $p : q$ and solve the equation. [4]

- 5 The diagram shows two isosceles triangles OAB and COD .

$OA = OB = 4$ cm, $OC = OD = 8$ cm,

angle $AOD = \frac{\pi}{2}$ and angle $COD = \theta$.

The sum of the areas of triangles OAB and OCD is S cm².



- (i) Show that S can be expressed in the form $p \sin \theta + q \cos \theta$, where p and q are constants. [3]

- (ii) Express S in the form $R \sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Given that the sum of the areas of triangles OAB and OCD is 32 cm², find the value of θ . [5]

[Turn over]

- 6 A rectangle of area $A \text{ m}^2$ has sides measuring $x \text{ m}$ by $(px + q) \text{ m}$, where p and q are constants and x and A are variables. Values of x and A are given in the table below.

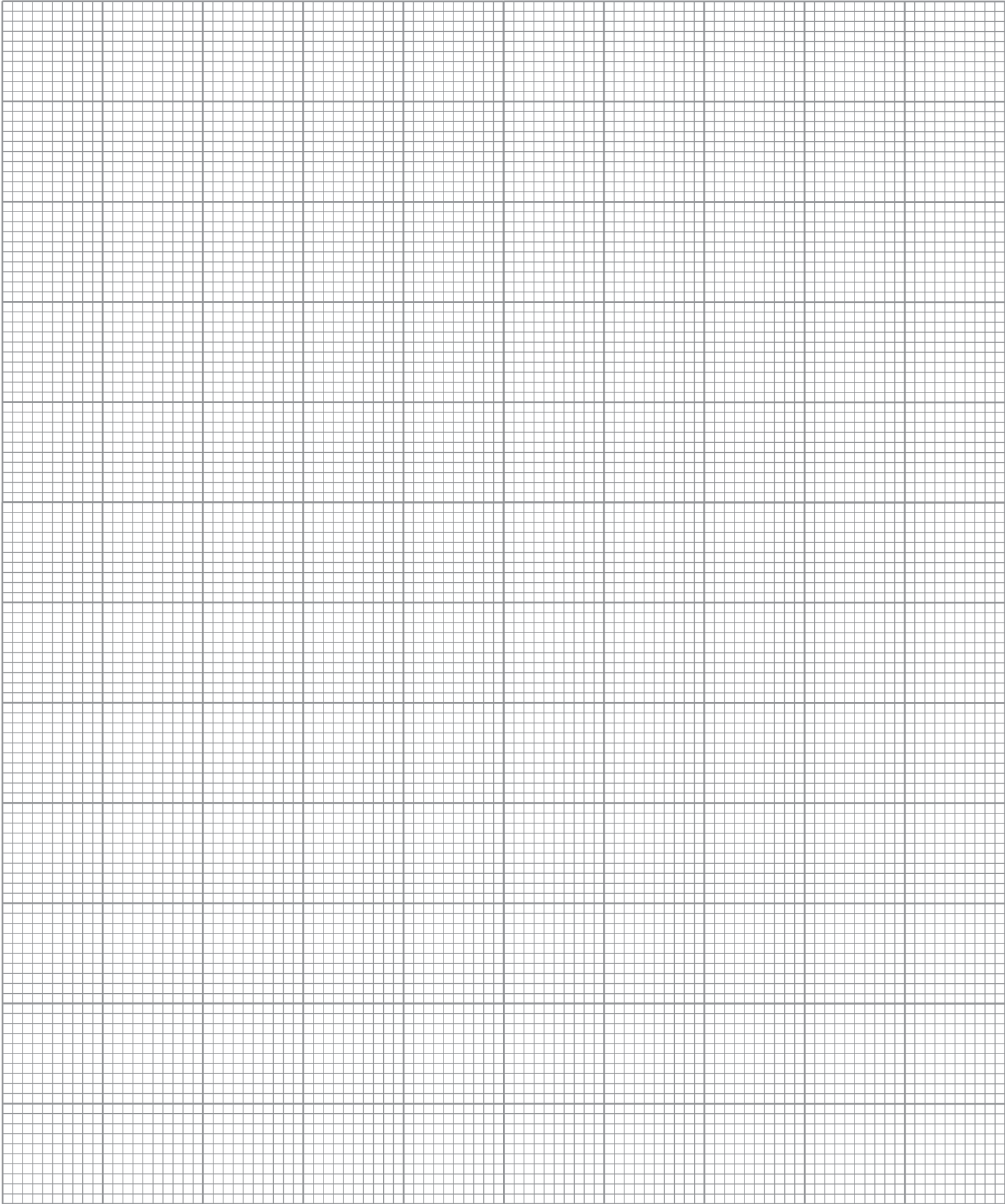
x	50	100	150	200	250
A	3800	10 500	20 400	33 000	48 500

- (i) Using suitable variables, draw, on the grid opposite, a straight line graph and hence estimate the value of each of the constants p and q .

[6]

- (ii) On the same diagram, draw the straight line representing the equation $A = x^2$ and explain the significance of the value of x given by the point of intersection of the two lines.

[3]



[Turn over

- 7 (a) Given that $\operatorname{cosec} x + \cot x = 4$ and x is acute, show that $\operatorname{cosec} x - \cot x = \frac{1}{4}$. [5]
Hence find the value of $\sec x$.

- (b) Given that $\frac{1+2\cos^2\theta}{\sin^2\theta} = \frac{43}{3}$ for $90^\circ \leq \theta \leq 180^\circ$, find the value of $\frac{\sin\theta}{1+\cos\theta}$,
giving your answer in the form $\frac{1}{3}(a+b\sqrt{c})$ where a , b and c are integers. [5]

8 The quadratic equation $2x^2 + 6x + k = 0$ has roots α and β .

(i) Find the value of $(\alpha - \beta)^2$ in terms of k . [3]

(ii) Find the smallest integer value of k if the equation has no x -intercept. [3]

(iii) Given that the value of k is 5, find a quadratic equation whose roots are

$\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [5]

[Turn over]

9 (a) Find the coefficient of $\frac{1}{x^4}$ in $\left(2x - \frac{1}{x}\right)^{14}$. [3]

(b) (i) Write down, without simplifying, the fourth term in the binomial expansion of $\left(ax + \frac{b}{x}\right)^n$. [2]

(ii) If this term is a constant term, find the value of n . [2]

(iii) The fourth term of the binomial expansion is 160 and $a - b = 1$ where both a and b are positive.

Use the value of n in part (ii) to calculate the value of a and of b . [4]

[Turn over]

- 10** A circle with centre C , passes through the origin and has x -intercept of 4 at point A and y -intercept of 3 at point B .

(i) Explain why the centre of the circle is $(2, 1.5)$. [1]

(ii) Determine the equation of the circle in the general form. [3]

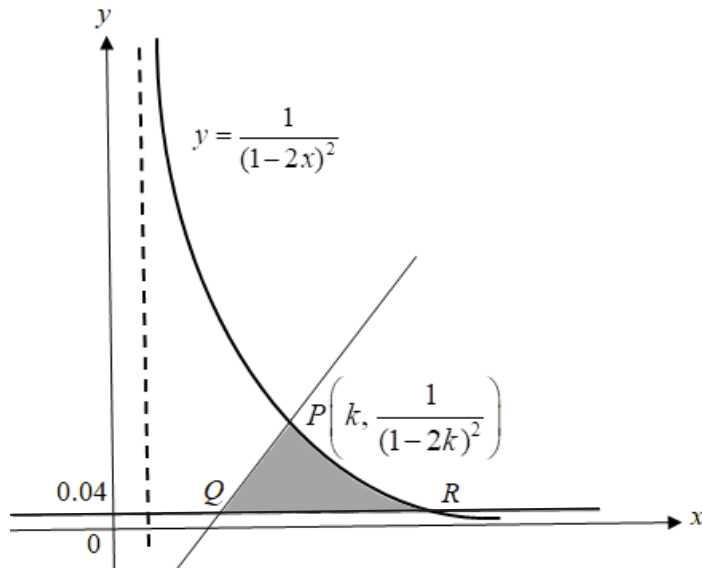
(iii) OD is a diameter of the circle. Find the coordinates of D . [2]

- (iv) The tangent at D cuts the x -axis at E . Find the area of triangle BCE .

[5]

[Turn over

11



The diagram (not drawn to scale) shows part of the curve $y = \frac{1}{(1-2x)^2}$ passing through the point $P\left(k, \frac{1}{(1-2k)^2}\right)$, where k is a constant. The normal to the curve at P is perpendicular to $2y = 1 - x$ and the normal meets the horizontal line $y = 0.04$ at point Q . The curve meets the same horizontal line at the point R .

(i) State the equation of the asymptote denoted by dotted lines. [1]

(ii) Prove that $k = 1.5$. [4]

- (iii) Show that the x -coordinates of Q and R are 1.395 and 3 respectively. [4]

- (iv) Find the area of the shaded region PQR , giving your answer to 2 significant figures. [5]

END OF PAPER

