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NAME: _____ ()

CLASS: 4 ()



**ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020**

S4

ADDITIONAL MATHEMATICS

4047/01

Paper 1

1 September 2020 Tuesday

2 hours

Candidates answer on the Question Paper

Additional Material: 1 Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiners' Use

Examiners' Use					
Question	Marks	Question	Marks	Table of Penalties	
1		7			
2		8		Units	
3		9		Presentation	
4		10		Accuracy	
5		11		Total:	
6		12			
Parent's Name & Signature:				80	
Date:					

This paper consists of **18** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \triangle = \frac{1}{2} ab \sin C$$

1 The roots of the quadratic equation $2x^2 - 6x + 1 = 0$ are α and β .

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Find the value of $\frac{\alpha^2 + \beta^2}{\alpha\beta}$. [1]

(iii) Form a quadratic equation with roots $\frac{\alpha}{\beta} + 2$ and $\frac{\beta}{\alpha} + 2$. [3]

[Turn over]

- 2 The mass, m grams, of a radioactive substance, present at time t days after first being observed, is given by the formula $m = 30 e^{-0.025t}$.

(i) Find the mass remaining after 30 days. [2]

(ii) Find the number of days required for the mass to drop to half of its initial value. Give your answer correct to the nearest integer. [2]

(iii) State the value m approaches when t becomes large. [1]

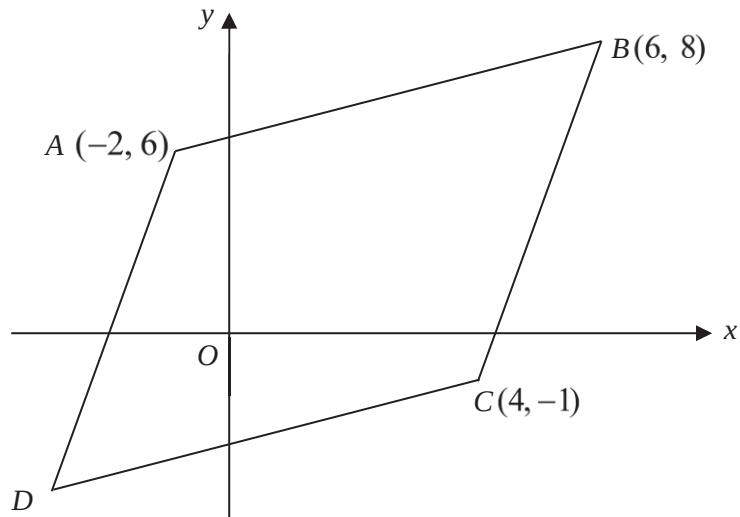
- 3 (a) Find, in ascending powers of x , the first three terms in the expansion of $(2-x)^7$. [2]

Hence, find the value of the constant a for which the coefficient of x^2 in the expansion of $(a-x)(2-x)^7$ is 616. [3]

- (b) In the expansion of $\left(x^2 - \frac{1}{2x^4}\right)^n$ in descending powers of x , the sixth term is independent of x . Find the value of n and the term independent of x . [4]

[Turn over]

4 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a parallelogram $ABCD$ in which $A(-2, 6)$, $B(6, 8)$ and $C(4, -1)$ are the coordinates of its vertices. Find the

(i) equation of AD , [2]

(ii) coordinates of D , [2]

(iii) equation of the perpendicular bisector of the line AD , [2]

(iv) area of the parallelogram $ABCD$, [2]

(v) acute angle the line AB makes with the y -axis. [2]

[Turn over

5 (i) Show that $\frac{d}{dx}(\tan^3 5x) = 15 \sec^4 5x - 15 \sec^2 5x$. [3]

(ii) Use your answers to part (i), find $\int \sec^4 5x \, dx$. [4]

- 6 (i) Given that $y = \frac{3x}{\sqrt{5-4x}}$, express $\frac{dy}{dx}$ in the form $\frac{ax+b}{\sqrt{(5-4x)^n}}$ where a , b and n are real constants. [4]

- (ii) Hence find the equation of the normal to the curve $y = \frac{3x}{\sqrt{5-4x}}$ at the point on the curve where $x = 1$. [2]

[Turn over]

7 (i) On the same diagram, sketch the graphs of $y = 16x^{\frac{5}{3}}$ and $y = \frac{9}{\sqrt[3]{x}}$ for $x > 0$. [3]

(ii) Find the x – coordinate of the intersection point of the two graphs. [2]

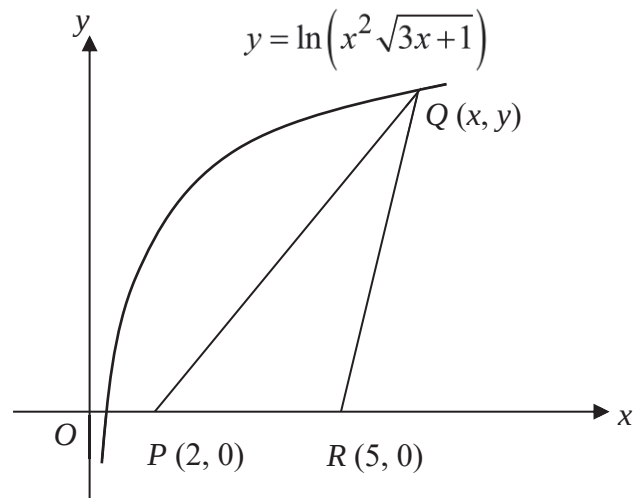
8 Given that $y = \operatorname{cosec} x \tan x$,

(i) show that $\frac{dy}{dx} = \sin x \sec^2 x$, and [2]

(ii) determine where y is decreasing for $0 \leq x \leq 2\pi$. [2]

[Turn over]

- 9 The diagram shows the curve $y = \ln(x^2\sqrt{3x+1})$ and three points $P(2, 0)$, $Q(x, y)$ and $R(5, 0)$. The point $Q(x, y)$ lies on the curve.

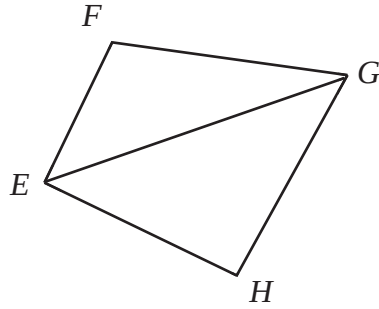


- (i) Show that the area, A units², of the triangle PQR is given by

$$A = 3 \ln x + \frac{3}{4} \ln(3x+1). \quad [2]$$

- (ii) Given that x is increasing at a rate of 0.2 units/s, find the rate at which the area, A , is changing at the instant when $x = 15$ units. [3]

[Turn over



$EFGH$ is a plot of land that comprises two smaller plots, triangle EFG and triangle EGH . EF and EH are perpendicular, angle $FEG = \theta$, $EH = 42$ m, $EG = 55$ m and $EF = 48$ m.

- (i) Show that the area, $A \text{ m}^2$, of $EFGH$ can be expressed as $A = 1320 \sin \theta + 1155 \cos \theta$. [2]

- (ii) Express A in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(iii) Find the value of θ if the area is 1231 m^2 .

[2]

[Turn over

- 11 (a) The variables x and y are related in such a way that when $\frac{x}{y}$ is plotted against $\frac{1}{x}$, a straight line is obtained. The line passes through (2, 9) and (5, 3). Find an expression for y in terms of x . [4]

- (b) The table shows experimental values of two variables, x and y .

x	2	4	6	8
y	8.48	5.99	4.90	4.24

It is known that x and y are related by the equation $x^n y = k$, where n and k are constants. Draw a suitable straight line graph to represent the above data and use it to estimate the values of n and k .

[6]

[Turn over

- 12 Solve the equation $3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)$, giving the principal values of x , in radians. [5]

End of paper

NAME: _____ ()

CLASS: 4 ()



**ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020**

S4

ADDITIONAL MATHEMATICS**4047/02**

Paper 2

28 August 2020 Friday**2 hours 30 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the space at the top of this page.

Write in dark blue or black pen.

You may use a HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiners' Use

Question	Marks	Question	Marks	Table of Penalties	
1		7			
2		8		Units	
3		9		Presentation	
4		10		Accuracy	
5		11		Total:	
6		12			
Parent's Name & Signature:				100	
Date:					

This document consists of **18** printed pages.**[Turn over**

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab \sin C$$

- 1 (a) The equation of a curve is $y = mx^2 + (m-3)x + m$, where m is a constant.

Find the range of values of m for which the curve lies completely above the x -axis. [5]

- (b) Given that $y = ax^2 - 4x + c$ is always negative, give an example of values of a and c which satisfy the condition. [2]

- 2 (a) Given that $2x^4 + 3x^3 + ax^2 - 9x + 9 = (x^2 - 1)(x - 2)Q(x) - 3x^2 + bx + c$ is an identity, state, with reason, the degree of $Q(x)$. [1]

- (b) Find the value of a , of b and of c . [5]

- (c) Hence, find the remainder when $2x^4 + 3x^3 + ax^2 - 9x + 9$ is divided by $(3x - 1)$. [1]

- 3 (a) Given that $p = 3^x$ and $q = 3^y$, express $\log_3 \frac{p^7 q}{243}$ in terms of x and y . [4]

- (b) Given that $\log_2 x - \log_x x^2 = \frac{1}{3} - \log_8 2x$, find the value of x , leaving your answer in index form. [4]

[Turn over]

- 4 (a) Without using a calculator, express $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}}$ in the form of $a\sqrt{10}+b\sqrt{3}$. [4]

- (b) Without the use of a calculator, solve the equation $\sqrt[3]{27^x} - 81^{x+1} = 0$. [3]

- 5 (a) (i) Given the curve $y = -|3x - x^2| + 4$, find the x -coordinates of the points where the curve meets the x -axis. [2]

- (ii) Sketch the curve $y = -|3x - x^2| + 4$, giving the coordinates of the turning point and of the points where the curve meets the axes. [3]

- (b) Explain why there are only two solutions to the equation $-|3x - x^2| = k - 4$ for $k < 1.75$. [2]

- (c) Determine the maximum value for m for which the line $y = mx + 1$ intersects the graph of $y = -|3x - x^2| + 4$ at three points. [1]

[Turn over

- 6 (i) Express $\frac{7x+11}{(x-1)(x+2)^2}$ in partial fractions. [4]

- (ii) Hence, find $\int \frac{7x+11}{2(x-1)(x+2)^2} dx$. [3]

- 7 A piece of wire which has a fixed length of k cm long is bent to form a rectangle. Show that the area of the rectangle is a maximum when it is a square. [5]

[Turn over

8 Given a circle with the equation $(2x+5)(x+2)+(2y+1)(y-5)=0$,

(i) Express the equation of the circle in standard form.

[5]

- (ii) Find the length of the chord when the line $y = -2x$ cuts the circle. [5]

[Turn over

9 (a) (i) Prove the identity $\sin x \cos x + \cot x \cos^2 x = \cot x$. [4]

(ii) Hence, solve $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1$ for $0 \leq x \leq \pi$. [3]

- (b) (i) On the same axes, sketch the graphs of $y = 3 \sin x - 1$ and $y = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$. [5]

- (ii) Hence, state the number of solutions of $3 \sin x - 1 = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$. [1]

[Turn over

- 10** Two particles, A and B , leave a point O at the same time and travel in the same direction along the same straight line.
Particle A starts with a velocity of 9 m/s and moves with a constant acceleration of 2 m/s^2 .
Particle B starts from rest and moves with an acceleration of $a \text{ m/s}^2$, where $a = 1 + \frac{t}{3}$ and t seconds is the time since leaving O . Find
- (a)** an expression for the velocity of each particle in terms of t , [3]

- (b)** an expression for the displacement of each particle in terms of t , [3]

(c) the distance from O at which particle B collides with A , [3]

(d) the speed of each particle at the point of collision. [2]

[Turn over

11 Given that $\cos A = p$ and that A is acute, express the following in terms of p .

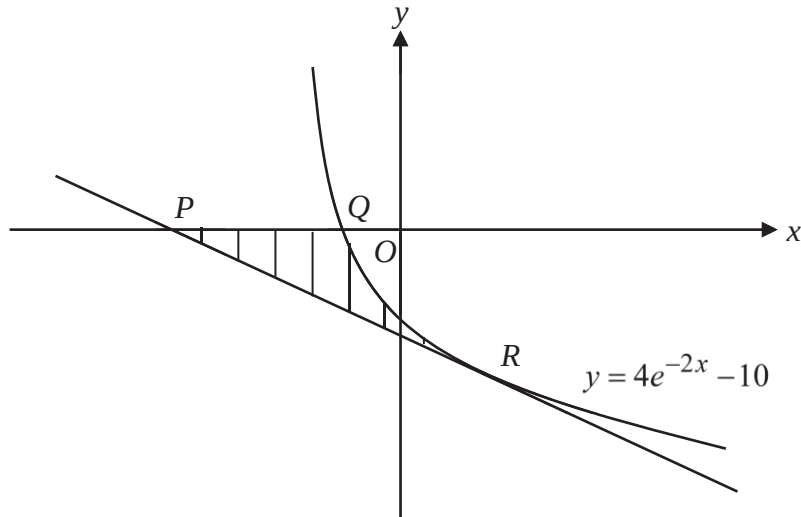
(i) $\sin 2A$ [3]

(ii) $\cos \frac{A}{2}$ [3]

- 12** The diagram shows the curve, $y = 4e^{-2x} - 10$. The curve crosses the x -axis at Q . The line PR is a tangent to the curve at R and intersects the x -axis at P . The x -coordinate of R is $\ln 2$.

Find the area of the shaded region, PQR , which is the region enclosed by curve, the x -axis and the line PR correct to 3 significant figures.

[11]



[Turn over

Continuation of working space for Question 12.

- END OF PAPER –

