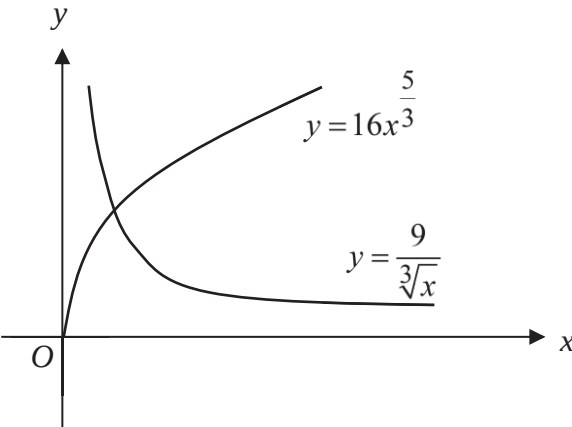
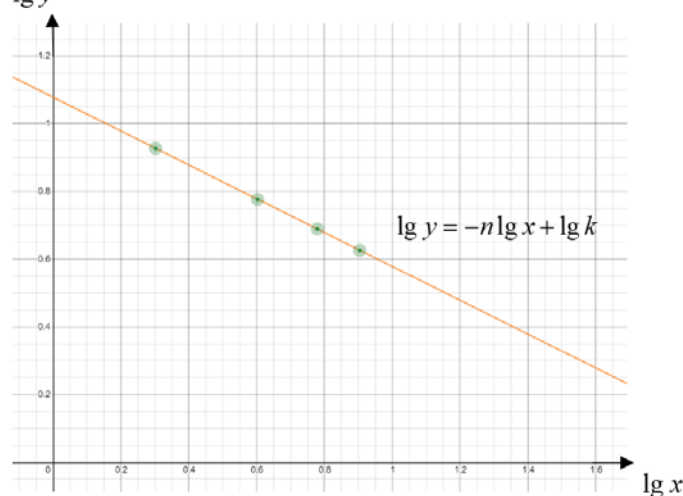
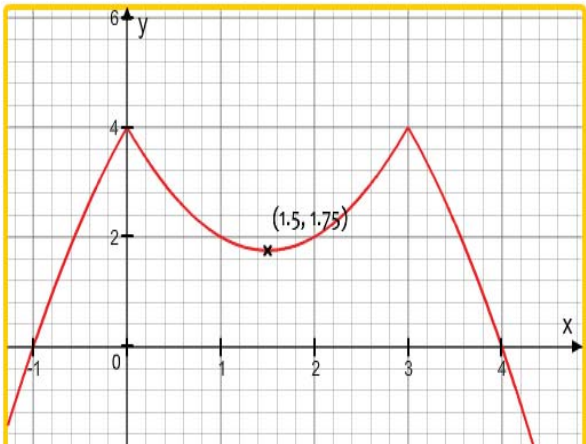
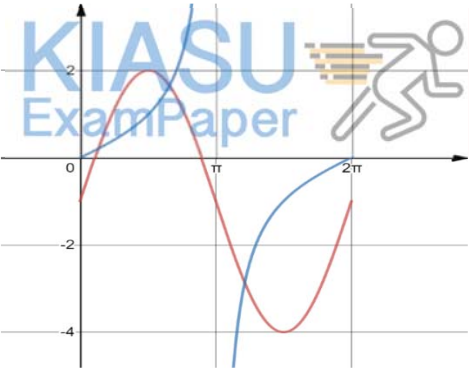


# Answer key for AM P1

|    |                                                                                                                                                                                                                                                                   |    |                                                                                                                                                                                                       |
|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1  | (i) $\alpha^2 + \beta^2 = 8$ (ii) $\frac{\alpha^2 + \beta^2}{\alpha\beta} = 16$<br>(iii) $x^2 + 20x + 37 = 0$                                                                                                                                                     | 2  | (i) The remaining mass after 30 days is 14.2g.<br>(ii) The number of days required is 28 days.<br>(iii) As $t \rightarrow \infty$ , $30e^{-0.025t} \rightarrow 0$ ,<br>the value of $m$ approaches 0. |
| 3  | (a) $a = \frac{1}{4}$ (b) $-\frac{3003}{32}$ or $-93\frac{27}{32}$                                                                                                                                                                                                | 4  | (i) $y = \frac{9}{2}x + 15$ (ii) $(-4, -3)$<br>(iii) $y = -\frac{2}{9}x + \frac{5}{6}$ (iv) 68 units <sup>2</sup> (v) 76.0° (1 d.p.)                                                                  |
| 5  | (ii) $\frac{1}{15}\tan^3 5x + \frac{1}{5}\tan 5x + C_2$                                                                                                                                                                                                           |    |                                                                                                                                                                                                       |
| 6  | (i) $\frac{15-6x}{\sqrt{(5-4x)^3}}$ (ii) $9y + x = 28$                                                                                                                                                                                                            | 7  | (i)<br>                                                                                                            |
| 8  | (ii) For a decreasing function, $\frac{dy}{dx} < 0$<br>For $0 \leq x \leq 2\pi$ ,<br>$\sec^2 x > 0$ , $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$ ,<br>$\sin x < 0$ for $\pi < x < 2\pi$<br>Hence, $y$ is decreasing for<br>$\pi < x < 2\pi$ , $x \neq \frac{3\pi}{2}$ |    | (ii) $x = \frac{3}{4}$ ( $x > 0$ )                                                                                                                                                                    |
| 9  | (ii) The area is increasing at 0.0498 units <sup>2</sup> /s.                                                                                                                                                                                                      |    |                                                                                                                                                                                                       |
| 10 | (ii) $A \approx \sqrt{3076425} \sin(\theta + 41.2^\circ)$ or $A \approx 165\sqrt{113} \sin(\theta + 41.2^\circ)$<br>(iii) $\theta = 3.4^\circ$                                                                                                                    |    |                                                                                                                                                                                                       |
| 12 | $\cot x = -\frac{1}{3}$ $\cot x = 2$<br>$\tan x = -3$ or $\tan x = \frac{1}{2}$<br>$x = -1.25$ $x = 0.464$                                                                                                                                                        | 11 | (i) $y = \frac{x^2}{13x-2}$<br>(ii)<br><br>$k = 12.0$ (11.5–12.6)<br>$n = 0.500$ (0.45–0.55)                      |

## Answer key for AM paper 2

|    |                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                             |
|----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1  | (a) $m > 1$<br>(b) $a = -1$ and $c = -5$ or<br>$a = -2$ and $c = -2$ or<br>$a = -5$ and $c = -1$ or any pairs of values that fulfill the above criteria                                                                                          | 2                                                                                                                                                                                                                                                                                                                                                 | (a) Since degree of dividend = degree of quotient + degree of divisor,<br>degree of $Q(x) = 1$<br><br>ALTERNATIVE: Must multiply with $x^3$ to give degree 4 in the polynomial on the left.<br><br>(b) $a = -19$ , $b = -6$ , $c = -5$ (c) $\frac{326}{81}$ |
| 3  | (a) $7x + y - 5$<br>(b) $x = 2^{\frac{3}{2}}$ or $x = 8^{\frac{1}{2}}$ or $x = 64^{\frac{1}{4}}$                                                                                                                                                 | 5                                                                                                                                                                                                                                                                                                                                                 | (a)(i) $x = 4$ or $x = -1$<br>(ii)                                                                                                                                                                                                                          |
| 4  | (a) $\frac{4}{13}\sqrt{10} - \frac{7}{13}\sqrt{3}$ (b) $x = -\frac{4}{3}$                                                                                                                                                                        |                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                             |
| 6  | (i) $\frac{2}{x-1} - \frac{2}{x+2} + \frac{1}{(x+2)^2}$<br>(ii) $\ln(x-1) - \ln(x+2) - \frac{1}{2(x+2)} + c$                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                             |
| 7  | $x = \frac{k}{4}$ , $\frac{d^2 A}{dx^2} = -2 < 0$<br>Therefore, since the stationary value occurs when the sides of the rectangle are $\frac{k}{4}$ cm, and it is a maximum value, the maximum area of the rectangle occurs when it is a square. | <p>(b)</p> $- 3x - x^2  = k - 4$ $- 3x - x^2  + 4 = k$ $y = k$ <p><math>y = k</math> is a <b>horizontal line</b> and when <math>k</math> is lesser than 2.25, it will be <b>below the turning point</b> and so it will only <b>intersect the curve at the two outer arms thereby giving two solutions only.</b></p> <p>(c) <math>m = 1</math></p> |                                                                                                                                                                                                                                                             |
| 8  | (i) $\left(x + \frac{9}{4}\right)^2 + \left(y - \frac{9}{4}\right)^2 = \frac{61}{8}$<br>(ii) 5.14 units (to 3 s.f.)                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                             |
| 9  | (a) (ii) $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$ rad<br>(b)(i)                                                                                                                                                                     |                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                             |
|    | (ii) From the sketch, the two functions intersect at three points. Hence there are three solutions for the equation for $0 \leq x \leq 2\pi$                                                                                                     |                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                             |
| 10 | (a) $v_B = t + \frac{1}{6}t^2$<br><br>(b) $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3$<br><br>(c) 486 m<br><br>(d) 72 m/s                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                             |

|    |                                                                                      |    |                                                                                                                                                                      |
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| 11 | <p>(i) <math>2p\sqrt{1-p^2}</math></p> <p>(ii) <math>\sqrt{\frac{p+1}{2}}</math></p> | 12 | <p><math>R(\ln 2, -9) \quad P\left(\ln 2 - \frac{9}{2}, 0\right) \quad Q\left(-\frac{1}{2} \ln \frac{5}{2}, 0\right)</math></p> <p>Area = 13.2 units<sup>2</sup></p> |
|----|--------------------------------------------------------------------------------------|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|



**ANGLICAN HIGH SCHOOL  
SECONDARY FOUR  
PRELIMINARY EXAMINATIONS 2020**



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**ADDITIONAL MATHEMATICS**

**4047/01**

Paper 1

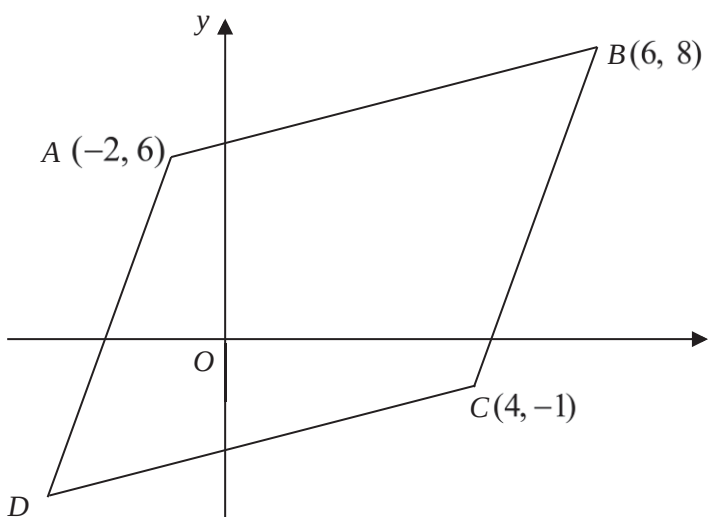
**1 September 2020 Tuesday  
2 hours**

**Marking Scheme**

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1     | <p>The roots of the quadratic equation <math>2x^2 - 6x + 1 = 0</math> are <math>\alpha</math> and <math>\beta</math>.</p> <p>(i) Find the value of <math>\alpha^2 + \beta^2</math>. [3]</p> <p>(ii) Find the value of <math>\frac{\alpha^2 + \beta^2}{\alpha\beta}</math>. [1]</p> <p>(iii) Form a quadratic equation with roots <math>\frac{\alpha}{\beta} + 2</math> and <math>\frac{\beta}{\alpha} + 2</math>. [3]</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 1(i)  | <div> <math display="block">\alpha + \beta = -\left(\frac{-6}{2}\right) = 3</math> <math display="block">\alpha\beta = \frac{1}{2}</math> <math display="block">(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2</math> <math display="block">\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta</math> <math display="block">\alpha^2 + \beta^2 = (3)^2 - 2\left(\frac{1}{2}\right)</math> <math display="block">\alpha^2 + \beta^2 = 8</math> </div> <div> <p>M1 for both roots</p> <p>M1</p> <p>A1</p> </div>                                                                                                                                                                                                                                                                                                                                                   |
| (ii)  | <div> <math display="block">\frac{\alpha^2 + \beta^2}{\alpha\beta} = 8 \div \frac{1}{2} = 16</math> </div> <div> <p>B1</p> </div>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| (iii) | <div> <math display="block">\frac{\alpha}{\beta} + 2 + \frac{\beta}{\alpha} + 2 = \frac{\alpha^2 + \beta^2}{\alpha\beta} + 4</math> <math display="block">\frac{\alpha}{\beta} + 2 + \frac{\beta}{\alpha} + 2 = 16 + 4 = 20</math> <math display="block">\left(\frac{\alpha}{\beta} + 2\right)\left(\frac{\beta}{\alpha} + 2\right) = 1 + \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} + 4</math> <math display="block">\left(\frac{\alpha}{\beta} + 2\right)\left(\frac{\beta}{\alpha} + 2\right) = 5 + \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}</math> <math display="block">\left(\frac{\alpha}{\beta} + 2\right)\left(\frac{\beta}{\alpha} + 2\right) = 5 + 2(16) = 37</math> <math display="block">x^2 - (20)x + 37 = 0</math> <math display="block">x^2 - 20x + 37 = 0</math> </div> <div> <p>M1 for sum of roots</p> <p>M1 for product of roots</p> <p>A1</p> </div> |

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2     | <p>The mass, <math>m</math> grams, of a radioactive substance, present at time <math>t</math> days after first being observed, is given by the formula <math>m = 30 e^{-0.025t}</math>.</p> <p>(i) Find the mass remaining after 30 days. [2]</p> <p>(ii) Find the number of days required for the mass to drop to half of its value at <math>t = 0</math>. Give your answer correct to the nearest integer. [2]</p> <p>(iii) State the value <math>m</math> approaches when <math>t</math> becomes large. [1]</p> |
| 2(i)  | <div> <math>m = 30 e^{-0.025(30)}</math><br/> <math>= 14.171</math><br/> <math>= 14.2</math><br/>           The remaining mass after 30 days is 14.2g.         </div> <div>           M1<br/><br/>           A1 – 0 mark for omission of unit in answer         </div>                                                                                                                                                                                                                                             |
| (ii)  | <div> <math>15 = 30 e^{-0.025t}</math><br/> <math>e^{-0.025t} = \frac{1}{2}</math><br/> <math>-0.025t = \ln \frac{1}{2}</math><br/> <math>t = 27.726</math><br/> <math>t = 28</math><br/>           The number of days required is 28 days.         </div> <div>           B1<br/><br/><br/><br/><br/><br/><br/><br/><br/><br/>           A1         </div>                                                                                                                                                        |
| (iii) | <div>           As <math>t \rightarrow \infty</math>, <math>30 e^{-0.025t} \rightarrow 0</math>,<br/>           the value of <math>m</math> approaches 0.         </div> <div>           A1         </div>                                                                                                                                                                                                                                                                                                         |



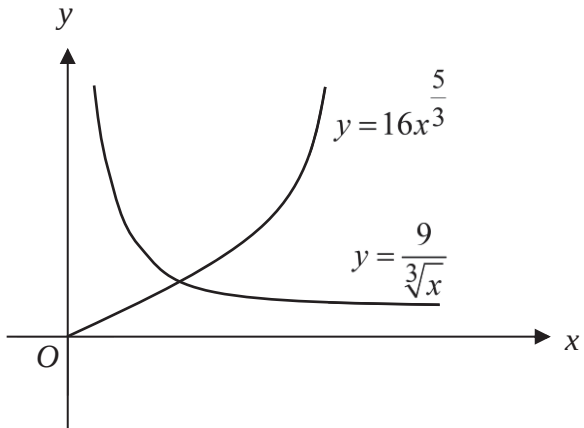
|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4    | <p><b>Solutions to this question by accurate drawing will not be accepted.</b></p>  <p>The diagram shows a parallelogram <math>ABCD</math> in which <math>A(-2, 6)</math>, <math>B(6, 8)</math> and <math>C(4, -1)</math> are the coordinates of its vertices. Find the</p> <p>(i) equation of <math>AD</math>, [2]<br/> (ii) coordinates of <math>D</math>, [2]<br/> (iii) equation of the perpendicular bisector of the line <math>AD</math>, [2]<br/> (iv) area of the parallelogram <math>ABCD</math>, [2]<br/> (v) acute angle the line <math>AB</math> makes with the <math>y</math>-axis. [2]</p> |
| 4(i) | <p>Gradient of <math>AD</math> = Gradient of <math>BC</math></p> $= \frac{8+1}{6-4}$ $= \frac{9}{2}$ <p>Equation of <math>AD</math> :</p> $\frac{y-6}{x+2} = \frac{9}{2}$ $y = \frac{9}{2}x + 15$ <p>M1</p> <p>A1</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| (ii) | <p>Coordinates of <math>D = (-2-2, 6-9)</math></p> $= (-4, -3)$ <p>M1</p> <p>A1</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

|       |                                                                                                                                                                                                                                                                                                                                                                        |                                                               |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------|
| (iii) | <p>Gradient of Perpendicular Bisector <math>= -\frac{2}{9}</math></p> <p>Midpoint of <math>AD = \left( \frac{-2+(-4)}{2}, \frac{-3+6}{2} \right)</math></p> <p><math>= \left( -3, \frac{3}{2} \right)</math></p> <p>Equation of Perpendicular Bisector</p> <p><math>y - \frac{3}{2} = -\frac{2}{9}(x+3)</math></p> <p><math>y = -\frac{2}{9}x + \frac{5}{6}</math></p> | <p>M1</p> <p>A1</p>                                           |
| (iv)  | <p>Area of <math>ABCD</math></p> <p><math>= \frac{1}{2} \begin{vmatrix} -2 &amp; -4 &amp; 4 &amp; 6 &amp; -2 \\ 6 &amp; -3 &amp; -1 &amp; 8 &amp; 6 \end{vmatrix}</math></p> <p><math>= \frac{1}{2} [(6+4+32+36) - (-24-12-6-16)]</math></p> <p><math>= 68 \text{ units}^2</math></p>                                                                                  | <p>M1</p> <p>clockwise and -68<br/>(max 1 mark)</p> <p>A1</p> |
| (v)   | <p>Angle made by line <math>AB</math> with <math>y</math>-axis</p> <p><math>= \tan^{-1} \left( \frac{6+2}{8-6} \right)</math></p> <p><math>= 76.0^\circ (1 \text{ d.p.})</math></p>                                                                                                                                                                                    | <p>M1</p> <p>A1 accept both<br/>degrees and radians</p>       |

|          |                                                                                                                                                                                                                              |                                                                                              |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| <b>5</b> | <p>(i) Show that <math>\frac{d}{dx}(\tan^3 5x) = 15 \sec^4 5x - 15 \sec^2 5x</math>. [3]</p> <p>(ii) Use your answers to part (i), find <math>\int \sec^4 5x \, dx</math>. [4]</p>                                           |                                                                                              |
| (i)      | <p><math>\frac{d}{dx}(\tan^3 5x) = 3(\tan^2 5x)(\sec^2 5x)(5)</math></p> <p><math>= 15(\tan^2 5x)(\sec^2 5x)</math></p> <p><math>= 15(\sec^2 5x - 1)(\sec^2 5x)</math></p> <p><math>= 15 \sec^4 5x - 15 \sec^2 5x</math></p> | <p>M1 – power</p> <p>M1 – chain rule</p> <p>differentiate <math>\tan 5x</math></p> <p>M1</p> |

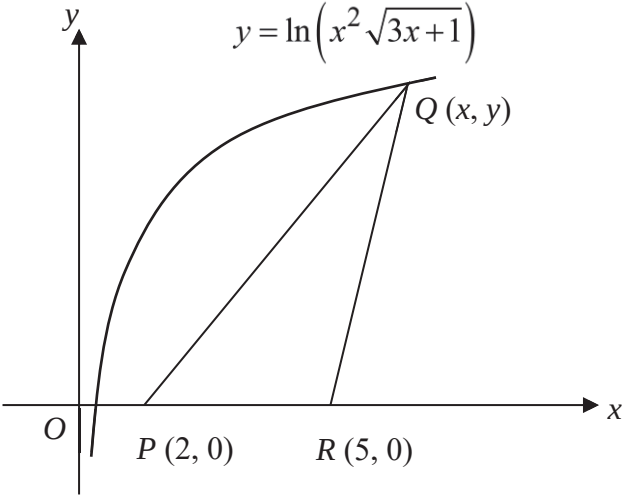
|      |                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                  |
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| (ii) | $\frac{d}{dx}(\tan^3 5x) = 15 \sec^4 5x - 15 \sec^2 5x$ <p>Integrate w.r.t. <math>x</math></p> $\tan^3 5x + C_1 = \int (15 \sec^4 5x - 15 \sec^2 5x) dx$ $\tan^3 5x + C_1 = 15 \int \sec^4 5x dx - 15 \int \sec^2 5x dx$ $\tan^3 5x + C_1 = 15 \int \sec^4 5x dx - 15 \left( \frac{\tan 5x}{5} \right)$ $\tan^3 5x + 3 \tan 5x + C_1 = 15 \int \sec^4 5x dx$ $\int \sec^4 5x dx = \frac{1}{15} \tan^3 5x + \frac{1}{5} \tan 5x + C_2$ | <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>If everything is C,<br/>minus 1 mark for<br/>overall presentation</p> |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 6    | <p>(i) Given that <math>y = \frac{3x}{\sqrt{5-4x}}</math>, express <math>\frac{dy}{dx}</math> in the form <math>\frac{ax+b}{\sqrt{(5-4x)^n}}</math> where <math>a</math>, <math>b</math> and <math>n</math> are real constants. [4]</p> <p>(ii) Hence find the equation of the normal to the curve <math>y = \frac{3x}{\sqrt{5-4x}}</math> at the point on the curve where <math>x = 1</math>. [2]</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 6(i) | <div style="display: flex; justify-content: space-between;"> <div style="width: 65%;"> <math display="block">y = \frac{3x}{\sqrt{5-4x}}</math> <math display="block">\frac{dy}{dx} = \frac{\sqrt{5-4x} \frac{d}{dx}(3x) - (3x) \frac{d}{dx} \sqrt{5-4x}}{(\sqrt{5-4x})^2}</math> <math display="block">= \frac{\sqrt{5-4x}(3) - (3x) \frac{1}{2}(5-4x)^{-\frac{1}{2}}(-4)}{5-4x}</math> <math display="block">= \frac{\sqrt{5-4x}(3) + \frac{(3x)2}{\sqrt{5-4x}}}{5-4x}</math> <math display="block">= \frac{\sqrt{5-4x}(3) + \frac{6x}{\sqrt{5-4x}}}{5-4x} \times \frac{\sqrt{5-4x}}{\sqrt{5-4x}}</math> <math display="block">= \frac{(5-4x)(3) + 6x}{(5-4x)\sqrt{5-4x}}</math> <math display="block">= \frac{15-6x}{\sqrt{(5-4x)^3}}</math> </div> <div style="width: 30%; text-align: center;"> <p>M1 – Quotient Rule<br/>M1 – Chain Rule</p> <p>M1</p> <p>A1</p> </div> </div> |
| (ii) | <div style="display: flex; justify-content: space-between;"> <div style="width: 65%;"> <p><math>x = 1, y = \frac{3(1)}{\sqrt{5-4(1)}} = 3</math></p> <p>Gradient of the tangent = <math>\frac{15-6(1)}{\sqrt{(5-4(1))^3}}</math></p> <p style="text-align: center;"><math>= 9</math></p> <p>Gradient of the normal = <math>-\frac{1}{9}</math></p> <p>Equation of normal</p> <math display="block">y - 3 = -\frac{1}{9}(x - 1)</math> <math display="block">y = -\frac{1}{9}x + \frac{28}{9}</math> <math display="block">9y + x = 28</math> </div> <div style="width: 30%; text-align: center;"> <p>M1 – find gradient of tangent and normal</p> <p>A1</p> </div> </div>                                                                                                                                                                                                           |

|      |                                                                                                                                                                                                                                                                       |                                                                                                                                                                                |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 7    | <p>(i) On the same diagram, sketch the graphs of <math>y = 16x^{\frac{5}{3}}</math> and <math>y = \frac{9}{\sqrt[3]{x}}</math> for <math>x &gt; 0</math>. [3]</p> <p>(ii) Find the <math>x</math> – coordinate of the intersection point of the two graphs. . [2]</p> |                                                                                                                                                                                |
|      |                                                                                                                                                                                      | <p>B1 – For shape of <math>y = \frac{9}{\sqrt[3]{x}}</math></p> <p>B1 – For asymptotes</p> <p>B1 – For shape of <math>y = 16x^{\frac{5}{3}}</math> and pass through origin</p> |
| (ii) | $\frac{9}{\sqrt[3]{x}} = 16x^{\frac{5}{3}}$ $\frac{9}{16} = x^{\frac{5}{3}} \times \sqrt[3]{x}$ $x^{\frac{5}{3}} \times x^{\frac{1}{3}} = \frac{9}{16}$ $x^2 = \frac{9}{16}$ $x = \frac{3}{4} \quad (x > 0)$                                                          | <p>M1</p> <p>A1</p>                                                                                                                                                            |

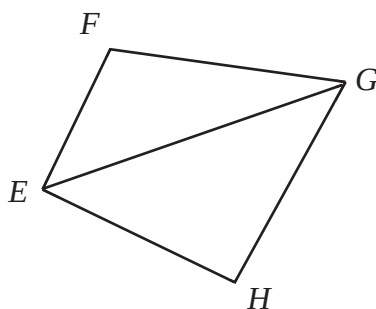


|      |                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                           |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
|      | $y = \operatorname{cosec} x \tan x$ $= \frac{\tan x}{\sin x}$ $\frac{dy}{dx} = \frac{\sin x \sec^2 x - \tan x \cos x}{\sin^2 x}$ $= \frac{\sin x \sec^2 x - \sin x}{\sin^2 x}$ $= \frac{\sin x (\sec^2 x - 1)}{\sin^2 x}$ $= \frac{\sin x (\tan^2 x)}{\sin^2 x}$ $= \frac{1}{\sin x} \left( \frac{\sin^2 x}{\cos^2 x} \right)$ $= \frac{1}{\cos^2 x} (\sin x)$ $= \sin x \sec^2 x$         | <p>M1</p> <p>M1</p>                                                                       |
| (ii) | <p>For a decreasing function, <math>\frac{dy}{dx} &lt; 0</math></p> <p>For <math>0 \leq x \leq 2\pi</math>, <math>\sec^2 x &gt; 0</math>, <math>x \neq \frac{\pi}{2}, \frac{3\pi}{2}</math>, <math>\sin x &lt; 0</math> for <math>\pi &lt; x &lt; 2\pi</math></p> <p>Hence <math>\frac{dy}{dx} &lt; 0</math> for <math>\pi &lt; x &lt; 2\pi</math>, <math>x \neq \frac{3\pi}{2}</math></p> | <p>B1</p> <p>B1 - if didn't write <math>x \neq \frac{3\pi}{2}</math>, its acceptable.</p> |

|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                     |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|
| 9    | <p>The diagram shows the curve <math>y = \ln(x^2\sqrt{3x+1})</math> and three points <math>P(2, 0)</math>, <math>Q(x, y)</math> and <math>R(5, 0)</math>. The point <math>Q(x, y)</math> lies on the curve.</p>  <p>(i) Show that the area, <math>A</math> units<sup>2</sup>, of the triangle <math>PQR</math> is given by</p> $A = 3 \ln x + \frac{3}{4} \ln(3x+1) \quad [2]$ <p>(ii) Given that <math>x</math> is increasing at a rate of 0.2 units/s, find the rate at which the area, <math>A</math>, is changing at the instant when <math>x = 15</math> units. <span style="float: right;">[3]</span></p> |                                                     |
| 9(i) | $A = \frac{1}{2} \times PR \times y = \frac{1}{2} \times 3 \times \ln(x^2\sqrt{3x+1})$ $= \frac{3}{2} \ln(x^2\sqrt{3x+1})$ $A = \frac{3}{2} (\ln x^2 + \ln \sqrt{3x+1})$ $= \frac{3}{2} \left( 2 \ln x + \ln(3x+1)^{\frac{1}{2}} \right)$ $= \frac{3}{2} \left( 2 \ln x + \frac{1}{2} \ln(3x+1) \right)$ $= 3 \ln x + \frac{3}{4} \ln(3x+1)$                                                                                                                                                                                                                                                                                                                                                      | <p>M1</p> <p>M1 – apply the laws for logarithms</p> |
| (ii) | <p><math>\frac{dx}{dt} = 0.2 \text{ m/s}</math> and <math>x = 15</math></p> $A = 3 \ln x + \frac{3}{4} \ln(3x+1)$ $\frac{dA}{dx} = 3 \left( \frac{1}{x} \right) + \frac{3}{4} \left( \frac{1}{3x+1} \right) (3)$ $= \frac{3}{x} + \frac{9}{4(3x+1)}$                                                                                                                                                                                                                                                                                                                                                                                                                                              | <p>M1 – differentiation of logarithms</p>           |

|  |                                                                                                                                                                                                                               |              |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|
|  | $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $= \left( \frac{3}{(15)} + \frac{9}{4(3(15)+1)} \right) \times (0.2)$ $= 0.0498 \text{ units}^2 / \text{s}$ <p>The area is increasing at 0.0498 units<sup>2</sup>/s.</p> | M1<br><br>A1 |
|--|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|

10



$EFGH$  is a plot of land that comprises two smaller plots, triangle  $EFG$  and triangle  $EGH$ .  $EF$  and  $EH$  are perpendicular, angle  $FEG = \theta$ ,  $EH = 42$  m,  $EG = 55$  m and  $EF = 48$  m.

- (i) Show that the area,  $A$  m<sup>2</sup>, of  $EFGH$  can be expressed as

$$A = 1320 \sin \theta + 1155 \cos \theta. \quad [2]$$

- (ii) Express  $A$  in the form  $R \sin(\theta + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [3]

- (iii) Find the value of  $\theta$  if the area is 1231 m<sup>2</sup>. [2]

10(i)

$$\begin{aligned} \text{Area of triangle } EFG &= \frac{1}{2} \times EF \times EG \sin \angle FEG \\ &= \frac{1}{2} \times 48 \times 55 \sin \theta \\ &= 1320 \sin \theta \end{aligned}$$

M1 – area of triangle

$$\begin{aligned} \text{Area of triangle } EGH &= \frac{1}{2} \times EH \times EG \sin \angle FEG \\ &= \frac{1}{2} \times 42 \times 55 \sin(90^\circ - \theta) \\ &= 1155(\sin 90^\circ \cos \theta - \cos 90^\circ \sin \theta) \\ &= 1155 \cos \theta \end{aligned}$$

M1 – use of trigonometry

$$\begin{aligned} A &= \text{Area of triangle } EFG + \text{Area of triangle } EGH \\ &= 1320 \sin \theta + 1155 \cos \theta \end{aligned}$$

(ii)

$$1320 \sin \theta + 1155 \cos \theta = R \sin(\theta + \alpha) = R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$$

$$R = \sqrt{1320^2 + 1155^2} = \sqrt{3076425}$$

$$\tan \alpha = \frac{1155}{1320}, \quad \alpha = \tan^{-1}\left(\frac{1155}{1320}\right) = 41.1859^\circ$$

$$A = \sqrt{3076425} \sin(\theta + 41.1859^\circ)$$

Or

$$A \approx 165\sqrt{113} \sin(\theta + 41.2^\circ)$$

M1 – finding  $R$  and  $\alpha$   
A1

A1

|       |                                                                                                                                                                                                                                                                                                                                                                  |                          |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|
| (iii) | $\sqrt{3076425} \sin(\theta + 41.1859^\circ) = 1231$<br>$\sin(\theta + 41.1859^\circ) = \frac{1231}{\sqrt{3076425}}$<br>Basic angle = $\sin^{-1}\left(\frac{1231}{\sqrt{3076425}}\right)$<br>$= 44.57449^\circ$<br>$\theta + 41.1859^\circ = 44.57449^\circ, 180 - 44.57449^\circ$<br>$\theta = 3.3885^\circ, 94.2397^\circ (\text{NA})$<br>$\theta = 3.4^\circ$ | M1<br><br><br><br><br>A1 |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|

|                                                                                                                                                                                                                                                                                                                                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                |                                         |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------|
| 11                                                                                                                                                                                                                                                                                                                                             | <p>(a) The variables <math>x</math> and <math>y</math> are related in such a way that when <math>\frac{x}{y}</math> is plotted against <math>\frac{1}{x}</math>, a straight line is obtained. The line passes through (2, 9) and (5, 3). Find an expression for <math>y</math> in terms of <math>x</math>. [4]</p>                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                |                                         |
| 11(i)                                                                                                                                                                                                                                                                                                                                          | <table border="1"> <tr> <td data-bbox="268 421 1129 990"> <p>Let <math>Y = \frac{x}{y}, X = \frac{1}{x}</math></p> <p>Gradient = <math>\frac{9-3}{2-5} = -2</math></p> <p><math>\therefore Y - 3 = -2(X - 5)</math></p> <p><math>Y = -2X + 13</math></p> <p><math>\frac{x}{y} = -\frac{2}{x} + 13</math></p> <p><math>\frac{x}{y} = \frac{13x-2}{x}</math></p> <p><math>y = \frac{x^2}{13x-2}</math></p> </td><td data-bbox="1129 421 1422 990"> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> </td></tr> </table> | <p>Let <math>Y = \frac{x}{y}, X = \frac{1}{x}</math></p> <p>Gradient = <math>\frac{9-3}{2-5} = -2</math></p> <p><math>\therefore Y - 3 = -2(X - 5)</math></p> <p><math>Y = -2X + 13</math></p> <p><math>\frac{x}{y} = -\frac{2}{x} + 13</math></p> <p><math>\frac{x}{y} = \frac{13x-2}{x}</math></p> <p><math>y = \frac{x^2}{13x-2}</math></p> | <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> |
| <p>Let <math>Y = \frac{x}{y}, X = \frac{1}{x}</math></p> <p>Gradient = <math>\frac{9-3}{2-5} = -2</math></p> <p><math>\therefore Y - 3 = -2(X - 5)</math></p> <p><math>Y = -2X + 13</math></p> <p><math>\frac{x}{y} = -\frac{2}{x} + 13</math></p> <p><math>\frac{x}{y} = \frac{13x-2}{x}</math></p> <p><math>y = \frac{x^2}{13x-2}</math></p> | <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                |                                         |

(b) The table shows experimental values of two variables,  $x$  and  $y$ .

|     |      |      |      |      |
|-----|------|------|------|------|
| $x$ | 2    | 4    | 6    | 8    |
| $y$ | 8.48 | 5.99 | 4.90 | 4.24 |

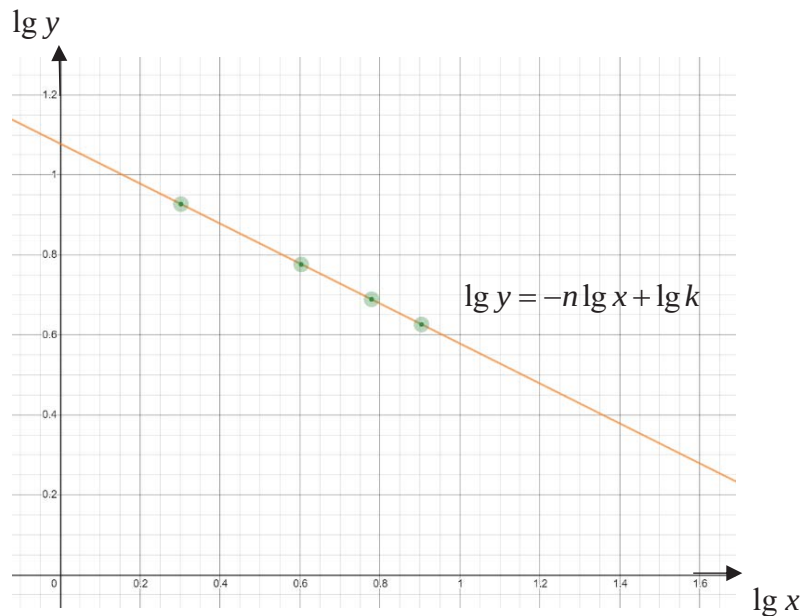
It is known that  $x$  and  $y$  are related by the equation  $x^n y = k$ , where  $n$  and  $k$  are constants. Draw a suitable straight line graph to represent the above data and use it to estimate the values of  $n$  and  $k$ . [6]

(ii)

$$x^n y = k$$

$$\lg y = -n \lg x + \lg k$$

|         |       |       |       |       |
|---------|-------|-------|-------|-------|
| $x$     | 2     | 4     | 6     | 8     |
| $y$     | 8.48  | 5.99  | 4.90  | 4.24  |
| $\lg x$ | 0.301 | 0.602 | 0.778 | 0.903 |
| $\lg y$ | 0.928 | 0.777 | 0.690 | 0.627 |



$$\lg k = 1.08 \quad (1.06 - 1.10)$$

$$k = 12.0 \quad (11.5 - 12.6)$$

$$-n = \frac{1.00 - 0.60}{0.15 - 0.95}$$

$$= -0.500$$

$$\therefore n = 0.500 \quad (0.45 - 0.55)$$

M1

B1

G1 – correct points

G1 – y-intercept

Deduct 1 mark for labels

A1

A1

|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 12 | Solve the equation $3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)$ , giving the principal values of $x$ , in radians. [5]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|    | <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <math display="block">3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)</math> <math display="block">3\operatorname{cosec}^2 x \sin x = 5\cos x + 5\sin x</math> <math display="block">3\operatorname{cosec}^2 x = \frac{5\cos x + 5\sin x}{\sin x}</math> <math display="block">= \frac{5\cos x}{\sin x} + \frac{5\sin x}{\sin x}</math> <math display="block">= 5\cot x + 5</math> <math display="block">3\operatorname{cosec}^2 x = 5\cot x + 5</math> <math display="block">3(\cot^2 x + 1) - 5\cot x - 5 = 0</math> <math display="block">3\cot^2 x + 3 - 5\cot x - 5 = 0</math> <math display="block">3\cot^2 x - 5\cot x - 2 = 0</math> <math display="block">(3\cot x + 1)(\cot x - 2) = 0</math> <math display="block">\cot x = -\frac{1}{3} \qquad \cot x = 2</math> <math display="block">\tan x = -3 \qquad \text{or} \qquad \tan x = \frac{1}{2}</math> <math display="block">x = -1.25 \qquad \qquad \qquad x = 0.464</math> </div> <div style="width: 50%;"> <p>M1 – use of <math>\cot x = \frac{\cos x}{\sin x}</math>,<br/> <math>\operatorname{cosec} x = \frac{1}{\sin x}</math>, <math>\tan x = \frac{\sin x}{\cos x}</math> etc</p> <p>M1 – use identity to change into an equation with single trigonometric term</p> <p>M1 – reach quadratic equation and factorize, or use formula</p> <p>A2 – deduct one mark if more than the principal values are given.</p> </div> </div> |



ADDITIONAL MATHEMATICS

Paper 2

4047/02

28 August 2020 Friday

2 hours 30 minutes

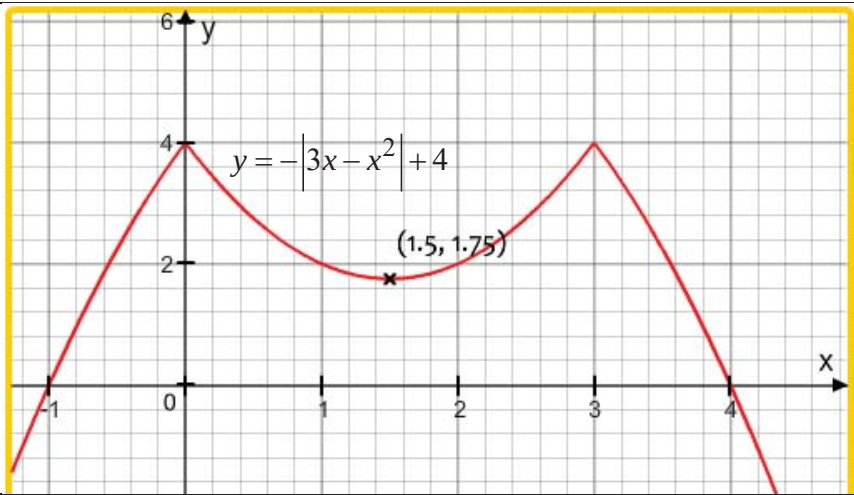
Marking Scheme

|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                   |
|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------|
| 1a | <p>The equation of a curve is <math>y = mx^2 + (m-3)x + m</math>, where <math>m</math> is a constant.<br/>Find the range of values of <math>m</math> for which the curve lies completely above the <math>x</math>-axis.</p>                                                                                                                                                                                                                                                                                                                                                                                 | [5]                                               |
|    | <p>Since the curve is completely above the <math>x</math>-axis,<br/>the curve must be a quadratic curve with minimum point, <math>a &gt; 0</math><br/><math>\therefore m &gt; 0</math></p> <p>And there will be no point of intersection with the <math>x</math>-axis, <math>b^2 - 4ac &lt; 0</math><br/> <math>(m-3)^2 - 4(m)(m) &lt; 0</math><br/> <math>m^2 - 6m + 9 - 4m^2 &lt; 0</math><br/> <math>-3m^2 - 6m + 9 &lt; 0</math><br/> <math>m^2 + 2m - 3 &gt; 0</math><br/> <math>(m+3)(m-1) &gt; 0</math><br/> <math>m &lt; -3</math> or <math>m &gt; 1</math></p> <p>Hence, <math>m &gt; 1</math></p> | <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> |
| 1b | <p>Given that <math>y = ax^2 - 4x + c</math> is always negative, give an example of values of <math>a</math> and <math>c</math> which satisfy the condition.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                            | [2]                                               |
|    | <p><math>b^2 - 4ac &lt; 0</math><br/> <math>(-4)^2 - 4ac &lt; 0</math><br/> <math>ac &gt; 4</math>, <math>a &lt; 0</math></p> <p><math>a = -1</math> and <math>c = -5</math> or<br/> <math>a = -2</math> and <math>c = -2</math> or<br/> <math>a = -5</math> and <math>c = -1</math> or any pairs of values that fulfill the above criteria</p>                                                                                                                                                                                                                                                             | <p>M1</p> <p>A1</p>                               |

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                             |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------|
|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                             |
| <b>2a</b> | Given that $2x^4 + 3x^3 + ax^2 - 9x + 9 = (x^2 - 1)(x - 2)Q(x) - 3x^2 + bx + c$ is an identity, state, with reason, the degree of $Q(x)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | [1]                                                                         |
|           | Since degree of dividend = degree of quotient + degree of divisor,<br>degree of $Q(x) = 1$<br><br>ALTERNATIVE: Must multiply with $x^3$ to give degree 4 in the polynomial on the left.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | B1 for reason and answer                                                    |
|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                             |
| <b>2b</b> | Find the value of $a$ , of $b$ and of $c$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | [5]                                                                         |
|           | $2x^4 + 3x^3 + ax^2 - 9x + 9 = (x^2 - 1)(x - 2)Q(x) - 3x^2 + bx + c$ <p>When <math>x = 1</math>,<br/> <math>2 + 3 + a - 9 + 9 = -3 + b + c</math><br/> <math>a = -8 + b + c</math> ----- (1)</p> <p>When <math>x = -1</math>,<br/> <math>2 - 3 + a + 9 + 9 = -3 - b + c</math><br/> <math>a = -20 - b + c</math> ----- (2)</p> <p>(1) = (2):<br/> <math>-20 - b + c = -8 + b + c</math><br/> <math>-12 = 2b</math><br/> <math>b = -6</math></p> <p>When <math>x = 2</math>,<br/> <math>32 + 24 + 4a - 18 + 9 = -12 - 12 + c</math><br/> <math>c = 47 + 4a + 24</math><br/> <math>c = 4a + 71</math> ----- (3)</p> <p>Sub (3) and <math>b = -6</math> into (2)<br/> <math>a = -20 + 6 + 4a + 71</math><br/> <math>a = 4a + 57</math><br/> <math>a = -19</math></p> <p>Sub <math>a = -19</math> into (2)<br/> <math>c = 4(-19) + 5</math><br/> <math>c = -5</math><br/> <math>a = -19, b = -6, c = -5</math></p> | <p>M1</p> <p>M1</p> <p>M1</p> <p>M1 for solving</p> <p>A1 for 3 answers</p> |
|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                             |
| <b>2c</b> | Hence, find the remainder when $2x^4 + 3x^3 + ax^2 - 9x + 9$ is divided by $(3x - 1)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | [1]                                                                         |
|           | Let $f(x) = 2x^4 + 3x^3 - 19x^2 - 9x + 9$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                             |

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                          |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
|           | $f\left(\frac{1}{3}\right) = 2\left(\frac{1}{3}\right)^4 + 3\left(\frac{1}{3}\right)^3 - 19\left(\frac{1}{3}\right)^2 - 9\left(\frac{1}{3}\right) + 9$ $f\left(\frac{1}{3}\right) = \frac{326}{81}$ <p>Therefore the remainder is <math>\frac{326}{81}</math>.</p>                                                                                                                                                               | B1                                                                                       |
| <b>3a</b> | Given that $p = 3^x$ and $q = 3^y$ , express $\log_3 \frac{p^7 q}{243}$ in terms of $x$ and $y$ .                                                                                                                                                                                                                                                                                                                                | [4]                                                                                      |
|           | $p = 3^x \Rightarrow x = \log_3 p \quad q = 3^y \Rightarrow y = \log_3 q$ $\log_3 \frac{p^7 q}{243} = 7 \log_3 p + \log_3 q - \log_3 3^5$ $\log_3 \frac{p^7 q}{243} = 7x + y - 5$                                                                                                                                                                                                                                                | [M1 for change to log]<br><br>M2 for using rules<br><br>A1                               |
| <b>3b</b> | Given that $\log_2 x - \log_x x^2 = \frac{1}{3} - \log_8 2x$ , find the value of $x$ by leaving your answer in index form.                                                                                                                                                                                                                                                                                                       | [4]                                                                                      |
|           | $\log_2 x - \log_x x^2 = \frac{1}{3} - \log_8 2x$ $\log_2 x - 2 \log_x x = \frac{1}{3} - (\log_8 2 + \log_8 x)$ $\log_2 x - 2(1) = \frac{1}{3} - \frac{1}{3} \log_8 8 - \frac{\log_2 x}{\log_2 8}$ $\log_2 x - 2 = \frac{1}{3} - \frac{1}{3} - \frac{\log_2 x}{3}$ $3 \log_2 x - 6 = -\log_2 x$ $4 \log_2 x = 6$ $\log_2 x = \frac{3}{2}$ $x = 2^{\frac{3}{2}} \text{ or } x = 8^{\frac{1}{2}} \text{ or } x = 64^{\frac{1}{4}}$ | M1 for power rule and product rule<br><br>M1 for change of base rule<br><br>M1<br><br>A1 |
| <b>4a</b> | Without using a calculator, express $\frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} + \sqrt{2}}$ in the form of $a\sqrt{10} + b\sqrt{3}$ .                                                                                                                                                                                                                                                                                                 | [4]                                                                                      |
|           |                                                                                                                                                                                                                                                                                                                                                                                                                                  | M1 for rationalization                                                                   |

|             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                   |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
|             | $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{(\sqrt{6}-\sqrt{5})(\sqrt{15}-\sqrt{2})}{15-2}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{\sqrt{90}-\sqrt{12}-\sqrt{75}+\sqrt{10}}{13}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{3\sqrt{10}-2\sqrt{3}-5\sqrt{3}+\sqrt{10}}{13}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{4\sqrt{10}-7\sqrt{3}}{13}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{4}{13}\sqrt{10} - \frac{7}{13}\sqrt{3}$ | M1 for expansion<br><br>M1 for simplifying<br><br>A1                                              |
| <b>4b</b>   | Without the use of a calculator, solve the equation $\sqrt[3]{27^x} - 81^{x+1} = 0$ .                                                                                                                                                                                                                                                                                                                                                                                                         | [3]                                                                                               |
|             | $\sqrt[3]{27^x} - 81^{x+1} = 0$ $27^{\frac{x}{3}} - 81^{x+1} = 0$ $(3^3)^{\frac{x}{3}} - (3^4)^{x+1} = 0$ $3^x - 3^{4x+4} = 0$ $3^x = 3^{4x+4}$ <p>By comparing powers,</p> $x = 4x + 4$ $x = -\frac{4}{3}$                                                                                                                                                                                                                                                                                   | M1 for converting all terms to base 3<br><br>M1 for using if $a^m = a^n$ then $m = n$<br><br>[A1] |
| <b>5ai</b>  | Given the curve $y = - 3x - x^2  + 4$ , find the $x$ -coordinates of the points where the curve meets the $x$ -axis.                                                                                                                                                                                                                                                                                                                                                                          | [2]                                                                                               |
|             | <p>When <math>y = 0</math>,</p> $0 = - 3x - x^2  + 4$ $3x - x^2 = 4 \quad \text{or} \quad 3x - x^2 = -4$ $3x - x^2 - 4 = 0 \quad \text{or} \quad 3x - x^2 + 4 = 0$ $x^2 - 3x + 4 = 0 \quad \text{or} \quad x^2 - 3x - 4 = 0$ $b^2 - 4ac = (-3)^2 - 4(1)(4) \quad \text{or} \quad (x-4)(x+1) = 0$ $b^2 - 4ac = -7 \quad \text{or} \quad x = 4 \quad \text{or} \quad x = -1$                                                                                                                    | M1 for considering positive and negative<br><br>A1 for answers                                    |
| <b>5aia</b> | Sketch the curve $y = - 3x - x^2  + 4$ , giving the coordinates of the maximum point and of the points where the curve meets the axes..                                                                                                                                                                                                                                                                                                                                                       | [3]                                                                                               |

|             |                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                                                      |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
|             |                                                                                                                                                                                                                                                                                                                                                      | <p>B1 for shape</p> <p>B1 for coordinates of max pt and x-intercepts.</p> <p>B1 for labelling.</p>                   |
| <b>5b</b>   | Explain why there are only two solutions to the equation $- 3x - x^2  = k - 4$ for $k < 1.75$ .                                                                                                                                                                                                                                                                                                                                        | [2]                                                                                                                  |
|             | $- 3x - x^2  = k - 4$ $- 3x - x^2  + 4 = k$ $y = k$ <p><math>y = k</math> is a <b>horizontal line</b> and when <math>k</math> is lesser than 2.25, it will be <b>below the turning point</b> and so it will only <b>intersect the curve at the two outer arms thereby giving two solutions only.</b></p>                                                                                                                               | <p>B1</p> <p>B1</p>                                                                                                  |
| <b>5c</b>   | Determine the maximum value of $m$ for which the line $y = mx + 1$ intersects the graph of $y = - 3x - x^2  + 4$ in three points.                                                                                                                                                                                                                                                                                                      | [1]                                                                                                                  |
|             | $m = \frac{4-1}{3-0}$ $m = 1$                                                                                                                                                                                                                                                                                                                                                                                                          | B1                                                                                                                   |
| <b>6(i)</b> | Express $\frac{7x+11}{(x-1)(x+2)^2}$ in partial fractions.                                                                                                                                                                                                                                                                                                                                                                             | [4]                                                                                                                  |
|             | <p>Let <math>\frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{7x+11}{(x-1)(x+2)^2}</math></p> $A(x+2)^2 + B(x-1)(x+2) + C(x-1) = 7x+11$ <p>when <math>x = 1</math>,<br/> <math>9A = 7 + 11</math><br/> <math>A = 2</math></p> <p>when <math>x = -2</math><br/> <math>-3C = -14 + 11</math><br/> <math>C = 1</math></p> <p>when <math>x = 0</math><br/> <math>4A - 2B - C = 11</math><br/> <math>4(2) - 2B - 1 = 11</math></p> | <p>[M1 combining fractions together and equating the numerator]</p> <p>[M1 for substitution or any other method]</p> |

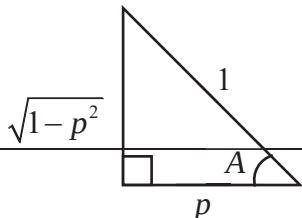
|       |                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                               |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|
|       | $-2B = 4$<br>$B = -2$<br>$\therefore \frac{7x+11}{(x-1)(x+2)^2} = \frac{2}{x-1} - \frac{2}{x+2} + \frac{1}{(x+2)^2}$                                                                                                                                                                                                                                                                        | [A2 for fully correct partial fractions. A1 only if at least two terms are correct]           |
| 6(ii) | Hence, find $\int \frac{7x+11}{2(x-1)(x+2)^2} dx$ .                                                                                                                                                                                                                                                                                                                                         | [3]                                                                                           |
|       | $\int \frac{7x+11}{2(x-1)(x+2)^2} dx = \frac{1}{2} \int \frac{2}{x-1} - \frac{2}{x+2} + \frac{1}{(x+2)^2} dx$ $= \ln(x-1) - \ln(x+2) - \frac{1}{2(x+2)} + c$                                                                                                                                                                                                                                | M1 for taking out coefficient<br><br>A2 for all correct, A1 if at least two terms are correct |
| 7     | A piece of wire which has a fixed length of $k$ cm long is bent to form a rectangle. Show that the area of the rectangle is a maximum when it is a square.                                                                                                                                                                                                                                  | [5]                                                                                           |
|       | Let the length of the rectangle be $x$ .<br>Breadth of rectangle = $\frac{k-2x}{2}$<br><br>Let area of the rectangle be $A$ .<br><br>$A = x \left( \frac{k-2x}{2} \right)$<br>$A = \frac{xk - 2x^2}{2}$<br>$A = \frac{1}{2} xk - x^2$<br>$\frac{dA}{dx} = \frac{1}{2} k - 2x$<br><br>At the stationary value, $\frac{dA}{dx} = 0$<br><br>$\frac{1}{2} k - 2x = 0$<br><br>$2x = \frac{k}{2}$ | [M1]<br><br><br><br><br><br><br>[M1]<br><br>[M1]                                              |

|              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                               |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
|              | $x = \frac{k}{4}$<br><br>$\frac{d^2 A}{dx^2} = -2 < 0$<br>Therefore, since the stationary value occurs when the sides of the rectangle are $\frac{k}{4}$ cm, and it is a maximum value, the maximum area of the rectangle occurs when it is a square.                                                                                                                                                                                                                                                                                                                                                                                                                                                       | [M1]<br><br>[A1]                                                                                                              |
| <b>8</b>     | Given a circle with the equation $(2x+5)(x+2) + (2y+1)(y-5) = 0$ ,                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                               |
| <b>8(i)</b>  | Express the equation of the circle in standard form.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | [5]                                                                                                                           |
|              | $(2x+5)(x+2) + (2y+1)(y-5) = 0$<br><br>$2x^2 + 9x + 10 + 2y^2 - 9y - 5 = 0$<br><br>$2x^2 + 2y^2 + 9x - 9y + 5 = 0$<br><br>$x^2 + y^2 + \frac{9}{2}x - \frac{9}{2}y + \frac{5}{2} = 0$<br><br>$x^2 + \frac{9}{2}x + \left(\frac{9}{4}\right)^2 + y^2 - \frac{9}{2}y + \left(\frac{9}{4}\right)^2 = -\frac{5}{2} + \left(\frac{9}{4}\right)^2 + \left(\frac{9}{4}\right)^2$<br><br>$\left(x + \frac{9}{4}\right)^2 + \left(y - \frac{9}{4}\right)^2 = \frac{61}{8}$<br><br>Coordinates of centre = $\left(-\frac{9}{4}, \frac{9}{4}\right)$<br><br>Radius of circle = $\sqrt{\frac{61}{8}}$ units<br><br>Equation of circle, $\left(x + \frac{9}{4}\right)^2 + \left(y - \frac{9}{4}\right)^2 = \frac{61}{8}$ | M1 for expansion and simplification<br><br><br><br>M1 – for getting the centre and radius<br><br>[A1]<br><br>[A1]<br><br>[A1] |
| <b>8(ii)</b> | Find the length of the chord when the line $y = -2x$ cuts the circle.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | [5]                                                                                                                           |
|              | $y = -2x$ <span style="float: right;">-(1)</span><br>$\left(x + \frac{9}{4}\right)^2 + \left(y - \frac{9}{4}\right)^2 = \frac{61}{8}$ <span style="float: right;">-(2)</span><br><br>Sub (1) into (2)<br><br>$\left(x + \frac{9}{4}\right)^2 + \left(-2x - \frac{9}{4}\right)^2 = \frac{61}{8}$                                                                                                                                                                                                                                                                                                                                                                                                             | [M1 for substitution]                                                                                                         |

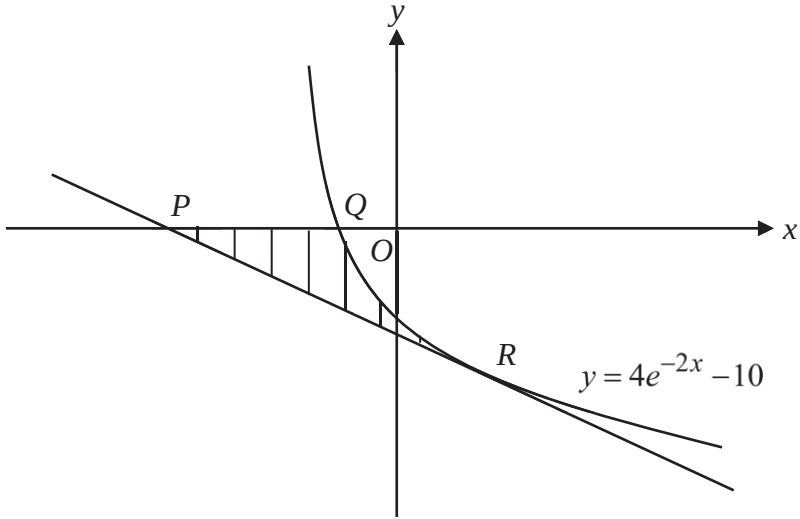
|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                     |
|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|            | $5x^2 + \frac{27}{2}x + \frac{5}{2} = 0$ $10x^2 + 27x + 5 = 0$ $(5x+1)(2x+5) = 0$ $x = -\frac{1}{5} \quad \text{or} \quad x = -\frac{5}{2}$ <p>when <math>x = -\frac{1}{5}, y = \frac{2}{5}</math></p> <p>when <math>x = -\frac{5}{2}, y = 5</math></p> <p>Thus the coordinates of the end points of the chord are <math>\left(-\frac{1}{5}, \frac{2}{5}\right)</math> and <math>\left(-\frac{5}{2}, 5\right)</math></p> <p>Length of chord = <math>\sqrt{\left(2\frac{3}{10}\right)^2 + \left(-4\frac{3}{5}\right)^2}</math><br/> <math>= 5.14</math> units (to 3 s.f.)</p>                                                                                                                 | <p>[M1 for solving]</p> <p>[A1 for correct coordinates]</p> <p>[M1]</p> <p>[A1]</p> |
| <b>9ai</b> | Prove the identity $\sin x \cos x + \cot x \cos^2 x = \cot x$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | [4]                                                                                 |
|            | <p>LHS = <math>\sin x \cos x + \cot x \cos^2 x</math></p> <p><math>= \cos x (\sin x + \cot x \cos x)</math></p> <p><math>= \cos x \left( \sin x + \frac{\cos x}{\sin x} \cos x \right)</math></p> <p><math>= \cos x \left( \sin x + \frac{\cos^2 x}{\sin x} \right)</math></p> <p><math>= \cos x \left( \frac{\sin^2 x + \cos^2 x}{\sin x} \right)</math></p> <p><math>= \cos x \left( \frac{1}{\sin x} \right)</math></p> <p><math>= \cot x = \text{RHS}</math></p> <p>Or</p> <p>LHS = <math>\sin x \cos x + \cot x \cos^2 x</math></p> <p><math>= \sin x \cos x + \frac{\cos x}{\sin x} \cos^2 x</math></p> <p><math>= \frac{\sin^2 x \cos x}{\sin x} + \frac{\cos^3 x}{\sin x}</math></p> | <p>[M1]</p> <p>[M1]</p> <p>[M1]</p> <p>[A1]</p> <p>[M1]</p>                         |

|              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                         |
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|              | $= \frac{\sin^2 x \cos x + \cos^3 x}{\sin x}$ $= \frac{\cos x (\sin^2 x + \cos^2 x)}{\sin x}$ $= \frac{\cos x (1)}{\sin x}$ $= \cot x = \text{RH}$                                                                                                                                                                                                                                                                                                                                                                                                                  | <br>[M1]<br>[M1]<br>[A1]                                                                                                                |
| <b>9a ii</b> | Hence, solve $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1$ for $0 \leq x \leq \pi$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | [3]                                                                                                                                     |
|              | <p>Since <math>\sin x \cos x + \cot x \cos^2 x = \cot x</math></p> <p>Therefore, <math>\sin 3x \cos 3x + \cot 3x \cos^2 3x = \cot 3x</math></p> <p>and</p> $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1 \Rightarrow \cot 3x = 1$<br>$0 \leq x \leq \pi$ $0 \leq 3x \leq 3\pi$<br><p>Let the basic angle be <math>\alpha</math></p> $\tan \alpha = 1$ $\alpha = \frac{\pi}{4}$ $3x = \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi + \frac{\pi}{4}$<br>$3x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4} \text{ rad}$ | <br>[M1]<br><br><br><br><br><br>[M1]<br><br><br>[A1]                                                                                    |
| <b>9bi</b>   | On the same axes, sketch the graphs of $y = 3 \sin x - 1$ and $y = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                     | [5]                                                                                                                                     |
|              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | <p>For tan curve,<br/>[B1] for shape,<br/>[B1] for asymptote</p> <p>For Sine curve,<br/>[B1] for shape<br/>[B1] for correct max and</p> |

|             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                            |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
|             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | <p>minimum values</p> <p>[B1] for correct period for both graphs</p> <p>[subtract 1 mark for wrong or no labels for axes or functions]</p> |
| <b>9bii</b> | Hence, state the number of solutions of $3 \sin x - 1 = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$ .                                                                                                                                                                                                                                                                                                                                                                                                                                | [1]                                                                                                                                        |
|             | From the sketch, the two functions intersect at three points. Hence there are three solutions for the equation for $0 \leq x \leq 2\pi$                                                                                                                                                                                                                                                                                                                                                                                             | [B1]                                                                                                                                       |
| <b>10a</b>  | <p>Two particles, A and B, leaves a point O at the same time and travel in the same direction along the same straight line.</p> <p>Particle A starts with a velocity of 9 m/s and moves with a constant acceleration of 2 m/s<sup>2</sup>.</p> <p>Particle B starts from rest and moves with an acceleration of <math>a</math> m/s<sup>2</sup>, where <math>a = 1 + \frac{t}{3}</math> and <math>t</math> seconds is the time since leaving O. Find an expression for the velocity of each particle in terms of <math>t</math>,</p> | [3]                                                                                                                                        |
|             | <p>For Particle A,</p> $v_A = \int 2 \, dt$ $v_A = 2t + c$ <p>When <math>t = 0</math>, <math>v_A = 9</math></p> $c = 9$ $v_A = 2t + 9$ <p>For Particle B,</p> $v_B = \int \left(1 + \frac{t}{3}\right) dt$ $v_B = t + \frac{1}{6}t^2 + c$ <p>When <math>t = 0</math>, <math>v_B = 0</math>,</p> $c = 0$ $v_B = t + \frac{1}{6}t^2$                                                                                                                                                                                                  | <p>B1</p> <p>M1</p> <p>A1</p>                                                                                                              |
| <b>10b</b>  | an expression for the displacement of each particle in terms of $t$ ,                                                                                                                                                                                                                                                                                                                                                                                                                                                               | [3]                                                                                                                                        |

|            |                                                                                                                                                                                                                                                                                                                                                                                 |                                            |
|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------|
|            | <p>For Particle A,</p> $s_A = \int (2t + 9) dt$ $s_A = t^2 + 9t + c$ <p>When <math>t = 0</math>, <math>s_A = 0</math></p> $c = 0$ $s_A = t^2 + 9t$ <p>For Particle B,</p> $s_B = \int \left( t + \frac{1}{6}t^2 \right) dt$ $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3 + c$ <p>When <math>t = 0</math>, <math>s_B = 0</math>,</p> $c = 0$ $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3$ | <p>B1</p> <p>M1</p> <p>A1</p>              |
| <b>10c</b> | the distance from O at which particle B collides with A,                                                                                                                                                                                                                                                                                                                        | [3]                                        |
|            | <p>When particle B collides with particle A,</p> $s_A = s_B$ $t^2 + 9t = \frac{1}{2}t^2 + \frac{1}{18}t^3$ $18t^2 + 162t = 9t^2 + t^3$ $t^3 - 9t^2 - 162t = 0$ $t(t^2 - 9t - 162) = 0$ $t(t - 18)(t + 9) = 0$ $t = 0(\text{N.A.}) \quad \text{or} \quad t = 18 \quad \text{or} \quad t = -9(\text{N.A.})$ <p>Distance from O = <math>(18)^2 + 9(18) = 486 \text{ m}</math></p>  | <p>M1</p> <p>M1</p> <p>A1</p>              |
| <b>10d</b> | the speed of each particle at the point of collision.                                                                                                                                                                                                                                                                                                                           | [2]                                        |
|            | Speed of particle A = $2(18) + 9 = 45 \text{ m/s}$                                                                                                                                                                                                                                                                                                                              | B1                                         |
|            | Speed of particle B = $(18) + \frac{1}{6}(18)^2 = 72 \text{ m/s}$                                                                                                                                                                                                                                                                                                               | B1                                         |
| <b>11</b>  | Given that $\cos A = p$ and that A is acute, express the following in terms of p.                                                                                                                                                                                                                                                                                               |                                            |
| <b>i</b>   | $\sin 2A$                                                                                                                                                                                                                                                                                                                                                                       | [3]                                        |
|            |                                                                                                                                                                                                                                                                                              | M1 for getting the length of opposite side |

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                  |
|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------|
|           | $\sin 2A = 2 \sin A \cos A$ $= 2p\sqrt{1-p^2}$ <p>Or</p> $\cos^2 A + \sin^2 A = 1$ $\sin^2 A = 1 - \cos^2 A$ $\sin^2 A = 1 - p^2$ $\sin A = \sqrt{1-p^2} \text{ (reject negative as } A \text{ is acute)}$ $\sin 2A = 2 \sin A \cos A$ $= 2p\sqrt{1-p^2}$                                                                                                                                                                                                                                                                         | M1<br>A1<br><br><br><br><br>M1<br>M1<br>A1       |
| <b>ii</b> | $\cos \frac{A}{2}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | [3]                                              |
|           | $\cos A = 2 \cos^2 \frac{A}{2} - 1$ $\cos \frac{A}{2} = \pm \sqrt{\frac{\cos A + 1}{2}}$ $\cos \frac{A}{2} = \pm \sqrt{\frac{p+1}{2}}$ $\cos \frac{A}{2} = -\sqrt{\frac{p+1}{2}} \text{ (rejected) or } \cos \frac{A}{2} = \sqrt{\frac{p+1}{2}}$                                                                                                                                                                                                                                                                                  | M1<br><br><br><br><br><br>M1 for rejection<br>A1 |
| <b>12</b> | <p>The diagram shows the curve, <math>y = 4e^{-2x} - 10</math>. The curve crosses the <math>x</math>-axis at <math>Q</math>. The line <math>PR</math> is a tangent to the curve at <math>R</math> and intersects the <math>x</math>-axis at <math>P</math>. The <math>x</math>-coordinate of <math>R</math> is <math>\ln 2</math>.</p> <p>Find the area of the shaded region, <math>PQR</math>, which is the region enclosed by curve, the <math>x</math>-axis and the line <math>PR</math> correct to 3 significant figures.</p> | [11]                                             |

|  |                                                                                                                                                                                                                                                                                                                                        |                                                    |
|--|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------|
|  |                                                                                                                                                                                                                                                      |                                                    |
|  | <p>At <math>R</math>, <math>x = \ln 2</math><br/> <math>y = 4e^{-2\ln 2} - 10 = -9</math><br/> <math>R(\ln 2, -9)</math></p>                                                                                                                                                                                                           | B1                                                 |
|  | <p><math>y = 4e^{-2x} - 10</math><br/> <math>\frac{dy}{dx} = 4(-2)e^{-2x} = -8e^{-2x}</math><br/>         Gradient of line <math>PR = -8e^{-2\ln 2} = -2</math><br/>         Equation of line <math>PR</math><br/> <math>y - (-9) = -2(x - \ln 2)</math><br/> <math>y = -2x + 2\ln 2 - 9</math><br/> <math>y = -2x - 7.6137</math></p> | M1<br><br>A1                                       |
|  | <p>At <math>P</math>, <math>y = 0</math><br/> <math>0 = -2x + 2\ln 2 - 9</math><br/> <math>x = \ln 2 - \frac{9}{2}</math><br/> <math>= -3.8069</math><br/> <math>P\left(\ln 2 - \frac{9}{2}, 0\right)</math></p>                                                                                                                       | B1                                                 |
|  | <p>At <math>Q</math>, <math>y = 0</math><br/> <math>0 = 4e^{-2x} - 10</math><br/> <math>4e^{-2x} = 10</math><br/> <math>e^{-2x} = \frac{5}{2}</math><br/> <math>-2x = \ln \frac{5}{2}</math><br/> <math>x = -\frac{1}{2} \ln \frac{5}{2} = -0.45815</math><br/> <math>Q\left(-\frac{1}{2} \ln \frac{5}{2}, 0\right)</math></p>         | M1 – change<br>index form<br>to log form<br><br>A1 |
|  | <p>Area of the shaded region, <math>PQR</math><br/> <math>= \text{Area of the triangle} - \text{Area between the curve and the } x\text{-axis}</math></p>                                                                                                                                                                              | M1                                                 |

|  |                                                                                                                                                                                                                  |    |
|--|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
|  | $\text{Area} = \frac{1}{2} \times \left( (\ln 2) - \left( \ln 2 - \frac{9}{2} \right) \right) \times (9) - \left  \int_{-\frac{1}{2} \ln \frac{5}{2}}^{\ln 2} (4e^{-2x} - 10) dx \right $                        | M1 |
|  | $= \frac{81}{4} - \left  \left[ -2e^{-2x} - 10x \right]_{-\frac{1}{2} \ln \frac{5}{2}}^{\ln 2} \right $                                                                                                          | M1 |
|  | $= \frac{81}{4} - \left  \left( \left[ -2e^{-2 \ln 2} - 10 \ln 2 \right] - \left[ -2e^{-2 \left( -\frac{1}{2} \ln \frac{5}{2} \right)} - 10 \left( -\frac{1}{2} \ln \frac{5}{2} \right) \right] \right) \right $ | M1 |
|  | $= \frac{81}{4} -  -7.01295 $                                                                                                                                                                                    |    |
|  | $= 13.23705 = 13.2 \text{ units}^2$                                                                                                                                                                              | A1 |
|  |                                                                                                                                                                                                                  |    |

