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Class	Index Number	Name
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BUKIT MERAH SECONDARY SCHOOL



END-OF-YEAR EXAMINATION 2023 SECONDARY 3 EXPRESS

MATHEMATICS

Paper 1

4052/01

4 October 2023

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

Calculator Model:

For Examiner's Use

--

Setter: Ms Lim LE

This document consists of **20** printed pages.

[Turn over]

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

- 1 (a) Evaluate $4.9 \times 10^6 - 5.64 \times 10^4$.

Give your answer in standard form, correct to 2 significant figures.

Answer [1]

- (b) 1.37×10^{-7} seconds can be written as k nanoseconds.

Find the value of k .

(1 nanosecond = 1×10^{-9} seconds)

Answer $k =$ [1]

- 2 Given that $0^\circ \leq \theta \leq 180^\circ$, find the value(s) of θ for $\sin \theta = 0.772$.

Answer $\theta =$ [2]

-
- 3 Find the value of p in $3^{25} - 3^{24} = 2 \times 3^{p+1}$.

Answer $p = \dots\dots\dots$ [2]

- 4 (a) Express 1008 as a product of its prime factors.

Answer $\dots\dots\dots$ [1]

- (b) Hence, find the smallest positive integer k such that $1008k$ is a perfect cube.

Answer $\dots\dots\dots$ [1]

5

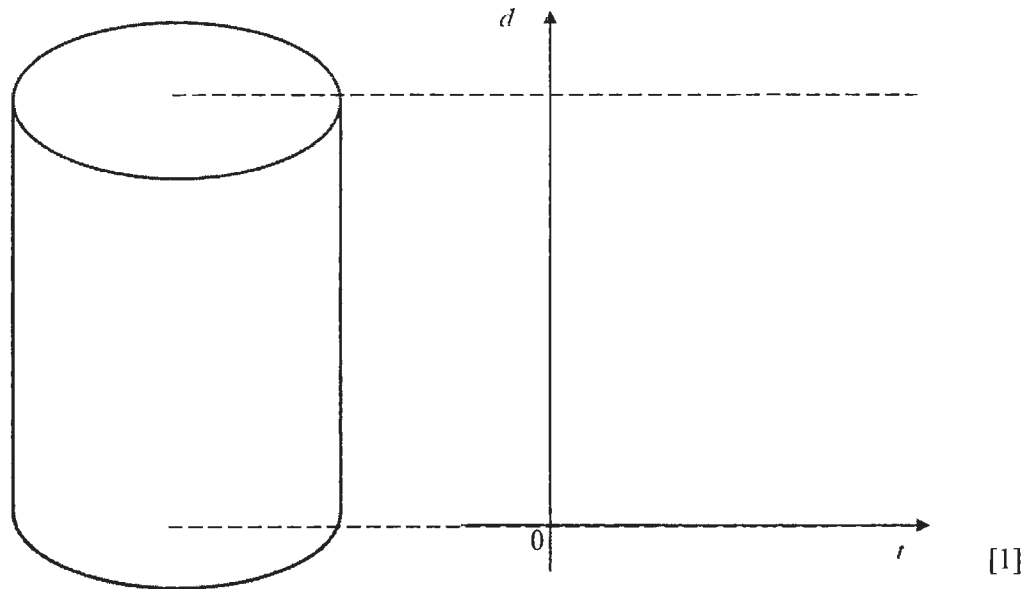
- 5 In country X, the Goods and Services Tax (GST) is predicted to rise from 7% to 8.5% in the future. Before the increase, the price of a tablet is \$1391 including 7% GST. Calculate the new price inclusive of GST, after the GST has increased to 8.5%.

Answer \$..... [2]

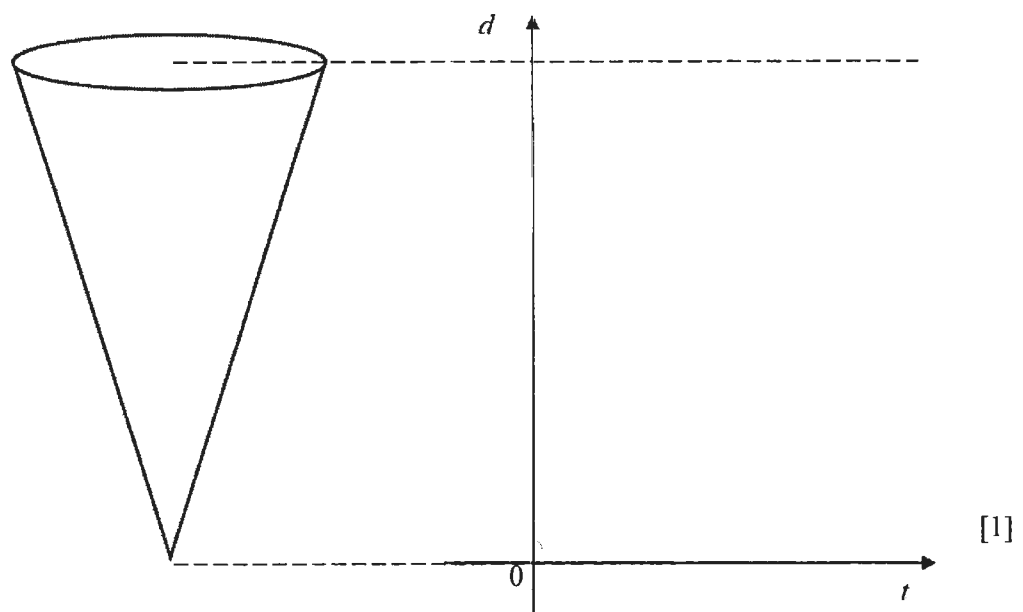
- 6 Simplify $m^{\frac{3}{2}}n^{-5} \div \left(\frac{n}{m}\right)^{-2}$, giving your answer in positive index notation.

Answer [3]

- 7 (a) Liquid is **flowing out of the full container** at a constant rate. After t seconds, the depth of water in the container is d cm. Sketch the graph of d against t .



- (b) Liquid is **poured into the empty container** at a constant rate. After t seconds, depth of water in the container is d cm. Sketch the graph of d against t .



- 8 Liquid detergent is sold in two sizes.



Which container is better value for money?

[3]

Explain your answer.

Answer

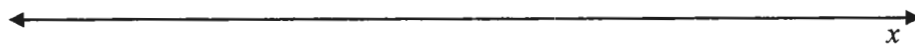
8

- 9 (a) Solve the inequalities $3x - 1 < 2x + 9 \leq \frac{1}{2}(7x - 6)$.

Answer [3]

- (b) Show your solution in (a) on the number line below.

Answer :



[1]

10 A map is drawn to a scale of 1 cm to 4 km.

- (a) Express the scale of the map in the form of 1 : n .

Answer [1]

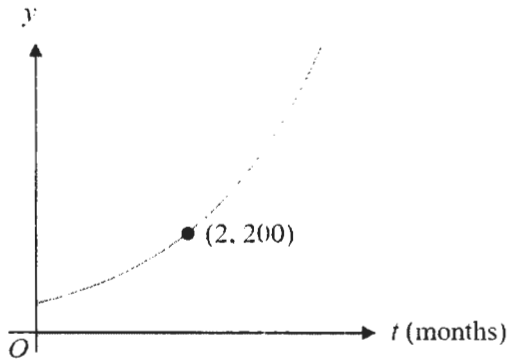
- (b) The distance between two schools is 8.4 km. Find the distance, in centimetres, on the map.

Answer cm [1]

- (c) One of the schools occupies an area of 2.5 cm^2 on the map. Calculate its actual area in square kilometres.

Answer km^2 [2]

- 11 (a) The number of a certain type of insects, y , on an island t months after the first observation is given by $y = k(2)^t$, where k is a positive constant. The point $(2, 200)$ lies on the graph of $y = k(2)^t$.



- (i) Find the value of k .

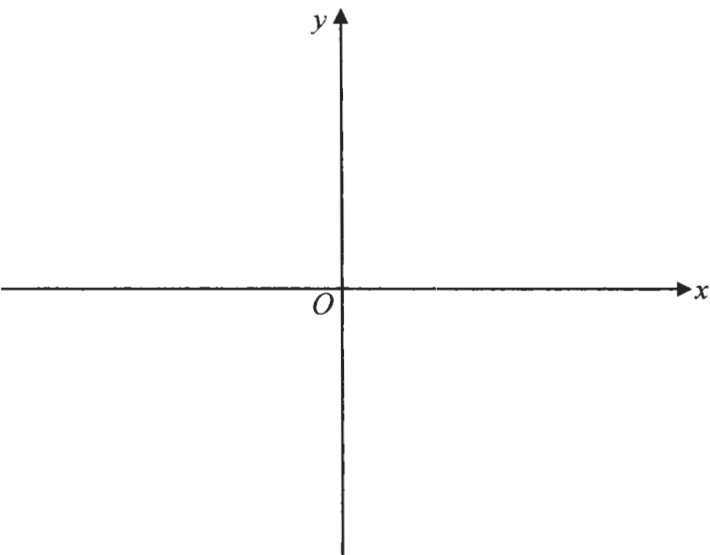
Answer $k = \dots\dots\dots$ [1]

- (ii) Find the initial value of y .

Answer $\dots\dots\dots$ [1]

- (b) Sketch the graph of $y = -(x+1)^2 + 9$, showing clearly the turning point and the intercepts.

Answer



[3]

12 Two jars are geometrically similar. Their heights are 8 cm and 14 cm respectively.

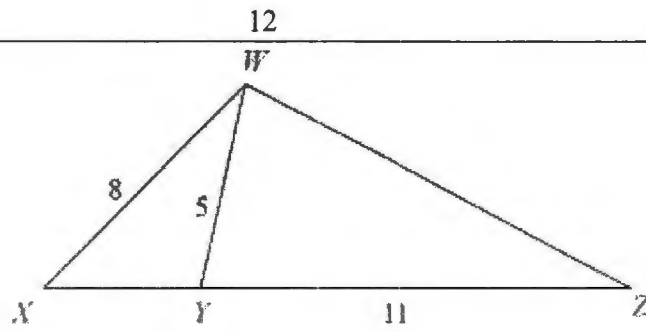
- (a) Calculate the volume of the smaller jar as a percentage of the volume of the larger jar.
Give your answer correct to the nearest whole number.

Answer% [2]

- (b) The base area of the larger jar is 140 cm^2 .
Find the base area of the smaller jar.

Answer cm^2 [2]

13



In triangle WXZ , $WX = 8$ cm.

Y is a point on XZ such that $WY = 5$ cm, $YZ = 11$ cm and $\sin \angle WYZ = \frac{4}{5}$.

(a) Find the area of triangle WYZ .

Answer cm^2 [2]

(b) Find the size of angle WXY .

Answer [2]

14 Factorise completely

(a) $x^2 - y^2 - 3x + 3y$,

Answer [2]

(b) $32a^2 - 50$.

Answer [2]

15 The n th term of a sequence is given by $T_n = n^2 + 3$.

- (a) Explain why 1010 is not a term in the sequence.

Answer

 [1]

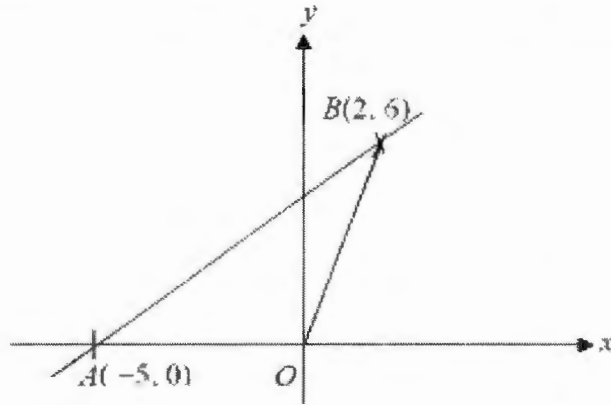
- (b) Find an expression, in terms of n , for $T_{n+1} - T_n$.

Answer [2]

- (c) Given that the difference between two successive terms in the sequence is 99, use your answer in part (b) to find the smaller term.

Answer [2]

- 16 In the diagram, a straight line passes through the points $A(-5,0)$ and $B(2, 6)$.



- (a) Find the equation of the line AB .

Answer [3]

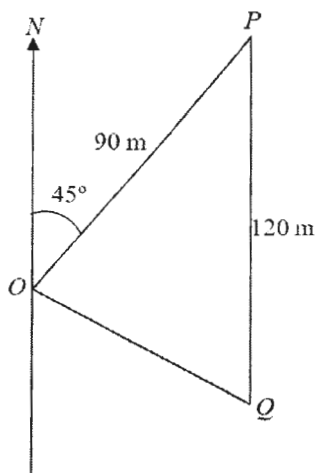
- (b) Find the length of AB .

Answer units [2]

- (c) Find the area of triangle ABO .

Answer units² [1]

- 17 The diagram shows three points O , P and Q .
 The bearing of P from O is 045° and P is due north of Q .
 $OP = 90$ m and $PQ = 120$ m.



- (a) Find the distance of OQ .

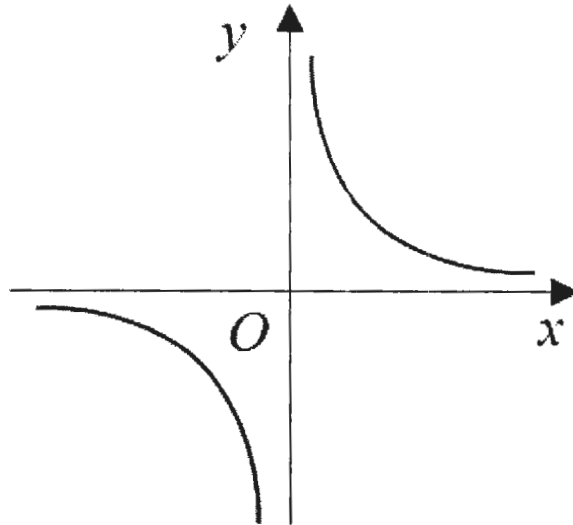
Answer m [3]

- (b) Calculate the bearing of Q from O .

Answer [3]

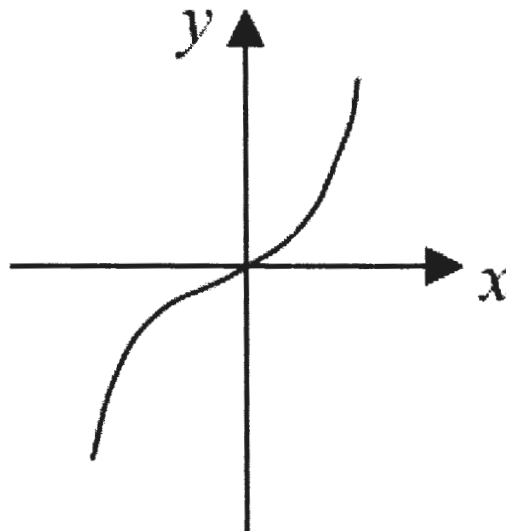
18 Given $y = kx^n$, where $k > 0$. State the value of n for the following graphs.

(a)



Answer $n = \dots\dots\dots$ [1]

(b)



Answer $n = \dots\dots\dots$ [1]

17

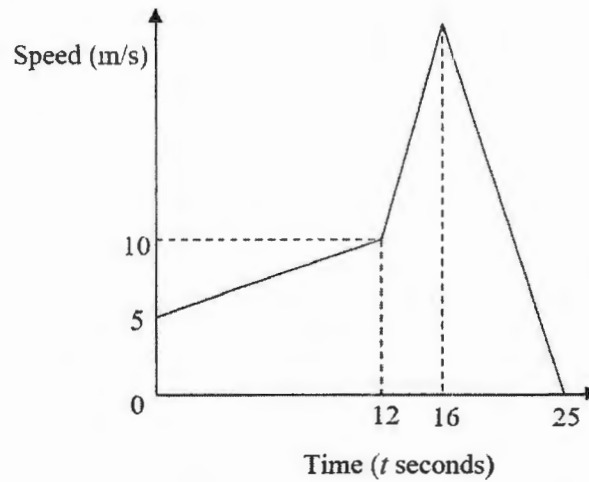
19 (a) Express $-x^2 + 4x + 1$ in the form $-(x + a)^2 + b$.

Answer [2]

(b) Hence, solve the equation $-x^2 + 4x + 1 = 0$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

- 20 The diagram shows the speed-time graph of an object moving in a straight line over a period of 25 seconds. The object travelled 90 m from $t = 12$ to $t = 16$.



- (a) Find the maximum speed of the object.

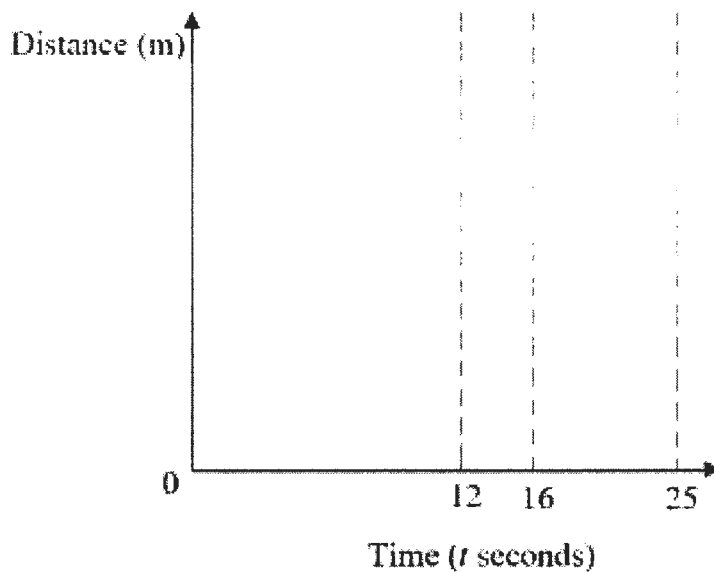
Answer m/s [2]

- (b) Find the speed of the object at $t = 6$.

Answer m/s [2]

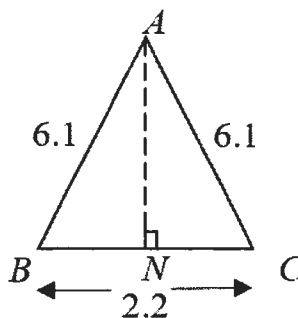
- (c) Sketch the distance-time graph for $0 \leq t \leq 25$.

Answer



[3]

- 21 Triangle ABC has $AB = AC = 6.1$ cm, $BC = 2.2$ cm.

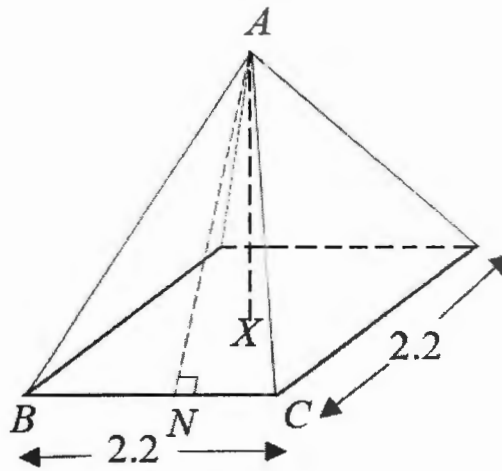


- (a) Show that $AN = 6$ cm.

Answer

[2]

- (b) Triangle ABC in (a), is one of the faces of a square-based pyramid.
 X is the centre of the base of the pyramid.



- (i) Calculate the total surface area of the pyramid.

Answer cm^2 [2]

- (ii) Calculate the volume of the pyramid.

Answer cm^3 [3]

~~~~~ END OF PAPER ~~~~~

## MARKING SCHEME FOR PAPER 1

|   |     |                                                                                                                                            |     |
|---|-----|--------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 1 | (a) | Evaluate $4.9 \times 10^6 - 5.64 \times 10^4$ .<br>Give your answer in standard form, correct to 2 significant figures.                    |     |
|   |     | <i>Answer</i> ..... $4.8 \times 10^6$ .....                                                                                                | [1] |
|   | (b) | $1.37 \times 10^{-7}$ seconds can be written as $k$ nanoseconds.<br>Find the value of $k$ .<br>(1 nanosecond = $1 \times 10^{-9}$ seconds) |     |
|   |     | $k \times 10^{-9} = 1.37 \times 10^{-7}$<br>$k = \frac{1.37 \times 10^{-7}}{10^{-9}}$<br>$= 137$                                           |     |
|   |     | <i>Answer</i> ..... 137 .....                                                                                                              | [1] |
| 2 |     | Given that $0^\circ \leq \theta \leq 180^\circ$ , find the value(s) of $\theta$ for $\sin \theta = 0.772$                                  |     |
|   |     | $\sin \theta = 0.772$<br>$\theta = 50.53$ or $\theta = 180 - 50.53$<br>$= 50.5$ $= 129.47$                                                 |     |
|   |     | <i>Answer</i> $\theta = ..50.5^0... \text{ or } ..129.5^0...$                                                                              | [2] |
| 3 |     | Find the value of $p$ in $3^{25} - 3^{24} = 2 \times 3^{p+1}$ .                                                                            |     |
|   |     | $3^{25} - 3^{24} = 2 \times 3^{p+1}$<br>$3^{24}(3 - 1) = 2 \times 3^{p+1}$ [M1]<br>$2 \times 3^{24} = 2 \times 3^{p+1}$<br>$p + 1 = 24$    |     |
|   |     | <i>Answer</i> $p = ...23.....$ [A1] .....                                                                                                  | [2] |

2

|   |     |                                                                                                                                                                                                                                                          |                                                         |
|---|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------|
| 4 | (a) | Express 1008 as a product of its prime factors.                                                                                                                                                                                                          |                                                         |
|   |     | $  \begin{array}{r l}  2 & 1008 \\  2 & 504 \\  2 & 252 \\  2 & 126 \\  3 & 63 \\  3 & 21 \\  7 & 7 \\  & 1  \end{array}  $                                                                                                                              |                                                         |
|   |     | $1008 = 2^4 \times 3^2 \times 7$                                                                                                                                                                                                                         | <i>Answer</i> ..... $2^4 \times 3^2 \times 7$ ..... [1] |
|   | (b) | Hence, find the smallest positive integer $k$ such that $1008k$ is a perfect cube.                                                                                                                                                                       |                                                         |
|   |     | $k = 2^2 \times 3 \times 7^2 = 588$                                                                                                                                                                                                                      | <i>Answer</i> .....588..... [1]                         |
| 5 |     | In country X, the Goods and Services Tax (GST) is predicted to rise from 7% to 8.5% in the future. Before the increase, the price of a tablet is \$1391 including 7% GST. Calculate the new price inclusive of GST, after the GST has increased to 8.5%. |                                                         |
|   |     | Price of tablet = \$1300<br><br>Price including 8.5% GST = \$1 410.50 [Answer must be given in 2 dp]<br><br>OR<br><br>Price including 8.5% GST = $\frac{108.5}{107} \times 1391$<br>= \$1 410.50 [Answer must be given in 2 dp]                          |                                                         |
|   |     | <i>Answer</i> \$...1 410.50.....                                                                                                                                                                                                                         | [2]                                                     |

3

6

Simplify  $m^{\frac{3}{2}}n^{-5} \div \left(\frac{n}{m}\right)^{-2}$ , giving your answer in positive index notation.

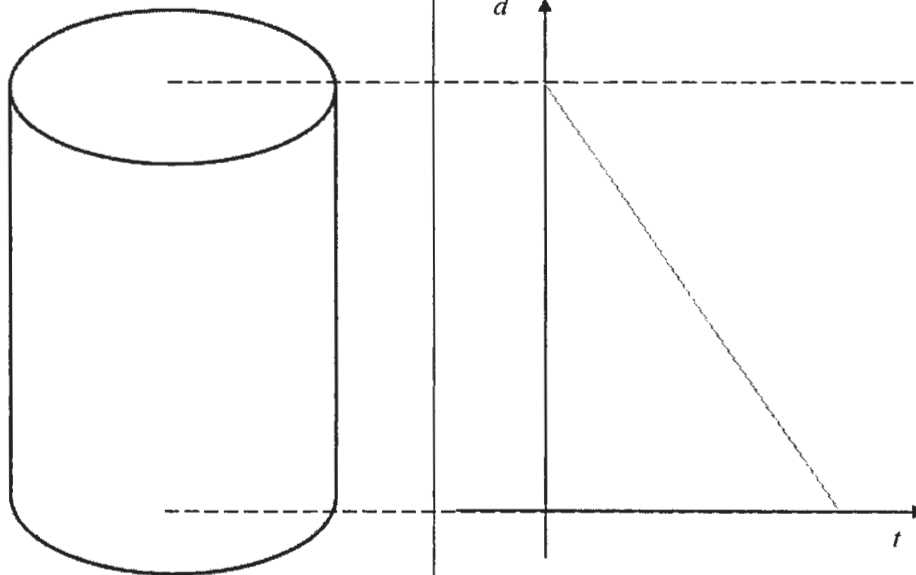
$$\begin{aligned}
 m^{\frac{3}{2}}n^{-5} \div \left(\frac{n}{m}\right)^{-2} &= m^{\frac{3}{2}}n^{-5} \times \left(\frac{n}{m}\right)^2 \\
 &= m^{\frac{3}{2}-2}n^{-5+2} \\
 &= m^{-\frac{1}{2}}n^{-3} \\
 &= \frac{1}{m^{\frac{1}{2}}n^3}
 \end{aligned}$$

Answer

..... [3]

7

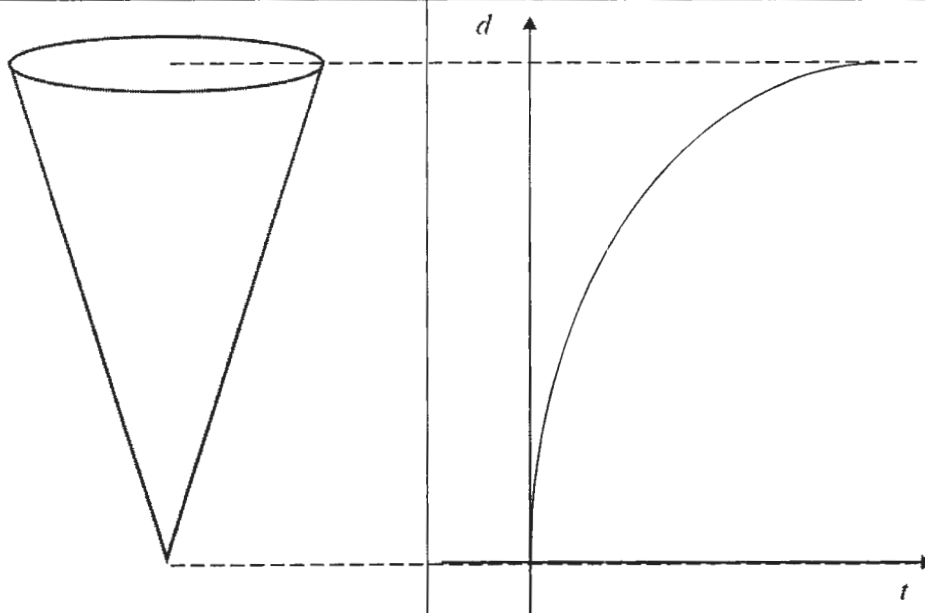
(a) Liquid is **flowing out of the full container** at a constant rate. After  $t$  seconds, the depth of water in the container is  $d$  cm. Sketch the graph of  $d$  against  $t$ .



[1]

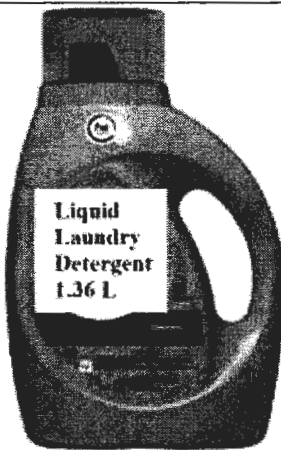
4

- (b) Liquid is poured into the empty container at a constant rate. After  $t$  seconds, the depth of water in the container is  $d$  cm. Sketch the graph of  $d$  against  $t$ .



[1]

- 8 Liquid detergent is sold in two sizes.

|                                                                                     |                                                                                      |
|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
|  |  |
| \$4.75                                                                              | \$13.71                                                                              |

Which container is better value for money?

[3]

Explain your answer.

*Answer*

Small bottle:

$$\text{Unit Cost} = \frac{475}{306} = 1.55 \text{ cents/ml}$$

Large bottle:

$$\text{Unit Cost} = \frac{1371}{1360} = 1.01 \text{ cents/ml}$$

The large bottle is better value for money because it costs 1.01 cents per ml (or \$0.0101 per ml) instead of 1.55 cents per ml (or \$0.0155 per ml).

OR

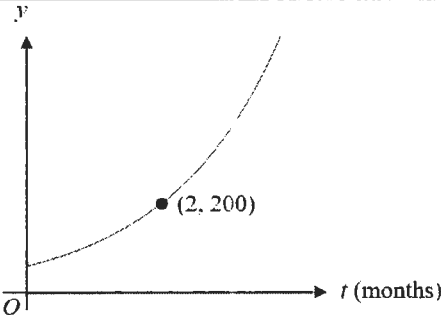
Small bottle: \$4.75 for 306 ml

$$\text{Big bottle: } \frac{13.71}{1360} \times 306 = \$3.08 \text{ for 306 ml}$$

The large bottle is better value for money because it costs less for the same amount of detergent.

|   |     |                                                                                                                             |                                       |
|---|-----|-----------------------------------------------------------------------------------------------------------------------------|---------------------------------------|
| 9 | (a) | Solve the inequalities $3x-1 < 2x+9 \leq \frac{1}{2}(7x-6)$ .                                                               |                                       |
|   |     | $3x-1 < 2x+9$ $x < 10$ <p>AND</p> $2x+9 \leq \frac{1}{2}(7x-6)$ $4x+18 \leq 7x-6$ $-3x \leq -24$ $x \geq 8$ $8 \leq x < 10$ | Deduct the answer mark if OR is used. |
|   | (b) | Show your solution in (a) on the number line below.                                                                         |                                       |
|   |     |                                         | [B1]                                  |

|           |                                            |                                                                                                                        |                              |     |
|-----------|--------------------------------------------|------------------------------------------------------------------------------------------------------------------------|------------------------------|-----|
| <b>10</b> | A map is drawn to a scale of 1 cm to 4 km. |                                                                                                                        |                              |     |
|           |                                            |                                                                                                                        |                              |     |
|           | <b>(a)</b>                                 | Express the scale of the map in the form of 1 : $n$ .                                                                  |                              |     |
|           |                                            | <i>Answer</i>                                                                                                          | ..... 1 : 400 000 .....      | [1] |
|           | <b>(b)</b>                                 | The distance between two schools is 8.4 km. Find the corresponding distance, in centimetres, on the map.               |                              |     |
|           |                                            | $\frac{8.4}{4} = 2.1 \text{ cm}$                                                                                       |                              |     |
|           |                                            | <i>Answer</i>                                                                                                          | .....2.1..... cm             | [1] |
|           | <b>(c)</b>                                 | One of the schools occupies an area of 2.5 cm <sup>2</sup> on the map. Calculate its actual area in square kilometres. |                              |     |
|           |                                            | 1 cm rep. 4 km<br>1 cm <sup>2</sup> rep 16 km <sup>2</sup><br>2.5 cm <sup>2</sup> rep 40 km <sup>2</sup>               |                              |     |
|           |                                            | <i>Answer</i>                                                                                                          | .....40..... km <sup>2</sup> | [2] |

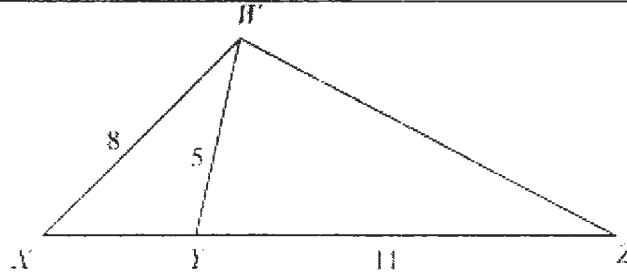
|           |            |                                                                                                                                                                                                                      |  |  |
|-----------|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|--|
| <b>11</b> | <b>(a)</b> | The number of a certain type of insects, $y$ , on an island $t$ months after the first observation is given by $y = k(2)^t$ , where $k$ is a positive constant. The point (2, 200) lies on the graph of $y = k(2)^t$ |  |  |
|           |            |                                                                                                                                  |  |  |
|           | <b>(i)</b> | Find the value of $k$ .                                                                                                                                                                                              |  |  |
|           |            | $200 = k(2)^2$<br>$k = 50$                                                                                                                                                                                           |  |  |

7

|    |                                                                                    |                                                                                                                                                                              |                                                                               |     |
|----|------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------|-----|
|    |                                                                                    | Answer                                                                                                                                                                       | $k = \dots\dots\dots 50 \dots\dots\dots$                                      | [1] |
|    |                                                                                    | (ii) Find the initial value of $y$ .                                                                                                                                         |                                                                               |     |
|    |                                                                                    | $y = 50(2)^0$<br>$y = 50$                                                                                                                                                    |                                                                               |     |
|    | (b)                                                                                | Sketch the graph of $y = -(x+1)^2 + 9$ .                                                                                                                                     |                                                                               |     |
|    |                                                                                    |                                                                                                                                                                              | B1 for shape of curve.<br>B1 for Turning point.<br>B1 for x-and y-intercepts. |     |
| 12 | Two jars are geometrically similar. Their heights are 8 cm and 14 cm respectively. |                                                                                                                                                                              |                                                                               |     |
|    | (a)                                                                                | Calculate the volume of the smaller jar as a percentage of the volume of the larger jar.<br>Give your answer correct to the nearest whole number.                            |                                                                               |     |
|    |                                                                                    | $\frac{V_1}{V_2} = \left(\frac{8}{14}\right)^3 = \frac{64}{343} \quad \text{[M1]}$ $\text{Percentage} = \frac{64}{343} \times 100\% = 18.65889\%$ $= 19\% \quad \text{[A1]}$ |                                                                               |     |
|    | (b)                                                                                | The base area of the larger jar is $140 \text{ cm}^2$ .<br>Find the base area of the smaller jar.                                                                            |                                                                               |     |
|    |                                                                                    | $\frac{A_s}{A_L} = \left(\frac{h_s}{h_L}\right)^2$ $\frac{A_s}{140} = \left(\frac{8}{14}\right)^2$ $A_s = \left(\frac{8}{14}\right)^2 \times 140$                            |                                                                               |     |

$$= 45.7 \text{ or } 45\frac{5}{7} \text{ cm}^2$$

13



In triangle  $WXZ$ ,  $WX = 8 \text{ cm}$ .

$Y$  is a point on  $XZ$  such that  $WY = 5 \text{ cm}$ ,  $YZ = 11 \text{ cm}$  and  $\sin \angle WYZ = \frac{4}{5}$ .

(a) Find the area of triangle  $WYZ$ .

$$\begin{aligned} \text{Area of triangle } WYZ &= \frac{1}{2} \times 5 \times 11 \times \frac{4}{5} \\ &= 22 \text{ cm}^2 \end{aligned}$$

Answer

.....22.....

[2]

(b) Find the exact size of angle  $WXY$ .

$$\frac{\sin \angle WXY}{5} = \frac{\sin \angle WYX}{8}$$

$$\frac{\sin \angle WXY}{5} = \frac{\sin \angle WYZ}{8}$$

$$\sin \angle WXY = \frac{5 \times \frac{4}{5}}{8}$$

$$\angle WXY = 30^\circ$$

Answer

.....30°.....

[2]

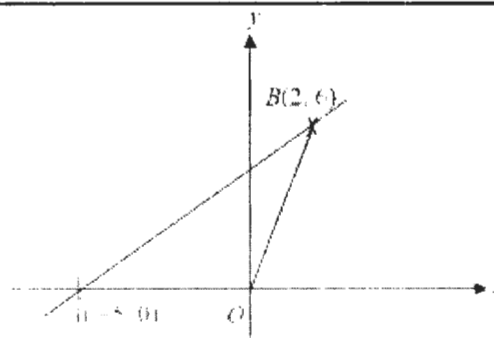
14 Factorise completely

(a)  $x^2 - y^2 - 3x + 3y,$

$$\begin{aligned} &x^2 - y^2 - 3x + 3y \\ &= (x + y)(x - y) - 3(x - y) \end{aligned}$$

|    |     |                                                                                                                                                                                                     |     |
|----|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
|    |     | $= (x - y)(x + y - 3)$                                                                                                                                                                              |     |
|    | (b) | $32a^2 - 50$                                                                                                                                                                                        |     |
|    |     | $32a^2 - 50$<br>$= 2(16a^2 - 25)$<br>$= 2(4a + 5)(4a - 5)$                                                                                                                                          |     |
| 15 |     | The $n$ th term of a sequence is given by $T_n = n^2 + 3$ .                                                                                                                                         |     |
|    | (a) | <p>Explain why 1010 is not a term in the sequence.</p> <p>Answer:</p> <p>If <math>1010 = n^2 + 3</math>, then <math>n = \sqrt{1007} = 31.7</math> (3 sf), which is not a positive integer. [B1]</p> |     |
|    | (b) | Find an expression, in terms of $n$ , for $T_{n+1} - T_n$ .                                                                                                                                         |     |
|    |     | $T_{n+1} - T_n = (n+1)^2 + 3 - [n^2 + 3]$<br>$= n^2 + 2n + 1 + 3 - n^2 - 3$<br>$= 2n + 1.$                                                                                                          |     |
|    |     | <div>Answer</div> <div>.....<math>2n + 1</math>.....</div>                                                                                                                                          | [2] |
|    | (c) | Given that the difference between two successive terms in the sequence is 99, use your answer in part (b) to find the smaller term.                                                                 |     |
|    |     | $2n + 1 = 99$<br>$n = 49$<br>$T_{49} = 49^2 + 3$<br>$= 2404.$                                                                                                                                       |     |
| 16 |     | In the diagram, a straight line passes through the points $A(-5, 0)$ and $B(2, 6)$ .                                                                                                                |     |

10



(a) Find the equation of the line AB,

$$\text{Gradient} = \frac{6 - 0}{2 - (-5)} = \frac{6}{7}$$

$$y - 6 = \frac{6}{7}(x - 2)$$

$$y = \frac{6}{7}x + \frac{30}{7}$$

$$7y = 6x + 30$$

OR

$$\text{Gradient} = \frac{6 - 0}{2 - (-5)} = \frac{6}{7}$$

$$\text{Sub } (2, 6) \text{ into } y = \frac{6}{7}x + c$$

$$6 = \frac{6}{7}(2) + c$$

$$c = \frac{30}{7}$$

$$y = \frac{6}{7}x + \frac{30}{7}$$

(b) Find the length of AB,

$$AB = \sqrt{(6 - 0)^2 + (2 + 5)^2} = 9.22 \text{ units}$$

(c) Find the area of triangle ABO.

$$\frac{1}{2} \times 5 \times 6 = 15 \text{ units}^2$$

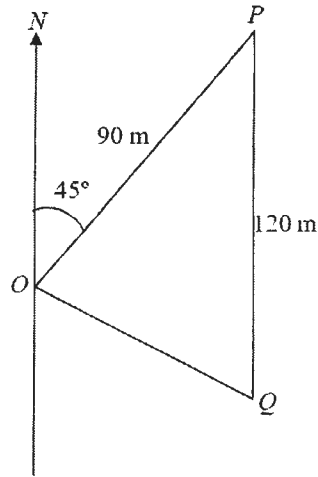
Answer

.....15..... units<sup>2</sup>

[1]

- 17 The diagram shows three points  $O$ ,  $P$  and  $Q$ .  
The bearing of  $P$  from  $O$  is  $045^\circ$  and  $P$  is due north of  $Q$ .  
 $OP = 90 \text{ m}$  and  $PQ = 120 \text{ m}$ .

11



(a) Find the distance of  $OQ$ .

$\angle OPQ = 45^\circ$  ( alt. angles,  $PQ \parallel ON$  )

$$OQ^2 = 90^2 + 120^2 - 2(90)(120)\cos 45^\circ$$

$$= 7226.49$$

$$OQ = 85.008785$$

$$= 85.0$$

Answer

..... m

[3]

(b) Calculate the bearing of  $Q$  from  $O$ .

$$\frac{\sin \angle POQ}{120} = \frac{\sin 45^\circ}{85.008785}$$

$$\sin \angle POQ = \frac{\sin 45^\circ}{85.008785} \times 120$$

$$\angle POQ = 86.529^\circ$$

$$\text{Bearing of } Q \text{ from } O = 45^\circ + 86.529^\circ$$

$$= 131.5^\circ$$

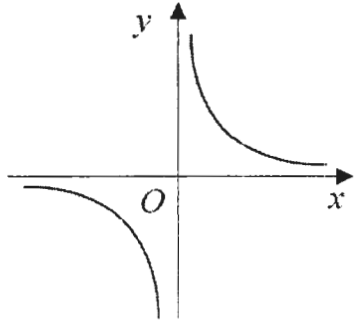
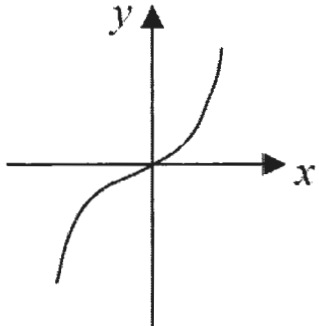
Answer

.....

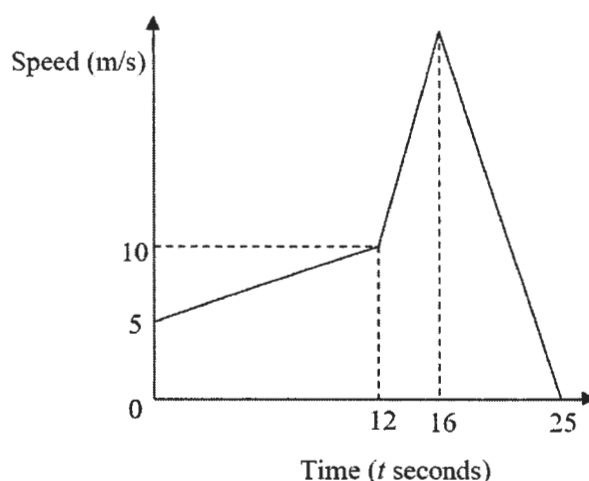
[3]

18

Given  $y = kx^n$ , where  $k > 0$ . State the value of  $n$  for the following graphs.

|     |                                                                                                                                              |                                                     |     |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|-----|
|     |                                                                                                                                              | 12                                                  |     |
| (a) |                                                             |                                                     |     |
|     | Answer                                                                                                                                       | $n = -1$                                            | [1] |
| (b) |                                                            |                                                     |     |
|     | Answer                                                                                                                                       | $n = 3$                                             | [1] |
| 19  | (a) Express $-x^2 + 4x + 1$ in the form $-(x+a)^2 + b$ .                                                                                     |                                                     |     |
|     | $  \begin{aligned}  & -x^2 + 4x + 1 \\  &= -(x^2 - 4x) + 1 \\  &= -(x-2)^2 + 4 + 1 \\  &= -(x-2)^2 + 5  \end{aligned}  $                     |                                                     |     |
|     | (b) Hence, solve the equation $-x^2 + 4x + 1 = 0$ .                                                                                          |                                                     |     |
|     | $  \begin{aligned}  & -(x-2)^2 + 5 = 0 \\  & x-2 = \pm\sqrt{5} \\  & x = \pm\sqrt{5} + 2 \\  & x = -0.236 \text{ or } 4.24  \end{aligned}  $ |                                                     |     |
|     | Answer                                                                                                                                       | $x = \dots -0.236\dots \text{ or } \dots 4.24\dots$ | [2] |

- 20 The diagram shows the speed-time graph of an object moving in a straight line over a period of 25 seconds. The object travelled 90 m from  $t = 12$  to  $t = 16$ .



- (a) Find the maximum speed of the object.

area under graph = distance travelled

$$\frac{1}{2}(4 \text{ s})(10 + v_{\text{max}}) = 90$$

$$v_{\text{max}} = 35 \text{ m/s}$$

Answer ..... m/s [2]

- (b) Find the speed of the object at  $t = 6$ .

Method 1:

$$a \text{ in first 12 s} = \frac{10 \text{ m/s} - 5 \text{ m/s}}{12 \text{ s}}$$

Method 2:

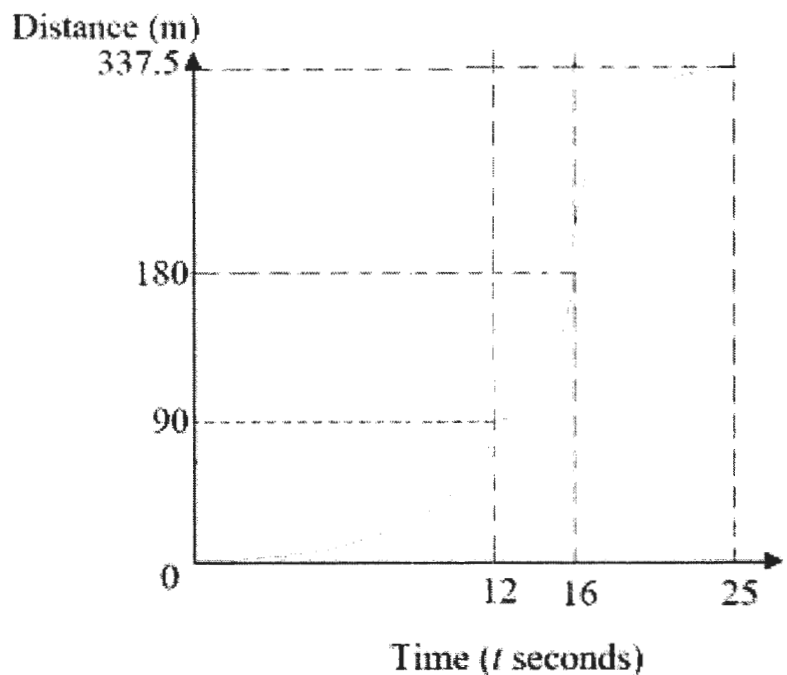
$$\frac{v - 5}{6 - 0} = \frac{10 - 5}{12 - 0}$$

$$v = 7.5 \text{ m/s}$$

$$= \frac{5}{12} \text{ m/s}^2$$
$$\text{speed at } t = 6 = 5 \text{ m/s} + (6 \text{ s})\left(\frac{5}{12} \text{ m/s}^2\right)$$
$$= 7.5 \text{ m/s}$$

Answer ..... m/s [2]

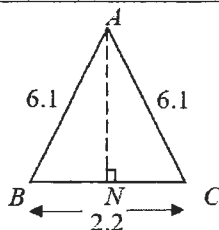
(c) Sketch the distance-time graph for  $0 \leq t \leq 25$ .



S1 for the correct shape for the 3 parts of the journey, without the distances on the vertical axis. Minus 1 mark for 1 wrong value of distance.

[3]

- 21 Triangle  $ABC$  has  $AB = AC = 6.1$  cm,  $BC = 2.2$  cm.



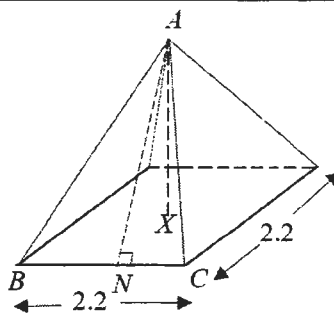
- (a) Show that  $AN = 6$  cm.

$$AC^2 = AN^2 + NC^2$$

$$AN = \sqrt{6.1^2 - 1.1^2} \quad [\text{M1}]$$

$$AN = 6 \quad [\text{A1}]$$

- (b) Triangle  $ABC$  in (a), is one of the faces of a square-based pyramid.  $X$  is the centre of the base of the pyramid.



- (i) Calculate the total surface area of the pyramid.

16

|  |  |                                                                                                                                                          |     |
|--|--|----------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
|  |  | Total surface area<br>$= (2.2 \times 2.2) + 4 \left( \frac{1}{2} \times 2.2 \times 6 \right) \quad [\text{M1}]$ $= 31.24 \text{ cm}^2 \quad [\text{A1}]$ |     |
|  |  | <i>Answer</i> .....31.24..... cm <sup>2</sup>                                                                                                            | [2] |
|  |  | (ii) Calculate the volume of the pyramid.                                                                                                                |     |
|  |  | $AX = \sqrt{6^2 - 1.1^2}$ $= 5.8983$ $\text{Vol of the pyramid} = \frac{1}{3} \times 2.2^2 \times 5.8983$ $= 9.52$                                       |     |
|  |  | <i>Answer</i> .....9.52..... cm <sup>3</sup>                                                                                                             | [3] |

|       |              |      |
|-------|--------------|------|
| Class | Index Number | Name |
|-------|--------------|------|

## BUKIT MERAH SECONDARY SCHOOL



### END-OF-YEAR EXAMINATION 2023 SECONDARY 3 EXPRESS

#### MATHEMATICS

Paper 2

**4052/02**

**6 October 2023**

**1 hour 30 minutes**

Candidates answer on the Question Paper.

#### READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 60.

**Calculator Model:**

**For Examiner's Use**

Setter: Ms Lim LE

This document consists of **16** printed pages and **2** blank pages.

**[Turn over**

---

*Mathematical Formulae*

---

*Compound Interest*

$$\text{Total amount} = P \left( 1 + \frac{r}{100} \right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left( \frac{\sum fx}{\sum f} \right)^2}$$

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- 1 Mr Zee drove from Singapore to Penang and then back to Singapore. The distance between Singapore and Penang is approximately 720 km via the North-South Highway.

- (a) During his journey from Singapore to Penang, he drove at an average speed of  $x$  km/h. Write down an expression, in terms of  $x$ , for the time Mr Zee took to travel from Singapore to Penang.

*Answer* ..... hours [1]

- (b) On his return journey, his average speed was reduced by 10 km/h. Write down an expression, in terms of  $x$ , for the time Mr Zee took to make the return journey.

*Answer* ..... hours [1]

- (c) Give that the difference in time for the two journeys was 105 minutes, form an equation in  $x$  and show that it reduces to  $7x^2 - 70x - 28800 = 0$ .

*Answer* [3]

5

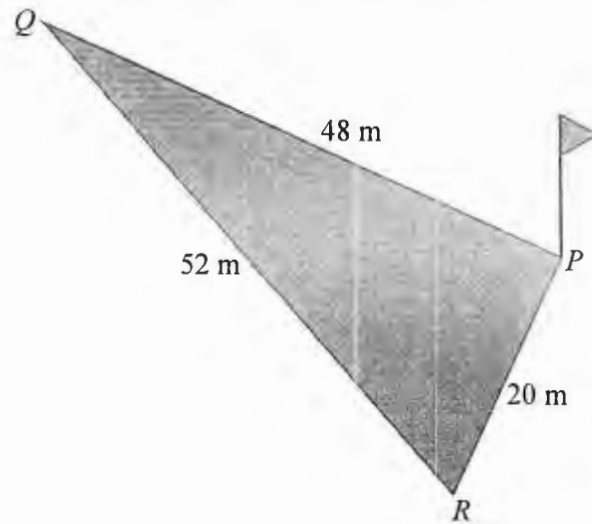
- 
- (d) Solve the equation  $7x^2 - 70x - 28800 = 0$ .

*Answer*  $x = \dots\dots\dots$  or  $\dots\dots\dots$  [3]

- (e) Find the time taken for the entire journey, giving your answer in hours, minutes and seconds, correct to the nearest second.

*Answer*  $\dots\dots\dots$  hours  $\dots\dots\dots$  minutes  $\dots\dots\dots$  seconds [2]

- 2 The diagram shows a triangular court  $PQR$ .  $PR = 20$  m,  $PQ = 48$  m and  $RQ = 52$  m.



- (a) Prove that  $\angle QPR$  is a right angle.

*Answer*

[2]

- (b) Hence, or otherwise, calculate angle  $PRQ$ .

*Answer*  $\angle PRQ = \dots\dots\dots$  [2]

---

(c) Find the shortest distance of  $P$  from the line  $RQ$ .

*Answer* ..... m [2]

(d) If a flag post of height 4.5 m is hoisted up at  $P$  and a bird was perched on top of the flag post, calculate

(i) the greatest angle of elevation of the bird from a wild chicken walking from  $Q$  to  $R$ ,

*Answer* ..... [2]

(ii) the greatest angle of depression of the wild chicken from the bird.

*Answer* ..... [1]

- 3 (a) Complete the table of values for  $y = \frac{1}{5}x^3 - \frac{4}{5}x^2$ .

|     |      |    |   |      |   |      |   |   |
|-----|------|----|---|------|---|------|---|---|
| $x$ | -1.5 | -1 | 0 | 1    | 2 | 3    | 4 | 5 |
| $y$ | -2.5 | -1 | 0 | -0.6 |   | -1.8 | 0 | 5 |

[1]

- (b) On the grid opposite, draw the graph of  $y = \frac{1}{5}x^3 - \frac{4}{5}x^2$  for  $-1.5 \leq x \leq 5$ .

[3]

- (c) Explain how your graph shows that there is only one solution of the equation  $\frac{1}{5}x^3 - \frac{4}{5}x^2 = 2$ .

[1]

*Answer*

.....  
 .....  
 .....

- (d) The equation  $x^3 - 4x^2 + 10x - 20 = 0$  can be solved by finding the points of intersection of the straight line  $y = ax + b$  and the curve  $y = \frac{1}{5}x^3 - \frac{4}{5}x^2$ .

- (i) Find the value of  $a$  and the value of  $b$ .

*Answer*  $a = \dots\dots\dots$ ,  $b = \dots\dots\dots$  [2]

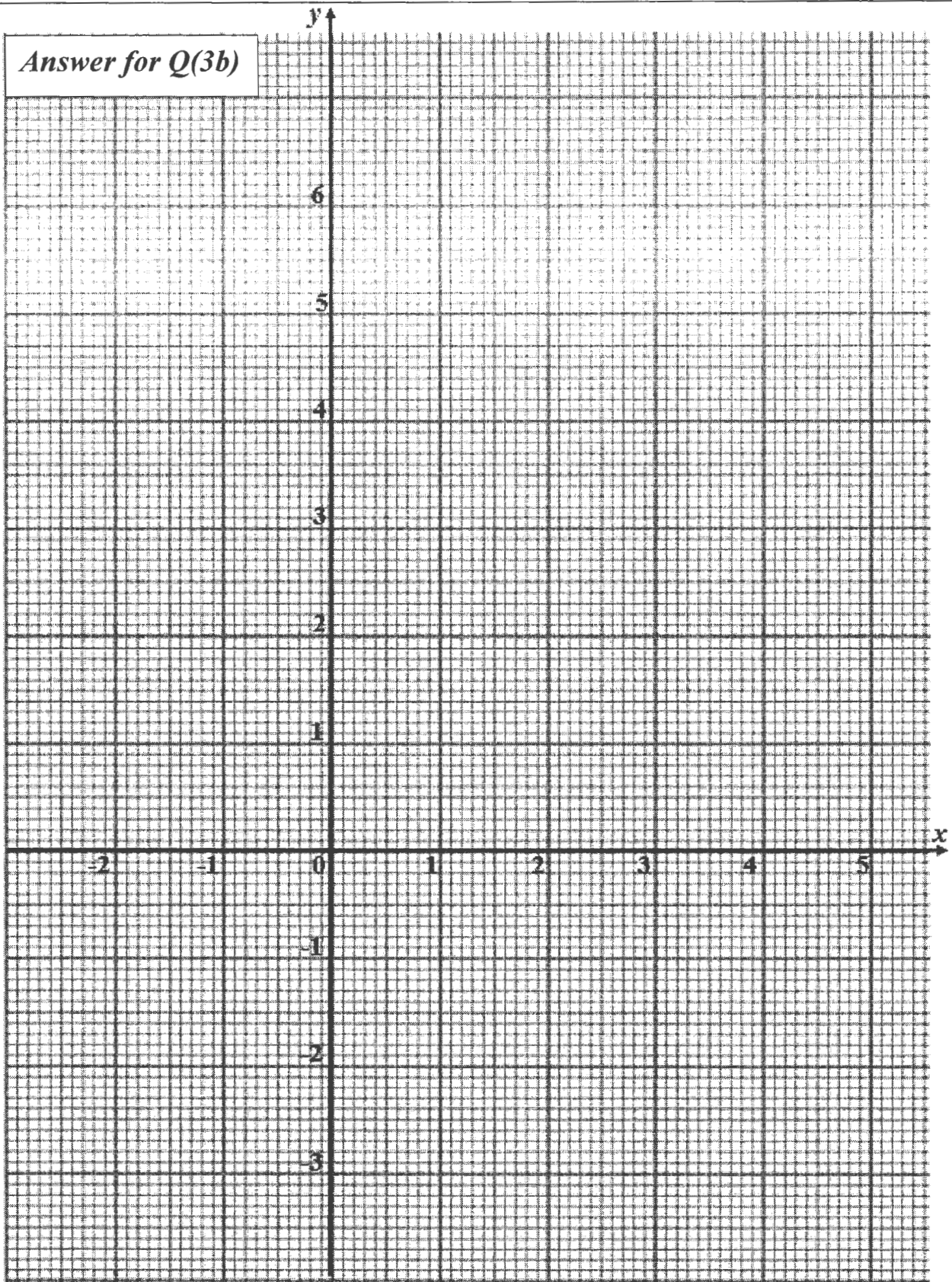
- (ii) By drawing the line  $y = ax + b$ , solve the equation  $x^3 - 4x^2 + 10x - 20 = 0$ .

*Answer*  $x = \dots\dots\dots$  [2]

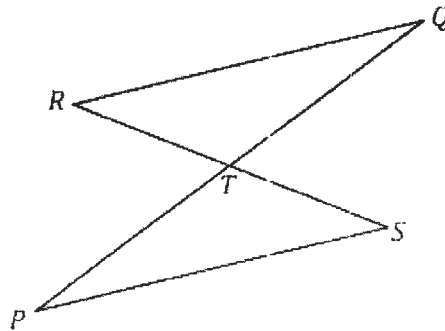
- (e) By drawing a tangent, find the gradient of the curve at  $(-1, -1)$ .

*Answer*  $\dots\dots\dots$  [2]

*Answer for Q(3b)*



4 (a)



The straight lines  $PQ$  and  $RS$  bisect each other at  $T$ .  $PQ = 26.2$  cm and  $RS = 18.6$  cm.

- (i) Using the dimensions given, show that area of triangle  $PST$  = area of triangle  $QRT$ . [2]

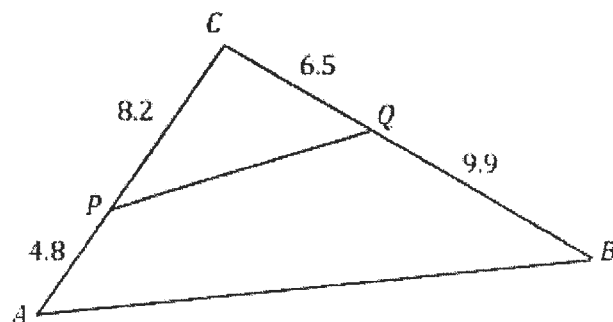
*Answer*

- (ii) By proving that the two triangles are congruent and stating your reasons clearly, explain why  $PS$  is parallel to  $RQ$ . [3]

*Answer*

11

(b)



$ABC$  is a triangle and  $P$  and  $Q$  lie on  $AC$  and  $BC$  respectively such that  $AP = 4.8$  cm,  $PC = 8.2$  cm,  $BQ = 9.9$  cm and  $QC = 6.5$  cm.

(i) Show that triangle  $ABC$  is similar to triangle  $QPC$ .

[3]

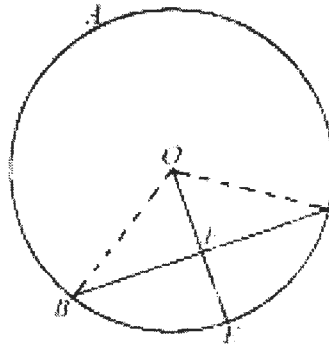
*Answer*

(ii) Hence, find the area of triangle  $QPC$ , given that the area of triangle  $ABC$  is  $100 \text{ cm}^2$ .

*Answer* .....  $\text{cm}^2$  [2]

12

- 5 The diagram below shows a circle of radius 7 cm, centre  $O$ .  
 $OE$  is perpendicular to the chord  $BC$  at the point  $F$ ,  $OF = 4$  cm.



- (a) Find, giving reasons for each answer, the length of chord  $BC$ .

Answer ..... cm [3]

- (b) (i) Find the angle  $BOC$ , in radians.

Answer ..... radians [2]

13

---

(ii) Express angle  $BOC$ , in degrees.

*Answer* ..... [1]

(iii) Find the area of the major segment  $BAC$ .

*Answer* .....  $\text{cm}^2$  [3]

- 6 (a) Betty has \$300 000 set aside for her investment plan with a bank. She is looking at a savings plan that guarantees a simple interest rate of 3.4% per annum for a period of 5 years. This savings plan requires a **minimum** amount of \$300 000 to be invested. She is proposed with two financial options, *A* and *B*, by her financial consultant. The table below shows the details of the two financial options.

|                 |                                                                                                                                                                                                                                                                                              |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Option <i>A</i> | Invest all her \$300 000 in the savings plan.                                                                                                                                                                                                                                                |
| Option <i>B</i> | <p>Betty will invest \$150 000 of her savings and borrow another \$150 000 from the bank at a simple interest rate of 2% per annum for 5 years.</p> <p>She will invest her remaining \$150 000 in a fixed deposit that guarantees a 3% compound interest, compounded yearly for 5 years.</p> |

Explain, with calculations, which option should Betty choose to maximise her investment earnings.

[5]

*Answer*

(Continued from page 14)

- (b) Betty wants to set aside a budget for a trip to Turkey with her sister and niece. She intends to spend 14 days in Turkey and in this trip fulfil her dream of taking the hot air balloon. She plans to take a hot air balloon package with video if the package does not exceed her budget of \$10 000 for the trip which she intends to pay for her sister and niece as well. They will only pay for their own shopping expenses. The following information below will help her work out a feasible budget for the trip.

[6]

| Daily Estimated Cost for 3 pax |                | Other Expenses          |                   |
|--------------------------------|----------------|-------------------------|-------------------|
| Item                           | Estimated cost |                         |                   |
| Food                           | \$ 90          | Air ticket (round trip) | \$1480 per person |
| Accommodation                  | \$150          | Shopping                | 10 000 TRY        |
| Car rental                     | \$45           |                         |                   |
| Petrol                         | \$20           |                         |                   |

| Hot Air Balloon Package Deal          | Price   |
|---------------------------------------|---------|
| 1 individual without video            | 200 USD |
| 1 individual with video               | 300 USD |
| 15% discount for 2 persons and above. |         |

| XYZ MONEY EXCHANGE   |        |         |
|----------------------|--------|---------|
| \$S1                 | We Buy | We Sell |
| USD                  | 0.75   | 0.74    |
| TRY                  | 17.53  | 17.30   |
| TRY for Turkish Lira |        |         |

Using the above information, suggest a minimum budget in **Singapore dollars** for Betty to set aside for this trip. Justify the decision she has to make and show your calculations clearly.

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(Continued from page 16)

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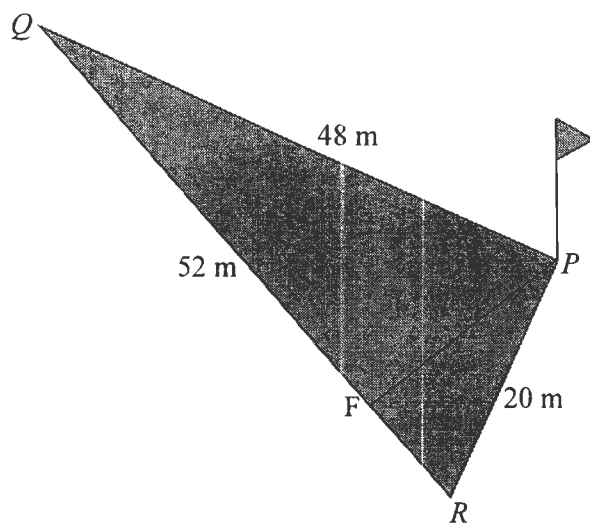
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LLE/BMSS/2023/3E/EOY/4052/2

## MARKING SCHEME FOR PAPER 2

|     |                                                                                                                                                                                                                                                |  |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| 1   | Mr Zee drove from Singapore to Penang and then back to Singapore. The distance between Singapore and Penang is approximately 720 km via the North-South Highway.                                                                               |  |
| (a) | During his journey from Singapore to Penang, he drove at an average speed of $x$ km/h. Write down an expression, in terms of $x$ , for the time Mr Zee took to travel from Singapore to Penang.                                                |  |
|     | Time for outward journey = $\frac{720}{x}$ h [B1]                                                                                                                                                                                              |  |
| (b) | On his return journey, his average speed was reduced by 10 km/h. Write down an expression, in terms of $x$ , for the time Mr Tan took to make the return journey.                                                                              |  |
|     | Time for return journey = $\frac{720}{x-10}$ h [B1]                                                                                                                                                                                            |  |
| (c) | Give that the difference in time for the two journeys was 105 minutes, form an equation in $x$ and show that it reduces to $7x^2 - 70x - 28800 = 0$ .                                                                                          |  |
|     | $\frac{720}{x-10} - \frac{720}{x} = \frac{105}{60}$ $4[720x - 720(x-10)] = 7x(x-10)$ $7x^2 - 70x - 28800 = 0$ <p>OR</p> $\frac{720x - 720(x-10)}{x(x-10)} = \frac{7}{4}$ $4[720x - 720(x-10)] = 7x(x-10)$ $7x^2 - 70x - 28800 = 0$             |  |
| (d) | <p>Solve the equation <math>7x^2 - 70x - 28800 = 0</math>.</p> $7x^2 - 70x - 28800 = 0$ $x = \frac{-(-70) \pm \sqrt{4900 - 4(7)(-28800)}}{2(7)}$ $= \frac{70 \pm \sqrt{811300}}{14}$ $= 69.337 \text{ or } -59.337$ $= 69.3 \text{ or } -59.3$ |  |

2

|   |     |                                                                                                                                                                                                                                                                                                                                                                                      |                                                                  |                                                                  |  |
|---|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------|------------------------------------------------------------------|--|
|   | (e) | Find the time taken for the entire journey, giving your answer in hours, minutes and seconds, correct to the nearest second.                                                                                                                                                                                                                                                         |                                                                  |                                                                  |  |
|   |     | $\frac{720}{59.337} + \frac{720}{69.337}$ $= 22.518 \text{ h}$ $= 22 \text{ h } 31 \text{ min } 5 \text{ sec}$                                                                                                                                                                                                                                                                       |                                                                  |                                                                  |  |
| 2 |     | <p>The diagram shows a triangular court <math>PQR</math>. <math>PR = 20 \text{ m}</math>, <math>PQ = 48 \text{ m}</math> and <math>RQ = 52 \text{ m}</math>.</p>  <p>(a) Prove that <math>\angle QPR</math> is a right angle.</p>                                                                 | [2]                                                              |                                                                  |  |
|   |     | $PR^2 + PQ^2 = 400 + 2304 \quad )$ $= 2704 \quad )$ $RQ^2 = 2704 \quad )$ <p>Since <math>PR^2 + PQ^2 = RQ^2</math>, <math>\angle QPR = 90^\circ</math> (by the converse of Pythagoras' Theorem)</p> <p>OR</p> $\cos \angle QPR = \frac{48^2 + 20^2 - 52^2}{2(48)(20)} = 0$ $\angle QPR = 90^\circ \text{ (shown)}$ <p>(b) Hence, or otherwise, calculate angle <math>PRQ</math>.</p> |                                                                  |                                                                  |  |
|   |     | $\angle PRQ = \sin^{-1} \frac{48}{52}$ $= 67.380$ $= 67.4^\circ$                                                                                                                                                                                                                                                                                                                     | $\angle PRQ = \cos^{-1} \frac{20}{52}$ $= 67.380$ $= 67.4^\circ$ | $\angle PRQ = \tan^{-1} \frac{48}{20}$ $= 67.380$ $= 67.4^\circ$ |  |

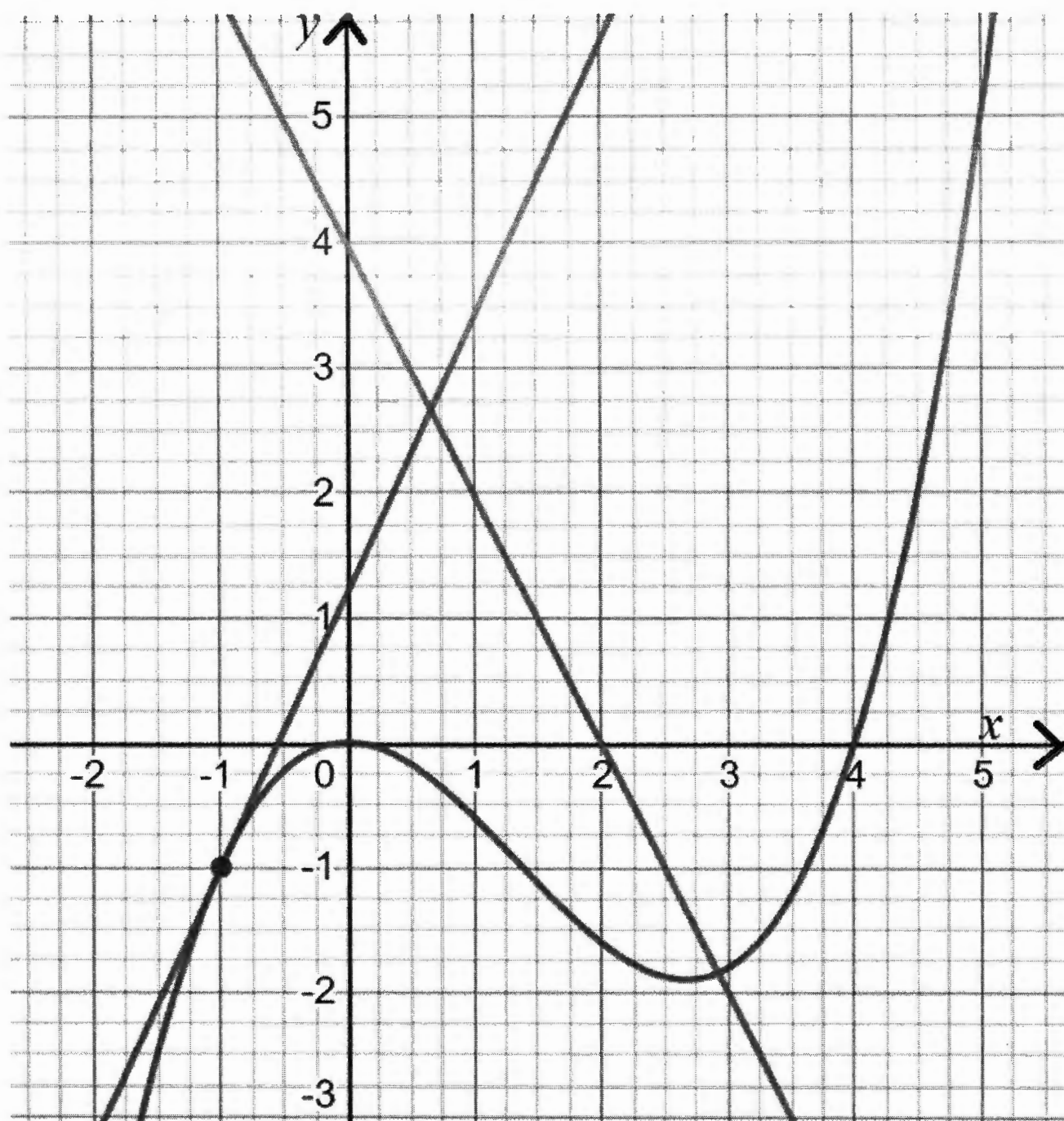
3

|  |  |                                                                                                                                                                                                                        |  |
|--|--|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
|  |  | (c) Find the shortest distance of $P$ from the line $RQ$ .                                                                                                                                                             |  |
|  |  | The shortest distance of $P$ from the line $RQ = 20\sin \angle PRQ$<br>$= 18.462$<br>$= 18.5 \text{ or } \frac{240}{13}$                                                                                               |  |
|  |  | (d) If a flag post of height 4.5 m is hoisted up at $P$ and a bird was perched on top of the flag post. calculate<br><br>(i) the greatest angle of elevation of the bird from a wild chicken walking from $Q$ to $R$ , |  |
|  |  | Greatest angle of elevation of the bird from the chicken $= \tan^{-1} \frac{4.5}{18.462}$<br>$= 13.7^\circ$                                                                                                            |  |
|  |  | (ii) the greatest angle of depression of the wild chicken from the bird.                                                                                                                                               |  |
|  |  | Greatest angle of depression of the chicken from the bird $= 13.7^\circ$                                                                                                                                               |  |

|     |      |                                                                                                                                                                                                                                                              |     |      |    |      |   |   |   |   |   |     |      |    |   |      |  |      |   |   |     |
|-----|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|------|----|------|---|---|---|---|---|-----|------|----|---|------|--|------|---|---|-----|
| 3   | (a)  | Complete the table of values for $y = \frac{1}{5}x^3 - \frac{4}{5}x^2$ .                                                                                                                                                                                     |     |      |    |      |   |   |   |   |   |     |      |    |   |      |  |      |   |   |     |
|     |      | <table><tr><td><math>x</math></td><td>-1.5</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td><math>y</math></td><td>-2.5</td><td>-1</td><td>0</td><td>-0.6</td><td></td><td>-1.8</td><td>0</td><td>5</td></tr></table> | $x$ | -1.5 | -1 | 0    | 1 | 2 | 3 | 4 | 5 | $y$ | -2.5 | -1 | 0 | -0.6 |  | -1.8 | 0 | 5 | [1] |
| $x$ | -1.5 | -1                                                                                                                                                                                                                                                           | 0   | 1    | 2  | 3    | 4 | 5 |   |   |   |     |      |    |   |      |  |      |   |   |     |
| $y$ | -2.5 | -1                                                                                                                                                                                                                                                           | 0   | -0.6 |    | -1.8 | 0 | 5 |   |   |   |     |      |    |   |      |  |      |   |   |     |
|     |      | -1.6 [B1]                                                                                                                                                                                                                                                    |     |      |    |      |   |   |   |   |   |     |      |    |   |      |  |      |   |   |     |
|     | (b)  | On the grid opposite, draw the graph of for $-2 \leq x \leq 5$ .                                                                                                                                                                                             | [3] |      |    |      |   |   |   |   |   |     |      |    |   |      |  |      |   |   |     |
|     |      | D3 for a smooth curve through 8 correct points<br>B2FT for 8 points plotted correctly<br>B1FT for 6 or 7 points plotted correctly<br><br>Tolerance $\pm 1\text{mm}$ for plotting points and drawing curve through points                                     |     |      |    |      |   |   |   |   |   |     |      |    |   |      |  |      |   |   |     |

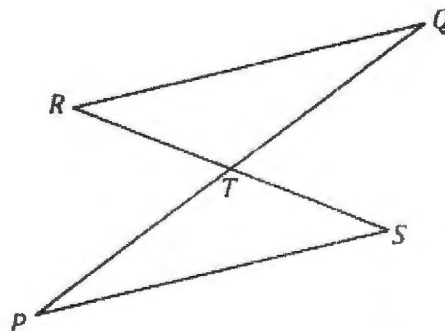
|  |     |                                                                                                                                                                                                                                                                                                          |     |
|--|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
|  | (c) | Explain how your graph shows that there is only one solution of the equation<br>$\frac{1}{5}x^3 - \frac{4}{5}x^2 = 2$                                                                                                                                                                                    | [1] |
|  |     | <p><i>Answer</i></p> <p>There is only <b>one point of intersection</b> between the curve <math>y = \frac{1}{5}x^3 - \frac{4}{5}x^2</math> and <b>the line</b> <math>y = 2</math>. [B1 for the 2 answers in bold]</p>                                                                                     |     |
|  | (d) | <p>The equation <math>x^3 - 4x^2 + 10x - 20 = 0</math> can be solved by finding the points of intersection of the straight line <math>y = ax + b</math> and the curve <math>y = \frac{1}{5}x^3 - \frac{4}{5}x^2</math>.</p> <p>(i) Find the value of <math>a</math> and the value of <math>b</math>.</p> |     |
|  |     | $x^3 - 4x^2 + 10x - 20 = 0$ $\frac{1}{5}x^3 - \frac{4}{5}x^2 + 2x - 4 = 0$ $\frac{1}{5}x^3 - \frac{4}{5}x^2 = -2x + 4$ $a = -2, b = 4$                                                                                                                                                                   |     |
|  |     | <p>(ii) By drawing the line <math>y = ax + b</math>, solve the equation <math>x^3 - 4x^2 + 10x - 20 = 0</math></p> <p>Correct straight line drawn on the grid [D1]</p> <p><math>2.9 \pm 0.1</math> [B1]</p>                                                                                              |     |
|  | (e) | By drawing a tangent, find the gradient of the curve at $(-1, -1)$ .                                                                                                                                                                                                                                     |     |
|  |     | <p>Straight tangent drawn at <math>(-1, -1)</math> [D1]</p> <p>Gradient of tangent = <math>\frac{-1 - 4.5}{-1 - 1.5}</math></p> <p><math>= 2.2 \pm 0.22</math> [1.98 to 2.42] [B1]</p>                                                                                                                   |     |

5



4

(a)



The straight lines  $PQ$  and  $RS$  bisect each other at  $T$ .  $PQ = 26.2$  cm and  $RS = 18.6$  cm.

- (i) Using the dimensions given, show that area of triangle  $PST$  = area of triangle  $QRT$ . [2]

*Answer*

$$\angle PTS = \angle QTR \text{ (vert. opp. } \angle\text{s)}$$

$$\frac{\text{Area of } \triangle PST}{\text{Area of } \triangle QRT} = \frac{\frac{1}{2} \times 13.1 \times 9.3 \sin \angle PTS}{\frac{1}{2} \times 13.1 \times 9.3 \sin \angle QTR} = 1$$

$$\text{Area of triangle } PST = \text{Area of triangle } QRT$$

- (ii) By proving that the two triangles are congruent and stating your reasons clearly, explain why  $PS$  is parallel to  $RQ$ . [3]

*Answer*

$$PT = QT \text{ (Given that } PQ \text{ and } RS \text{ bisect each other at } T)$$

$$RT = ST \text{ (Given)}$$

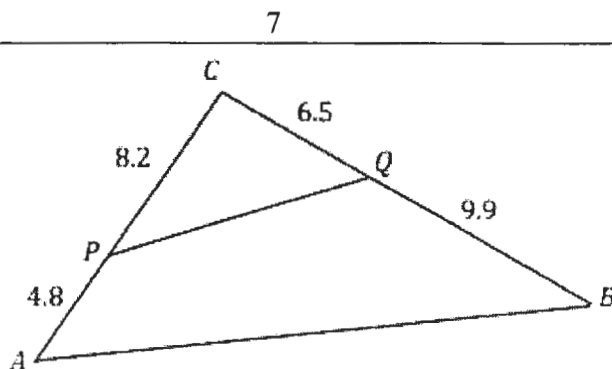
$$\angle PTS = \angle QTR \text{ (vert. opp. } \angle\text{s)}$$

$$\triangle PST \equiv \triangle QRT \text{ (SAS)}$$

$$\angle PST = \angle QRT \text{ (alt. } \angle\text{s)} \text{ OR } \angle SPT = \angle RQT \text{ (alt. } \angle\text{s)}$$

$$\text{Therefore, } PS \text{ is parallel to } RQ$$

(b)



$ABC$  is a triangle and  $P$  and  $Q$  lie on  $AC$  and  $BC$  respectively such that  $AP = 4.8$  cm,  $PC = 8.2$  cm,  $BQ = 9.9$  cm and  $QC = 6.5$  cm.

(i) Show that triangle  $ABC$  is similar to triangle  $QPC$ .

[3]

*Answer*

$$\frac{AC}{QC} = \frac{13}{6.5} = 2 \quad \text{and} \quad \frac{BC}{PC} = \frac{16.4}{8.2} = 2$$

$\angle ACB = \angle QCP$  (Common angle)

$\therefore \triangle ABC$  and  $\triangle QPC$  are similar (SAS)

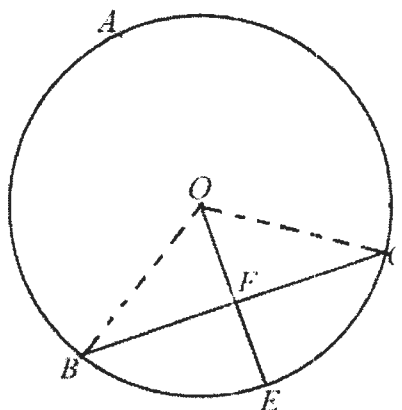
(b) (ii) Hence, find the area of triangle  $QPC$ , given that the area of triangle  $ABC$  is  $100 \text{ cm}^2$ .

$$\frac{\text{area of } \triangle QPC}{\text{area of } \triangle ABC} = \frac{1}{4}$$

$$\begin{aligned} \text{area of } \triangle QPC &= \frac{1}{4} \times 100 \\ &= 25 \text{ cm}^2 \end{aligned}$$

8

- 5 The diagram below shows a circle of radius 7 cm, centre  $O$ .  
 $OE$  is perpendicular to the chord  $BC$  at the point  $F$ ,  $OF = 4$  cm.



Find, giving reasons for each answer,

- (a) the length of chord  $BC$ .

Radius  $OB = 7$  cm

$$\begin{aligned} BF &= \sqrt{7^2 - 4^2} \\ &= \sqrt{33} \end{aligned}$$

Since  $OF \perp BC$ ,  $BF = FC$  (Perpendicular from centre bisects chord) )

OR Since  $OF \perp BC$ ,  $BF = FC$  (Ppty of isos. triangle) )

$$\begin{aligned} BC &= 2 \times \sqrt{33} \\ &= 11.5 \text{ cm} \end{aligned}$$

- (b) (i) Find the angle  $BOC$ , in radians.

OR

$$\begin{aligned} \angle BOC &= \cos^{-1} \frac{7^2 + 7^2 - (2\sqrt{33})^2}{2 \times 7 \times 7} \\ &= 1.9251 \\ &= 1.93 \text{ rad} \end{aligned}$$

$$\begin{aligned} \angle BOC &= 2 \cos^{-1} \frac{4}{7} \\ &= 1.9251 \\ &= 1.93 \text{ rad} \end{aligned}$$

- (ii) Express angle  $BOC$ , in degrees.

$$\angle BOC = 110.3^\circ$$

(iii) Find the area of the major segment  $BAC$ .

$$\begin{aligned}\text{Area of the major segment } BAC &= \frac{1}{2} \times 7^2 \times \sin 1.9251 + \frac{1}{2} \times (2\pi - 1.9251) \times 7^2 \\ &= 22.978 + 106.773 \\ &= 130 \text{ cm}^2\end{aligned}$$

OR

$$\begin{aligned}\text{Area of the major segment } BAC &= \frac{1}{2} \times 7^2 \times \sin 110.300^\circ + \frac{360 - 110.300}{360} \times \pi \times 7^2 \\ &= 22.978 + 106.773 \\ &= 130 \text{ cm}^2\end{aligned}$$

- 6 (a) Betty has \$300 000 set aside for her investment plan with a bank. She is looking at a savings plan that guarantees a simple interest rate of 3.4% per annum for a period of 5 years. This savings plan requires a **minimum** amount of \$300 000 to be invested. She is proposed with two financial options,  $A$  and  $B$ , by her financial consultant. The table below shows the details of the two financial options.

|            |                                                                                                                                                                                                                                                                                              |
|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Option $A$ | Invest all her \$300 000 in the savings plan.                                                                                                                                                                                                                                                |
| Option $B$ | <p>Betty will invest \$150 000 of her savings and borrow another \$150 000 from the bank at a simple interest rate of 2% per annum for 5 years.</p> <p>She will invest her remaining \$150 000 in a fixed deposit that guarantees a 3% compound interest, compounded yearly for 5 years.</p> |

Explain, with calculations, which option should Betty choose to maximise her investment earnings.

[5]

*Answer*

10

For Option A:

$$I = \frac{300000 \times 3.4 \times 5}{100}$$

$$= \$51\,000$$

∴ Earnings after 5 years is \$51 000.

For Option B:

$$\text{Interest to the bank} = \frac{150000 \times 2 \times 5}{100}$$

$$= \$15\,000$$

$$\text{Amount} = 150000 \left( 1 + \frac{3}{100} \right)^5$$

$$[\text{M1, no marks awarded if this is taken as the interest}]$$

$$= \$173\,891.11$$

$$\begin{aligned} \text{Total earnings after 5 years} \\ &= 51\,000 + (173\,891.11 - 150\,000) - 15\,000 \\ &= \$59\,891.11 > \$51\,000 \end{aligned}$$

Since the total earnings for Option B is higher than that of Option A, Betty should choose Option B to maximise her earnings.

OR

Since the total amount for Option B (\$359 891.11) is higher than that of Option A (\$351 000), Betty should choose Option B to maximise her earnings.

- (b) Betty wants to set aside a budget for a trip to Turkey with her sister and niece. She intends to spend 14 days in Turkey and in this trip fulfil her dream of taking the hot air balloon. She plans to take a hot air balloon package with video if the package does not exceed her budget of \$10 000 for the trip which she intends to pay for her sister and niece as well. They will only pay for their own shopping expenses. The following information below will help her work out a feasible budget for the trip.

[6]

| Daily Estimated Cost for 3 pax |                | Other Expenses          |                   |
|--------------------------------|----------------|-------------------------|-------------------|
| Item                           | Estimated cost | Air ticket (round trip) | \$1480 per person |
| Food                           | \$90           | Shopping                | 10 000 TRY        |
| Accommodation                  | \$150          |                         |                   |
| Car rental                     | \$45           |                         |                   |
| Petrol                         | \$20           |                         |                   |

| Hot Air Balloon Package Deal          | Price   | XYZ MONEY EXCHANGE   |        |         |
|---------------------------------------|---------|----------------------|--------|---------|
| 1 individual without video            | 200 USD | USD                  | We Buy | We Sell |
| 1 individual with video               | 300 USD | USD                  | 0.75   | 0.74    |
| 15% discount for 2 persons and above. |         | TRY                  | 17.53  | 17.30   |
|                                       |         | TRY for Turkish Lira |        |         |

Using the above information, suggest a minimum budget in **Singapore dollars** for Betty to set aside for this trip. Justify the decision she has to make and show your calculations clearly.

*Answer*

$$\text{Betty's shopping expenses in S\$} = \frac{10000}{17.30} = \text{S\$}578.035$$

$$\begin{aligned} \text{Total expenses excluding Betty's shopping expenses} \\ &= 14(90 + 150 + 45 + 20) + 3 \times (1480) \\ &= \$8710 \end{aligned}$$

$$\begin{aligned} \text{Total expenses} \\ &= 8710 + 578.035 \\ &= \$9\,288.035 \end{aligned}$$

$$\begin{aligned} \text{Package without video} \\ &= \frac{200}{0.74} \times 3 \times 0.85 \quad ) \\ &= \$689.189 \\ \text{Package with video} \\ &= \frac{300}{0.74} \times 3 \times 0.85 \quad ) \\ &= \$1\,033.784 \end{aligned}$$

$$\text{Expenses, including the package without the video} = 9\,288.035 + 689.189 = \$9\,977.22$$

$$\text{The minimum budget in Singapore dollars for Betty to set aside for this trip} = \$9\,977.22$$

$$\text{Expenses, including the package with the video} = 9\,288.035 + 1\,033.784 = \$10\,321.82 > \$10\,000$$

Betty will take up the package without the video for this trip as the one with the video exceeds her budget of \$10 000.

