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Name	Class				Index Number			
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BROADRICK SECONDARY SCHOOL

SECONDARY 3 EXPRESS / SECONDARY 3 NORMAL (ACADEMIC) (SBB)

END-OF-YEAR EXAMINATION 2023

MATHEMATICS

4052/01

Paper 1

Oct 2023

Candidates answer on the Question Paper

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use		
Reason	Question Number	Marks Deducted
Rounding-off		
Premature Rounding		
Others		

For Examiner's Use

This document consists of **18** printed pages.

[Turn Over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved Surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

3

Answer **all** the questions.

- 1 Calculate $\sqrt[3]{\frac{8.47^2 - 68}{180 \times 0.0076}}$, and give your answer correct to 3 decimal places.

Answer [1]

- 2 (a) Simplify $6 - 5(x + 2)$.

Answer [1]

- (b) Solve $4x + 10 = -8$.

Answer $x =$ [1]

- 3 Macy is 10% taller than Grace.

If the sum of their heights is 336 cm, find Grace's height.

Answer cm [2]

4 The table shows the sizes of a batch of eggs collected at a farm.

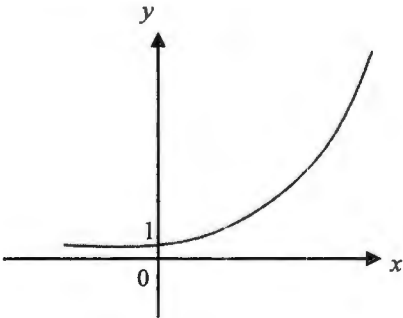
	Number of eggs	Sector Angle
Small	24	
Medium	30	
Large	18	

A pie chart is to be drawn to represent this information.

Calculate the angle of each sector of the pie chart.
Write your answers in the table above.

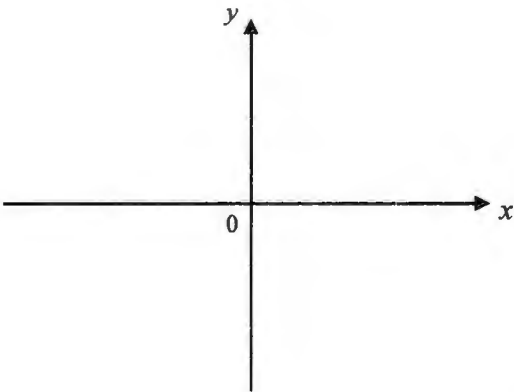
[3]

5 (a) Write down a possible equation for the following graph.



Answer [1]

(b) Sketch the graph of $y = -x^{-2}$.



[1]

- 6 Given that $6 \sin x = 4$, find the two possible values for angle x , where $0^\circ \leq x \leq 180^\circ$.

Answer $x = \dots\dots\dots^\circ$ or $\dots\dots\dots^\circ$ [2]

- 7 The side of an equilateral triangle is approximated to 8 cm to the nearest centimetre.
Find the shortest possible perimeter of the triangle.

Answer $\dots\dots\dots$ cm [2]

- 8 Factorise completely $4ab - 8b - ac + 2c$.

Answer $\dots\dots\dots$ [2]

- 9 A card is selected at random from 20 cards numbering 1 to 20.

Find the probability that the number on the card

- (a) is an odd number.

Answer [1]

- (b) is even and divisible by 3.

Answer [1]

- 10 A box measures 96 cm by 60 cm by 48 cm.
The box is completely filled with identical cubes.

Find the minimum number of cubes required.

Answer [2]

7

11 Solve the following pair of simultaneous equations.

$$5x + 2y - 8 = 0$$

$$8x - y = 38$$

Answer $x = \dots\dots\dots$

$y = \dots\dots\dots$ [3]

12 Two rooms are available for rent.

The rental rate of Room A is \$40 per day.

The rental rate of Room B is \$32 per day with a one-time booking charge of \$50.

By forming and solving an inequality, find the minimum number of complete days of rental required such that total amount payable for Room B becomes cheaper than the total amount payable for Room A.

Answer $\dots\dots\dots$ days [3]

- 13 (a) Expand $(3x+2p)^2$.

Answer [2]

- (b) Given that $(3x+2p)^2 = 9x^2 + 60x + 100$, find the value of p .

Answer $p =$ [1]

- 14 The kinetic energy, E Joules, of an object is directly proportional to $(v+1.5)^2$, where v is the velocity in m/s. Given that the difference between the values of E when $v=10$ and when $v=12$ is 750 Joules,

- (a) express E in terms of v , [2]

Answer (a)

- (b) find the value of E when $v = 20$.

Answer (b) Joules [1]

- 15 If $\frac{a-3b}{4} = \frac{b}{3}$, find the ratio of a to b .

Answer : [3]

- 16 The marks of a Physics class test of 8 students are given below.

16 18 10 24 10 18 x y

- (a) Given that the mean mark is 17 and the modal mark is 10, find a possible pair of values of x and y .

Answer $x =$

Answer $y =$ [3]

- (b) Hence, determine the median mark for the Physics class test.

Answer [1]

17 It is given that $a = \frac{b-c}{1+bc}$.

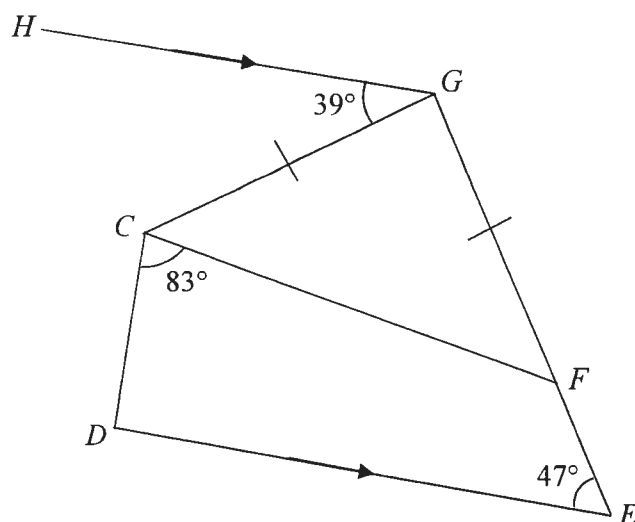
(a) Find the value of a when $b = 0.5$ and $c = -3$.

Answer $a = \dots\dots\dots$ [1]

(b) Express b in terms of a and c .

Answer $\dots\dots\dots$ [3]

- 18 In the diagram, GFE is a straight line, $CG = GF$, $HG \parallel DE$, angle $HGC = 39^\circ$, angle $DCF = 83^\circ$ and angle $DEF = 47^\circ$.



- (a) Find angle CGF , giving reasons for your answer.

Answer $^\circ$ [1]

- (b) Angie claims that CD is perpendicular to DE .

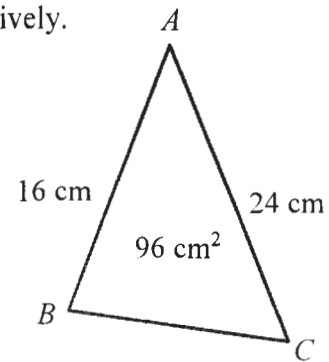
Do you agree with her?

Justify your answer by showing the relevant working.

Answer

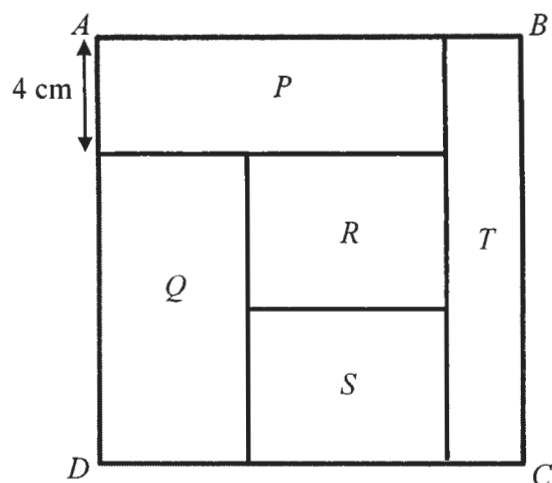
[3]

- 19 The area of triangle ABC is 96 cm^2 .
Two of the sides are of length 24 cm and 16 cm respectively.
Find the length of the third side.



Answer cm [4]

20



A square $ABCD$ is formed with 5 rectangles, P , Q , R , S and T . Each of these rectangles has the same area.

Given that the width of rectangle P is 4 cm, find the area of the square.

Answer cm^2 [4]

- 21 (a) The n th term of a sequence is $T_n = 70 - 4n$.

State the largest number in the sequence.

Answer [1]

- (b) The first four terms of another sequence K_n are 3, -2, -7, -12...

Find a formula for the n th term. Express your answer in the simplest form.

Answer [2]

- (c) The sequences in parts (a) and (b) are added to form a third sequence F_n , such that $F_n = T_n + K_n$.

Explain why all the terms of F_n are divisible by 3.
Show your working clearly.

Answer

[2]

- 22 (a) The diameter of a micro-organism is 0.000 001 256 cm.

Express the diameter of the micro-organism in standard form.

Answer cm [1]

- (b) The population of Town X is 5.2×10^5 and the population of Town Y is 3.8×10^4 .

- (i) Determine how many more people live in Town X than in Town Y.
Give your answer in standard form.

Answer [2]

- (ii) The population density of a town is calculated using the formula given below.

$\text{population density} = \frac{\text{population}}{\text{area}}$

If the area of Town X is $3.6 \times 10^2 \text{ km}^2$ and the area of Town Y is 25 km^2 , determine which town has a higher population density. Show your working clearly.

Answer Town [3]

23 A man jogged from point A to point B in a park.
He jogged for 52 minutes and 40 seconds at an average speed of 9 km/h.

(a) Write down the time taken to go from A to B , in hours.

Answer h [1]

(b) The distance between point A and B is 19.75 cm when represented on a map.

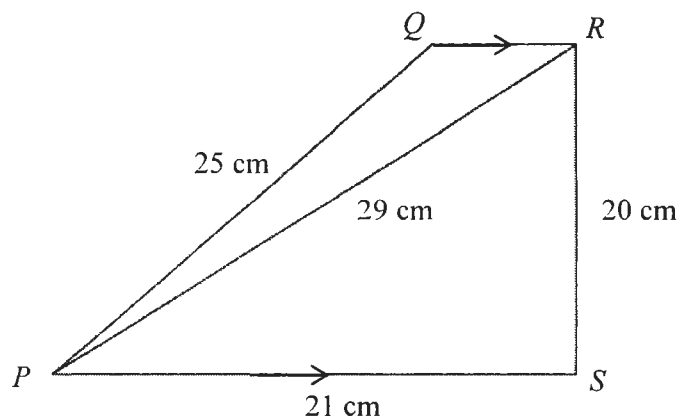
Find the scale of the map in the form 1 : n .

Answer [3]

(c) Hence, find the area on the map, in square centimetres, that represents an actual area of 6 km².

Answer cm² [2]

- 24 In trapezium $PQRS$, $PQ = 25$ cm, $PS = 21$ cm, $PR = 29$ cm and $RS = 20$ cm. QR is parallel to PS .



- (a) Show that triangle PRS is a right-angled triangle.

Answer

[2]

- (b) (i) Write down the value of $\tan \angle PRS$.

Answer $\tan \angle PRS = \dots\dots\dots$ [1]

- (ii) Find angle PRQ .

Answer $\angle PRQ = \dots\dots\dots^\circ$ [2]

(iii) Hence, or otherwise, find angle QPR , given that angle PQR is obtuse.

Answer $\angle QPR = \dots\dots\dots^\circ$ [3]

End of Paper

Answer Key

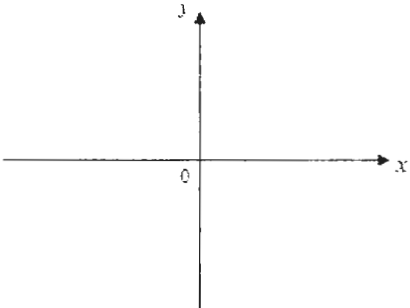
No.	Answer
1	1.398 (to 3 d.p.)
2a	$-5x - 4$
2b	$x = -\frac{9}{2}$
3	160 cm
4	Small sector angle = 120° Medium sector angle = 150° Large sector angle = 90°
5a	$y = a^x$ where a can be any positive number, $a > 1$.
5b	Graph
6	$x = 41.8^\circ$ $x = 138.2^\circ$
7	22.5 cm
8	$(a-2)(4b-c)$
9a	$P(\text{odd}) = \frac{1}{2}$
9b	$P(\text{even and divisible by 3}) = \frac{3}{20}$
10	Minimum number of cubes = 160
11	$x = 4$ $y = -6$
12	Minimum number of days = 7
13a	$9x^2 + 12xp + 4p^2$
13b	$p = 5$
14a	$E = 15(v+1.5)^2$
14b	6933.75 Joules
15	$a : b = 13 : 3$
16a	$x = 10$ $y = 30$
16b	Median = 17
17a	$a = -7$
17b	$b = \frac{a+c}{1-ac}$
18a	Angle $CGF = 94^\circ$
18b	I disagree with Angie because angle CDE is 93° , hence CD is not perpendicular to DE .
19	$BC = 12.9$ cm
20	Area of square = $16 \times 16 = 256$ cm ²

21a	66
21b	$8 - 5n$
21c	Since the n th term of the third sequence can be expressed as $3(26 - 3n)$, the terms are all multiples of 3 and hence are divisible by 3.
22a	$1.256 \times 10^{-6} \text{ cm}$
22bi	4.82×10^5
22bii	Town Y has a higher population density
23a	$\frac{79}{90}h$
23b	1 : 40000
23c	37.5 cm^2
24a	$PR^2 = 29^2 = 841$ $RS^2 + PS^2 = 20^2 + 21^2 = 841$ $PR^2 = RS^2 + PS^2$ therefore $\angle RSP$ is a right angle and $\triangle PRS$ is a right-angled triangle (by the converse of Pythagoras theorem)
24bi	$\tan \angle PRS = \frac{21}{20}$
24bii	$\angle PRQ = 43.6^\circ$
24biii	$\angle QPR = 9.5^\circ$

End of Answer Key

2023 Sec 3 Maths EOY Paper 1 Marking Scheme

No.	Solution	Marks
1	1.398 (to 3 d.p.)	B1
2a	$6 - 5(x + 2)$ $= 6 - 5x - 10$ $= -5x - 4$	B1
2b	$4x + 10 = -8$ $4x = -8 - 10$ $4x = -18$ $x = \frac{-18}{4}$ $x = -\frac{9}{2}$	B1
3	Grace = 100% Macy = 110% 100% + 110% = 336 210% = 336 $100\% = \frac{100}{210} \times 336$ $= 160 \text{ cm}$ OR Let Grace be x cm and Macy is $1.1x$ cm $x + 1.1x = 336$ $2.1x = 336$ $x = \frac{336}{2.1} = 160 \text{ cm}$	M1 A1
4	Small sector angle = $\frac{24}{24 + 30 + 18} \times 360^\circ = 120^\circ$ Medium sector angle = $\frac{30}{24 + 30 + 18} \times 360^\circ = 150^\circ$ Large sector angle = $\frac{18}{24 + 30 + 18} \times 360^\circ = 90^\circ$	B1 B1 B1

5a	$y = a^x$ where a can be any positive number, $a > 1$.	B1
5b		B1
6	$6 \sin x = 4$ $\sin x = \frac{4}{6}$ $x = \sin^{-1}\left(\frac{4}{6}\right) = 41.8^\circ$ $x = 180^\circ - 41.8^\circ = 138.2^\circ$	B1 B1
7	Shortest perimeter $= 7.5 + 7.5 + 7.5$ $= 22.5 \text{ cm}$	M1 A1 or B2
8	$4ab - 8b - ac + 2c$ $= 4b(a - 2) - c(a - 2)$ $= (a - 2)(4b - c)$	M1 A1
9a	Odd Numbers: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19	
9b	$P(\text{odd}) = \frac{10}{20} = \frac{1}{2}$ Even and divisible by 3: 6, 12, 18 $P(\text{even and divisible by 3}) = \frac{3}{20}$	B1 B1

10	<table><tr><td>2</td><td>96</td><td>60</td><td>48</td></tr><tr><td>2</td><td>48</td><td>30</td><td>24</td></tr><tr><td>3</td><td>24</td><td>15</td><td>12</td></tr><tr><td></td><td>8</td><td>5</td><td>4</td></tr></table> Minimum number of cubes = $8 \times 5 \times 4 = 160$ OR $96 = 2^5 \times 3$ $60 = 2^2 \times 3 \times 5$ $48 = 2^4 \times 3$ $HCF = 2^2 \times 3 = 12$ $\text{min no.} = \frac{96 \times 60 \times 48}{12^3} = 160$	2	96	60	48	2	48	30	24	3	24	15	12		8	5	4	M1 A1
2	96	60	48															
2	48	30	24															
3	24	15	12															
	8	5	4															
11	$5x + 2y - 8 = 0$ $8x - y = 38$ $5x + 2y = 8$ $16x - 2y = 76$ $21x = 84$ $x = \frac{84}{21} = 4$ $5(4) + 2y = 8$ $20 + 2y = 8$ $2y = 8 - 20$ $2y = -12$ $y = \frac{-12}{2} = -6$	M1 A1 A1																

12	$50 + 32(x) < 40x$ $50 < 40x - 32x$ $50 < 8x$ $8x > 50$ $x > \frac{50}{8}$ $x > 6.25$ Minimum number of days = 7	M1 – must form inequality M1 A1
13a	$(3x + 2p)^2$ $= 9x^2 + 12xp + 4p^2$ OR $(3x + 2p)(3x + 2p)$ $= 9x^2 + 6xp + 6px + 4p^2$ $= 9x^2 + 12px + 4p^2$	B2 (M1) (A1)
13b	$9x^2 + 12xp + 4p^2 = 9x^2 + 60x + 100$ $12xp = 60x$ $12p = 60$ $p = \frac{60}{12} = 5$ OR $4p^2 = 100$ $p^2 = \frac{100}{4} = 25$ $p = \sqrt{25} = 5$	B1 (allow ECF)

14a	$E = k(v + 1.5)^2$ When $v = 10$ m/s, $E = k(10 + 1.5)^2 = 132.25k$ When $v = 12$ m/s $E = k(12 + 1.5)^2 = 182.25k$ $182.25k - 132.25k = 750$ $50k = 750$ $k = 15$ $E = 15(v + 1.5)^2$	M1 – for forming equations in k A1
14b	When $v = 20$ m/s, $E = 15(20 + 1.5)^2 = 6933.75 \text{ Joules}$	A1
15	$\frac{a-3b}{4} = \frac{b}{3}$ $3(a-3b) = 4b$ $3a-9b = 4b$ $3a = 4b + 9b$ $3a = 13b$ $\frac{a}{b} = \frac{13}{3}$ $a : b = 13 : 3$	M1 M1 - fraction A1

16a	$x = 10$ $\frac{16+18+10+24+10+18+10+y}{8} = 17$ $\frac{106+y}{8} = 17$ $106+y = 136$ $y = 136 - 106$ $y = 30$ OR $y = 10$ $x = 30$	M1 – forming equation A1 A1
16b	10 10 10 16 18 18 24 30 Median = $\frac{16+18}{2} = 17$	 B1
17a	$a = \frac{b-c}{1+bc}$ $a = \frac{0.5 - (-3)}{1 + (0.5)(-3)} = -7$	 B1
17b	$a = \frac{b-c}{1+bc}$ $a(1+bc) = b-c$ $a+abc = b-c$ $abc - b = -c - a$ $b(ac-1) = -c-a$ $b = \frac{-c-a}{ac-1}$ OR $b = \frac{a+c}{1-ac}$	 M1 – linearise M1 – factorise A1

18a	Angle $CGF = 180 - 39 - 47 = 94^\circ$ (Int. $\angle$, $HG \parallel DE$)	B1
18b	Angle $GCF = \frac{180 - 94}{2} = 43^\circ$ (base $\angle$ of isos. Δ)	M1
	Angle $CDE = 360 - 94 - 43 - 83 - 47 = 93^\circ$ ($\angle$ sum of quad.)	M1
	I disagree with Angie because angle CDE is 93° , hence CD is not perpendicular to DE .	A1
19	Area $= \frac{1}{2}(24)(16)\sin \angle BAC = 96$	M1
	$\sin \angle BAC = \frac{96}{\frac{1}{2}(24)(16)}$	
	$\angle BAC = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$	M1
	$BC^2 = 16^2 + 24^2 - 2(16)(24)\cos(30^\circ)$	M1
	$BC^2 = 166.89248$	
	$BC = \sqrt{166.89248} = 12.91868 = 12.9 \text{ cm}$	A1
20	Let length of P be x Let length of Q be y	
	Area of $P = 4x$	M1 - area
	Area of $Q = \text{Area of } R = \text{Area of } S = \text{Area of } P = 4x$	
	Area of $(Q + R + S) = 3(4x) = 12x = xy$	M1 - equating
	$y = 12 \text{ cm}$	
	Length of square $= 4 + 12 = 16 \text{ cm}$	M1 - length
	Area of square $= 16 \times 16 = 256 \text{ cm}^2$	A1

21a	66	B1
21b	$3 + (n-1)(-5)$ $= 3 - 5n + 5$ $= 8 - 5n$	M1 A1 (or B2)
21c	$70 - 4n + 8 - 5n$ $= 78 - 9n$ $= 3(26 - 3n)$ Since the n th term of the third sequence can be expressed as $3(26 - 3n)$, the terms are all multiples of 3 and hence are divisible by 3.	M1 A1
22a	$0.000\ 001\ 256\ \text{cm} = 1.256 \times 10^{-6}\ \text{cm}$	B1
22bi	Difference $= 5.2 \times 10^5 - 3.8 \times 10^4$ $= 482000 = 4.82 \times 10^5$	M1 A1
22bii	Town X population density $= \frac{5.2 \times 10^5}{3.6 \times 10^2} = 1444.44444$ Town Y population density $= \frac{3.8 \times 10^4}{2.5 \times 10^1} = 1520$ Town Y has a higher population density	M1 M1 A1

23a	$40s = \frac{40}{60} \text{ min} = \frac{2}{3} \text{ min}$ Time $= \frac{52\frac{2}{3}}{60} h = \frac{79}{90} h$	B1
23b	Distance $= 9 \times \frac{79}{90} = 7.9 \text{ km}$ 19.75 cm : 7.9 km 19.75 cm : 7.9 × 100000 cm 19.75 : 790000 1 : 40000	M1 M1 - conversion A1
23c	Area scale 1 cm : 0.4 km 1 cm² : 0.16 km² Area on map $= \frac{6}{0.16} = 37.5 \text{ cm}^2$	M1 – area scale A1

24a	$PR^2 = 29^2 = 841$ $RS^2 + PS^2 = 20^2 + 21^2 = 841$ $PR^2 = RS^2 + PS^2$ therefore $\angle RSP$ is a right angle and $\triangle PRS$ is a right-angled triangle (by the converse of Pythagoras theorem)	M1 A1
24bi	$\tan \angle PRS = \frac{21}{20}$	B1
24bii	$\tan \angle PRS = \frac{21}{20}$	
	$\angle PRS = \tan^{-1}\left(\frac{21}{20}\right) = 46.39718^\circ$	M1
	$\angle PRQ = 90 - 46.39718 = 43.60281^\circ = 43.6^\circ$	A1
	OR	
	$\tan \angle RPS = \frac{20}{21}$	
	$\angle RPS = \tan^{-1}\left(\frac{20}{21}\right) = 43.60281^\circ$	(M1)
	$\angle PRQ = 43.6^\circ$ (alt. $\angle$, $//$ lines)	(A1)
24biii	$\frac{\sin 43.60281}{25} = \frac{\sin \angle PQR}{29}$ $\sin \angle PQR = \frac{29 \sin 43.60281}{25}$ $\angle PQR = \sin^{-1}\left(\frac{29 \sin 43.60281}{25}\right) = 53.13008$ $\text{obtuse } \angle PQR = 180 - 53.13008 = 126.86991$ $\angle QPR = 180 - 126.86991 - 43.60281 = 9.52727 = 9.5^\circ$ OR $\angle QPR = \sin^{-1}\left(\frac{20}{25}\right) - 43.6$ $= 9.53010 = 9.5^\circ$	M1 M1 – obtuse angle A1 (M2) (A1)

End of Marking Scheme

Name	Class				Index Number			
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**BROADRICK SECONDARY SCHOOL
SECONDARY 3 EXPRESS /
3 NORMAL (ACADEMIC) SBB
END-OF-YEAR EXAMINATION 2023**

MATHEMATICS

Paper 2

4052/02

October 2023

Candidates answer on the Question Paper.

2 hours

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For Examiner's Use		
Error In	Question Number	Marks Deducted
Rounding-off		
Reasoning		
Presentation		

This document consists of **20** printed pages.

Setter(s) : Mrs Vasughi Santhos

[Turn Over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

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$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1 (a) Simplify $\frac{6x^2 - x - 15}{(3x - 5)^2}$.

Answer [2]

(b) Solve $(3^{2x+2})(3^{3x}) = 729$.

Answer $x =$ [2]

(c) Simplify $\left(\frac{2a^2b}{c^3}\right)^0 \div (2a^{-2})^3$.

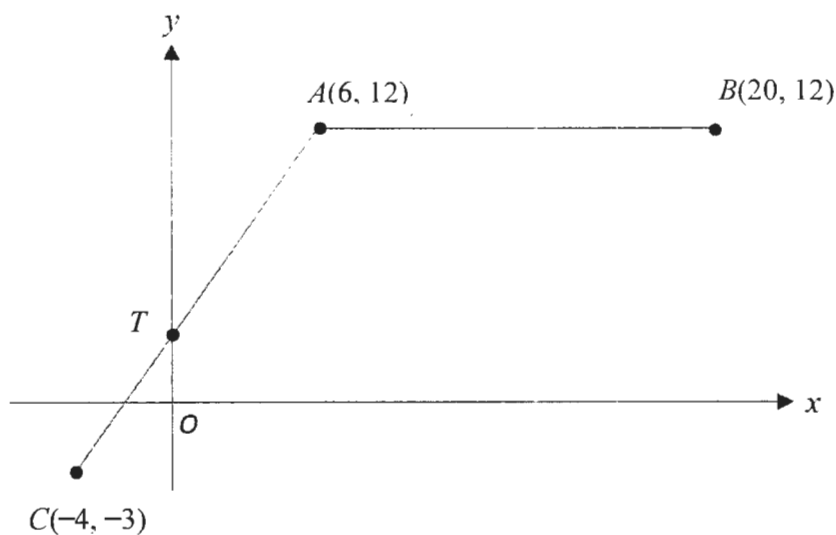
Leave your answer in positive index.

Answer [2]

- 2 The diagram shows two lines AC and AB .

Line AC intersects the y -axis at T .

Points A , B , C are $(6, 12)$, $(20, 12)$ and $(-4, -3)$ respectively.



- (a) Find the equation of the line AC .

Answer [2]

- (b) Line l , parallel to AB , passes through point T .
State the equation of line l .

Answer [1]

- (c) Find the area of triangle ABC .

Answerunits² [1]

- (d) Find the coordinates of D , such that $ABDC$ forms a parallelogram.

Answer (.....,) [1]

- 3 (a) Express $x^2 - 6x + 14$ in the form $(x - p)^2 + q$, where p and q are constants.

Answer [2]

- (b) Hence solve the equation $x^2 - 6x = 16$.

Answer $x =$ [2]

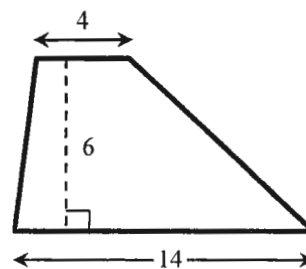
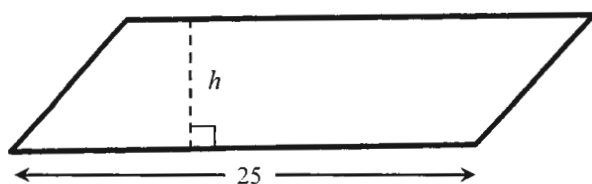
- (c) State the equation of line of symmetry of the graph $y = x^2 - 6x + 14$.

Answer [1]

- (d) State the turning point of the graph $y = x^2 - 6x + 14$.

Answer (.....,) [1]

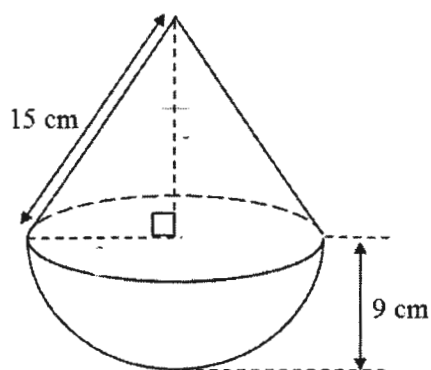
- 4 (a) The ratio of the area of the parallelogram to the area of the trapezium is 3:2.
All lengths are measured in centimetres.



Calculate the height, h , of the parallelogram.

Answer $h = \dots\dots\dots$ [2]

- (b) The diagram shows a solid made from a cone and a hemisphere.
Calculate the volume of the solid.



Answer $\dots\dots\dots\text{cm}^3$ [3]

- 5 (a) The population of turtles in an island is estimated to decrease by 1.5% every year.
There are 2500 turtles in the island in 2023.
What is the population of turtles in 2027?

Answer [2]

- (b) (i) Julian is traveling to Germany.
The current exchange rate between Singapore dollars (\$) and euros (€) is $\$1 = €0.7132$.
He decides to prepare €4500 for the trip.
Calculate the amount of Singapore dollars he needs.
Give your answer correct to the nearest cent.

Answer \$..... [2]

- (b) (ii) He has €850 left after the trip.
He decides to change the money back to Singapore dollars.
The current exchange rate is $\$1 = €(0.3156 + x)$.
He receives \$1167.
Find the value of x . Give your answer correct to 4 significant figures.

Answer $x =$ [2]

- 6 (a) The diagram shows two geometrically similar vases.
The ratio of the cost of painting the smaller container to the cost of painting the bigger vase is 16: 25.



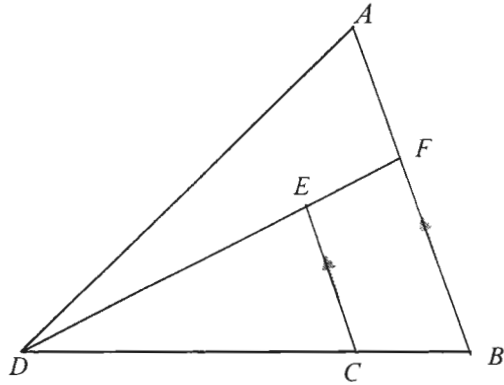
- (i) The base of the larger vase has a radius of 5.6 cm.
Find the radius of the base of smaller vase.

Answer cm [2]

- (ii) The capacity of the smaller vase is 4.5 litres.
Find the capacity of the larger vase, giving your answer correct to 3 decimal places.

Answerlitres [2]

- (b) In the figure, BA and CE are parallel lines.
 F lies on the line BA such that $BF : BA = 4 : 7$.
 E lies on the line FD such that $FE : FD = 1 : 3$.



- (i) Show that triangles CED and BFD are similar.

Answer

.....

.....

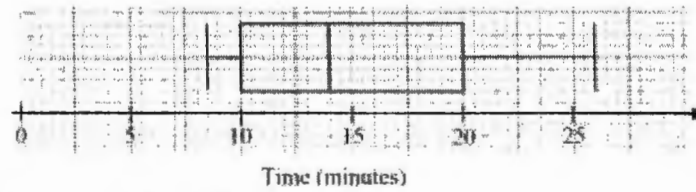
.....

[2]

- (ii) If the area of triangle CED is 36 cm^2 , calculate the area of triangle ABD .

Answer cm^2 [3]

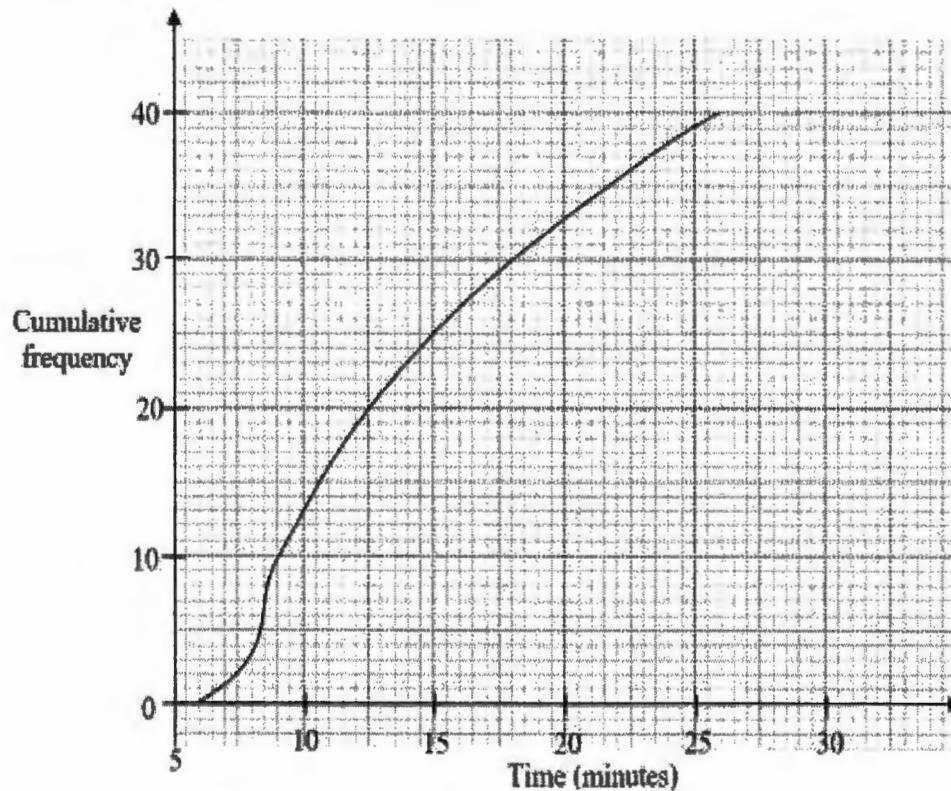
- 7 The box-and-whiskers plot shows the distribution of time taken by a group of 40 girls to complete a Sudoku puzzle.



- (a) Find the range of time taken.

Answer min [1]

The cumulative frequency graph shows the distribution of the time taken by a group of 40 boys to complete the same Sudoku puzzle.



- (b) Use the curve to estimate
(i) the median time,

Answer min [1]

- (ii) the inter-quartile range of the time,

Answer min [2]

(iii) the 40th percentile of the time.

Answer min [1]

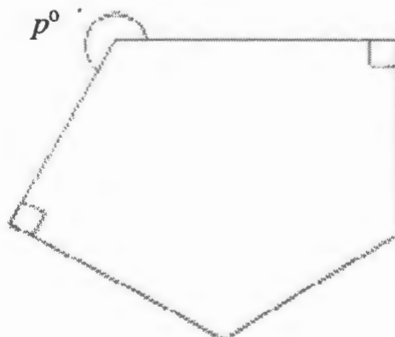
- (c) (i) Dylan claims that the boys were faster than the girls in completing the puzzles.
Do you agree? Explain your answer.

Answer
.....
.....
..... [1]

- (ii) Cash prizes are awarded to those who took 14 minutes or less to complete the puzzle.
James said that a higher percentage of boys than girls received the cash prize.
Do you agree? Explain your answer.

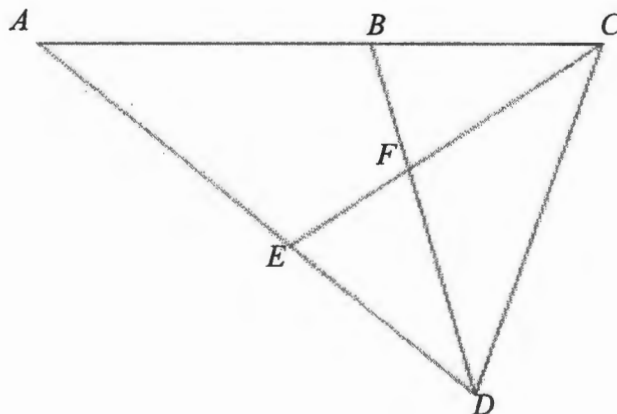
Answer
.....
..... [1]

- 8 (a) The diagram shows a pentagon.
Two of its interior angles are right angles and the remaining three interior angles are equal.
Find the value of p .



Answer $p = \dots\dots\dots$ [3]

- (b) In the diagram, $AC = AD$, CE bisects angle ACD and BD bisects angle CDA .
Prove that triangles ACE and ADB are congruent.



Answer

.....

.....

.....

.....

.....

[3]

- 9 The variables x and y are connected by the equation $y = -x^3 + 4x^2 - 3$.
Some corresponding values of x and y are given in the table below.

x	-2	-1.5	-1	0	1	2	2.5	3	4
y	21	a	2	-3	0	5	6.4	6	-3

- (a) Find the value of a , correct to 1 decimal place.

Answer $a = \dots\dots\dots$ [1]

- (b) Using the grid on the next page, draw the graph of $y = -x^3 + 4x^2 - 3$ for $-2 \leq x \leq 4$. [2]

- (c) By drawing a tangent, find the gradient of the curve at the point where $x = -1$.

Answer $\dots\dots\dots$ [2]

- (d) The horizontal line $y = k$ touches the curve $y = -x^3 + 4x^2 - 3$ at only one point.

State one possible positive integer value of k .

Answer $k = \dots\dots\dots$ [1]

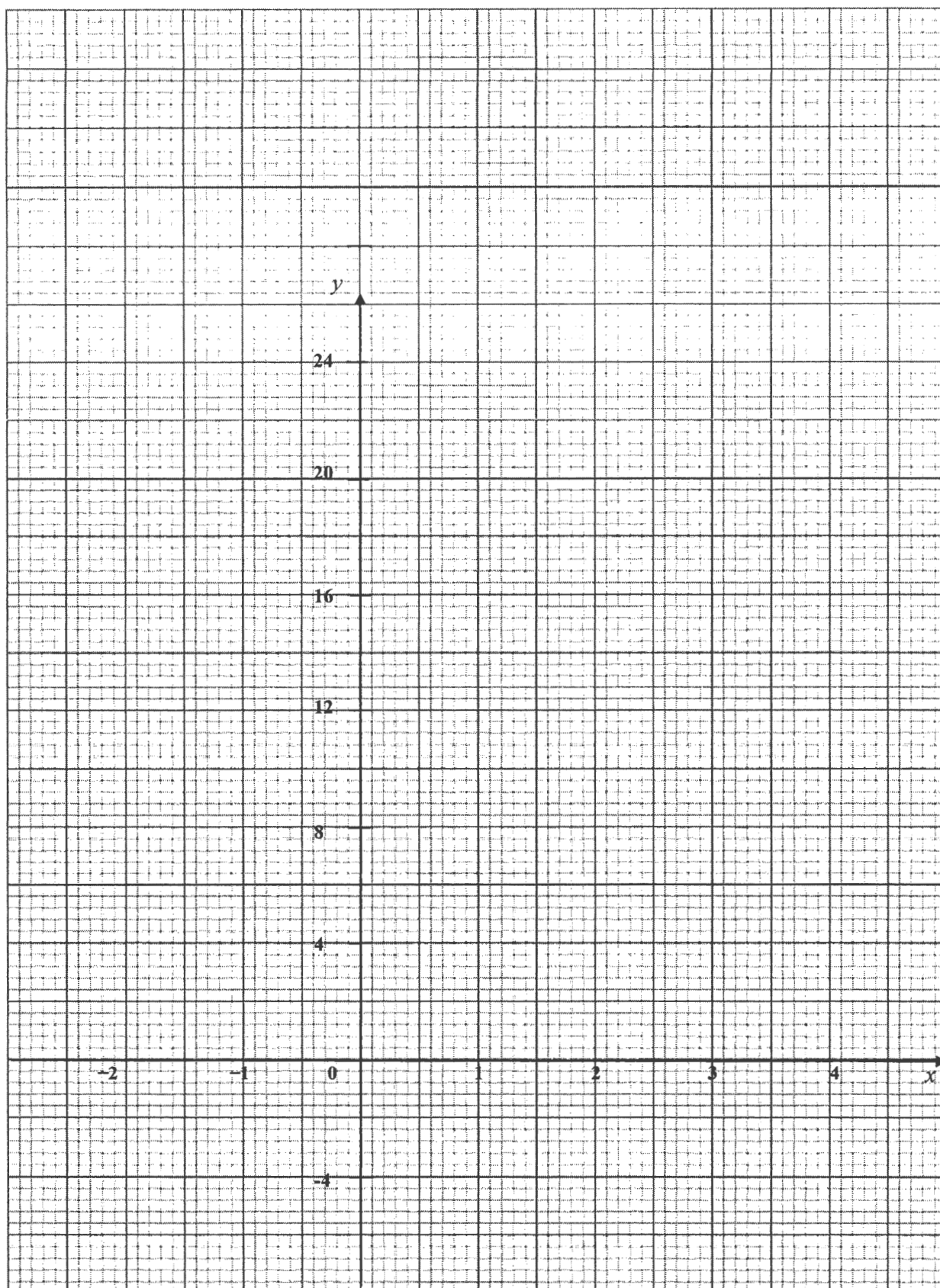
- (e) The equation $-2x^3 + 8x^2 - 3x - 4 = 0$ can be solved by drawing a suitable straight line on the grid.

- (i) Find the equation of this straight line.

Answer $\dots\dots\dots$ [1]

- (ii) By drawing the line in (e)(i), find the three solutions to the equation $-2x^3 + 8x^2 - 3x - 4 = 0$.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ or $\dots\dots\dots$ [3]



- 10 Benny, a wholesaler always spends \$300 a month to buy grapes.

In January, the price of grapes was \$ x per kilogram.

- (a) Write down an expression in terms of x for the number of kilograms of grapes he bought.

Answer kg [1]

In February, the price of grapes increased by 30 cents per kilogram.

- (b) Write down an expression in terms of x for the number of kilograms he bought for \$300 in February.

Answerkg [1]

- (c) Given that he bought 5 kg less grapes in February than January, form an equation in x and show that it reduces to $10x^2 + 3x - 180 = 0$.

Answer

[3]

- (d) Solve $10x^2 + 3x - 180 = 0$, giving your answer(s) correct to 2 decimal places.

Answer $x = \dots\dots\dots$ [3]

In March, the price of grapes was \$ k less per kilogram than in February.

Benny was able to buy 4 kg more of grapes in March than in February for \$300.

(e) Show that $k = 0.24$, correct to 2 decimal places.

Answer

[3]

- 11 Mr Gan wants to start a small tuition centre for secondary school students. The tables provide information on the cost of running the centre. The centre is closed on Mondays.

Start-up Cost	
Renovation	\$40 000
Furniture	\$5100
Computers and ICT Equipment	\$5000

Operational Cost Per Month	
Rent	\$8000
Utilities Bill (Electricity, Water)	\$1000
Advertising	\$300
Printing and Stationery	\$645
Interest of Renovation Loan	(b)

Labour Cost		
	Number of staff	Salary
Administrative Staff	1	\$1500 per month
Teaching Staff	1 per class [#]	\$45 per hour (Weekday)
		\$50 per hour (Weekend)
Cleaner	1	\$40 per day (3 hr)

Mr Gan intends to fix the tuition charges as follows:

Revenue			
	Total Number of Weekday Classes# Per Week	Total Number of Weekend Classes# Per Week	Monthly Tuition Fee Per Student
Lower Secondary	6	4	\$200
Upper Secondary	8	5	\$300

number of students per class: 5 to 8

*1 month has 4.3 weeks or 30 days

duration of each class is 2 hours

- (a) Calculate the total start-up cost.

Answer \$ [1]

- (b) Mr Gan took a loan for the renovation cost from a finance company that charges 2% simple interest per annum for 3 years.
Calculate the interest he paid on the renovation loan per month.

Answer \$..... [2]

- (c) Mr Gan has 2 goals:
First Goal : To recover the total start up cost (including the interest incurred) within 7 months of operating his tuition centre.

Second Goal : To make a profit of at least \$8000 per month after the 7 months.

Will he hit his goals (assuming that the minimum number of students per class is met)?

Justify by showing your calculation clearly.

Answer

You may continue your workings on the next page

[6]

~End of Paper~

Answer Key

1(a)	$\frac{2x+3}{3x-5}$		7(c)(i)	Yes
1(b)	$\frac{4}{5}$		(ii)	Yes
1(c)	$\frac{a^6}{8}$		8(a)	240
2(a)	$y = 1.5x + 3$		8(b)	AAS
2(b)	$y = 3$		9(a)	9.4
2(c)	105		9(c)	-11
2(d)	(10, -3)		9(d)	8
3(a)	$(x-3)^2 + 5$		9(e)(i)	$y = 1.5x - 1$
3(b)	(8, -2)		9(e)(ii)	3.4, 1.1, -0.5
3(c)	$x = 3$		10(a)	$\frac{300}{x}$
3(d)	(3, 5)		10(b)	$\frac{300}{x + 0.3}$
4(a)	3.24		10(d)	4.09, -4.40
4(b)	2540		11(a)	50100
5(a)	2353		11(b)	66.67
5(b)(i)	6309.59		11(c)	yes he will hit both his goals
5(b)(ii)	0.4128			
6(a)(i)	4.48			
6(a)(ii)	8.789			
6(b)(i)	AAA			
6(b)(ii)	141.75			
7(a)	17.5			
7(b)(i)	12.5			
7(b)(ii)	9			
7(b)(iii)	11			

Name	Marking Scheme	Class				Index Number			
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**BROADRICK SECONDARY SCHOOL
SECONDARY 3 EXPRESS /
3 NORMAL (ACADEMIC) SBB
END-OF-YEAR EXAMINATION 2023**

MATHEMATICS

Paper 2

4052/02

October 2023

Candidates answer on the Question Paper.

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Write the question number attempted in the left column in the box provided.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is 80.

For Examiner's Use		
Error In	Question Number	Marks Deducted
Rounding-off		
Reasoning		
Presentation		

For Candidate's Use	For Examiner's Use
Question Number	Marks Obtained
Total Marks	/80

This document consists of 20 printed pages.

Setter(s) : Mrs Vasughi Santhos

[Turn Over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved Surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer all the questions.

1 (a) Simplify $\frac{6x^2 - x - 15}{(3x-5)^2}$.

$$\frac{(\cancel{3x-5})(2x+3)}{(3x-5)(\cancel{3x-5})} \quad (M1)$$

Answer $\frac{2x+3}{3x-5}$ (A1) [2]

(b) Solve $(3^{2x+2})(3^{3x}) = 729$.

$$3^{2x+2+3x} = 3^6$$

$$2x+2+3x = 6 \quad (M1)$$

$$5x = 4$$

$$x = \frac{4}{5}$$

Answer $x = \frac{4}{5}$ (A1) [2]

(c) Simplify $\left(\frac{2a^2b}{c^3}\right)^0 \div (2a^{-2})^3$.

Leave your answer in positive index.

$$\downarrow (M1)$$

$$1 \div 2^3 a^{-6}$$

$$= \frac{1}{8a^{-6}}$$

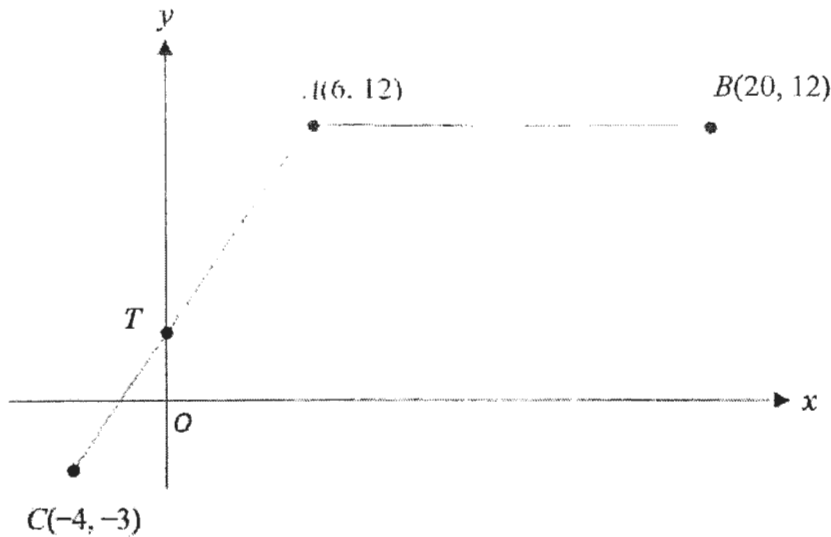
$$= \frac{a^6}{8}$$

Answer $\frac{a^6}{8}$ (A1) [2]

- 2 The diagram shows two lines AC and AB .

Line AC intersects the y -axis at T .

Points A , B , C are $(6, 12)$, $(20, 12)$ and $(-4, -3)$ respectively.



- (a) Find the equation of the line AC .

$$\begin{aligned} \text{Gradient} &= \frac{12 - (-3)}{6 - (-4)} \\ &= 1.5 \text{ (ml)} \end{aligned}$$

$$\begin{aligned} \text{Sub } x=6, y=12 \\ y &= 1.5x + c \end{aligned}$$

Answer

$$\begin{aligned} 12 &= 1.5(6) + c \\ c &= 3 \end{aligned}$$

$$y = 1.5x + 3 \quad [2]$$

(A1)

- (b) Line l , parallel to AB , passes through point T .
State the equation of line l .

$$\text{Answer } y = 3 \quad [1]$$

(B1)

- (c) Find the area of triangle ABC .

$$\begin{aligned}\text{Area of } \triangle &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 14 \times 15\end{aligned}$$

Answer 105 (B1) // units² [1]

- (d) Find the coordinates of D , such that $ABDC$ forms a parallelogram.

Answer (..... 10 -3) (B1) // [1]

6

- 3 (a) Express $x^2 - 6x + 14$ in the form $(x-p)^2 + q$, where p and q are constants.

$$x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 14$$

$$= (x-3)^2 + 5$$

Answer $\underbrace{(x-3)^2}_{(B1)} + \underbrace{5}_{(B1)}$ // [2]

- (b) Hence solve the equation $x^2 - 6x = 16$.

$$x^2 - 6x + 14 = 16 + 14$$

$$(x-3)^2 + 5 = 30$$

$$(x-3)^2 = 25 \quad (M1)$$

$$x-3 = \pm\sqrt{25}$$

$$x = 5+3 \quad \text{or} \quad x = -5+3$$

$$= 8 \quad \quad \quad = -2$$

Answer $x = \dots 8, -2 //$ (A1) [2]

- (c) State the equation of line of symmetry of the graph $y = x^2 - 6x + 14$.

$$\frac{8+(-2)}{2} = 3$$

Answer $x = 3 //$ (B1) [1]

- (d) State the turning point of the graph $y = x^2 - 6x + 14$.

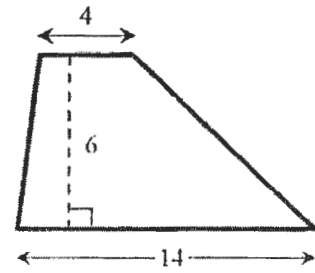
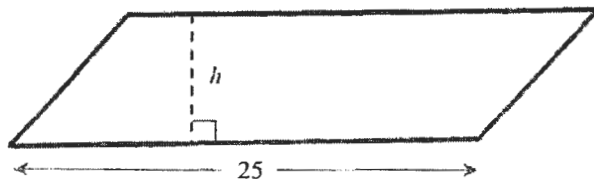
When $x=3$

$$y = 3^2 - 6(3) + 14$$

$$= 5$$

Answer (..... 3 5 //) (B1) [1]

- 4 (a) The ratio of the area of the parallelogram to the area of the trapezium is 3:2.
All lengths are measured in centimetres.



Calculate the height, h , of the parallelogram.

$$\frac{\text{Area of } \parallel\text{gram}}{\text{Area of trapezium}} = \frac{3}{2}$$

$$\frac{25 \times h}{\frac{1}{2} \times (4 + 14) \times 6} = \frac{3}{2} \text{ (mi)}$$

$$\frac{25h}{54} = \frac{3}{2}$$

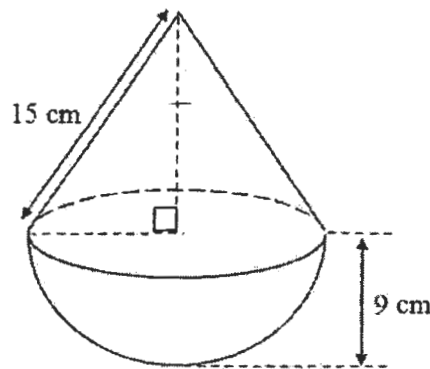
$$25h = 81$$

$$h = 3.24$$

Answer $h = \dots\dots\dots 3.24 // \dots\dots\dots$ [2]

(A1)

- (b) The diagram shows a solid made from a cone and a hemisphere.
Calculate the volume of the solid.



$$15^2 = h^2 + 9^2$$

$$h^2 = 144$$

$$h = 12$$

$$\begin{aligned} \text{Vol. of cone} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \times \pi \times 9^2 \times 12 \\ &= 324\pi \text{ cm}^3 \text{ (mi)} \end{aligned}$$

$$\begin{aligned} \text{Vol. of hemi-sphere} &= \frac{4}{3} \pi r^3 \times \frac{1}{2} \\ &= \frac{4}{3} \pi (9)^3 \times \frac{1}{2} = 486\pi \text{ cm}^3 \text{ (mi)} \end{aligned}$$

$$\text{Total Vol} = 324\pi + 486\pi$$

Answer $\dots\dots\dots 2545.02 // \text{cm}^3$ [3]

accept 2540

(A1)

- 5 (a) The population of turtles in an island is estimated to decrease by 1.5% every year.
There are 2500 turtles in the island in 2023.
What is the population of turtles in 2027?

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$A = 2500\left(1 - \frac{1.5}{100}\right)^4 \quad (\text{ml})$$

$$\approx 2353.341$$

Answer 2353 [2]
accept 2350 (A1)

- (b) (i) Julian is traveling to Germany.
The current exchange rate between Singapore dollars (\$) and euros (€) is \$1 = €0.7132.
He decides to prepare €4500 for the trip.
Calculate the amount of Singapore dollars he needs.
Give your answer correct to the nearest cent.

$$€ 0.7132 = \$1$$

$$€ 4500 = \frac{4500 \times 1}{0.7132} \quad (\text{ml})$$

Answer \$..... 6309.59 (A1) [2]

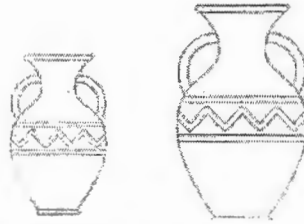
- (b) (ii) He has €850 left after the trip.
He decides to change the money back to Singapore dollars.
The current exchange rate is \$1 = €(0.3156 + x).
He receives \$1167.

Find the value of x. Give your answer correct to 4 significant figures. (ml)

$$\begin{aligned} € 0.3156 + x &= \$1 \\ 1167 \times (0.3156 + x) &= \$1167 \end{aligned} \quad \left\{ \begin{aligned} 850 &= 1167(0.3156 + x) \\ \frac{850}{1167} &= 0.3156 + x \\ x &= 0.4128 \end{aligned} \right.$$

Answer x = 0.4128 [2]
(A1)

- 6 (a) The diagram shows two geometrically similar vases.
The ratio of the cost of painting the smaller container to the cost of painting the bigger vase is 16: 25.



- (i) The base of the larger vase has a radius of 5.6 cm.
Find the radius of the base of smaller vase.

$$\frac{A_1}{A_2} = \left(\frac{L_1}{L_2}\right)^2$$

$$\frac{16}{25} = \left(\frac{L_1}{5.6}\right)^2 \text{ (m)}$$

$$\frac{L_1}{5.6} = \sqrt{\frac{16}{25}}$$

$$\frac{L_1}{5.6} = \frac{4}{5}$$

$$5L_1 = 4 \times 5.6$$

$$L_1 = 4.48$$

Answer 4.48 (A1) cm [2]

- (ii) The capacity of the smaller vase is 4.5 litres.

Find the capacity of the larger vase, giving your answer to 3 decimal places.

$$\frac{V_1}{V_2} = \left(\frac{L_1}{L_2}\right)^3$$

$$\text{(m)} \quad \frac{4.5}{V_2} = \left(\frac{4.48}{5.6}\right)^3$$

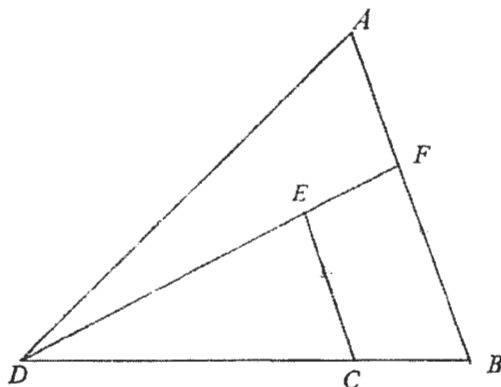
$$\frac{4.5}{V_2} = \frac{64}{125}$$

$$64V_2 = 562.5$$

$$V_2 = 8.789$$

Answer 8.789 litres [2]
(A1)

- (b) In the figure, BA and CE are parallel lines.
 F lies on the line BA such that $BF : BA = 4 : 7$.
 E lies on the line FD such that $FE : FD = 1 : 3$.



- (i) Show that triangles CED and BFD are similar.

Answer $\left\{ \begin{array}{l} \angle BFD = \angle CED \text{ (corr. } \angle\text{s, } BF \parallel CE) \\ \angle ECD = \angle FBD \text{ (corr. } \angle\text{s, } BF \parallel CE) \\ \angle CDE = \angle BDF \text{ (common } \angle) \end{array} \right.$
 $\therefore \triangle CED$ is similar to $\triangle BFD$ (AAA)

[2]

- (ii) If the area of triangle CED is 36 cm^2 , calculate the area of triangle ABD .

$$\frac{\text{Area of } \triangle CED}{\text{Area of } \triangle BFD} = \left(\frac{2}{3}\right)^2 = \frac{4}{9} \quad (\text{ml})$$

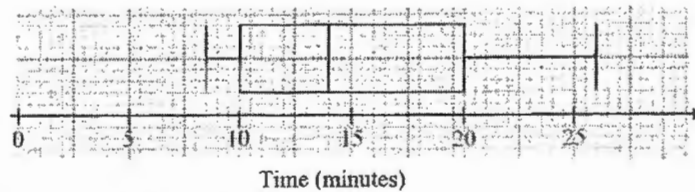
$$\frac{\text{Area of } \triangle BFD}{\text{Area of } \triangle ABD} = \frac{\frac{1}{2} \times 4 \times h}{\frac{1}{2} \times 7 \times h} = \frac{4}{7}$$

$$\begin{aligned} 4 \text{ units} &\rightarrow 36 \text{ cm}^2 \\ 9 \text{ units} &\rightarrow \frac{9 \times 36}{4} \\ &= 81 \text{ cm}^2 \quad (\text{ml}) \end{aligned}$$

$$\begin{aligned} 4 \text{ units} &\rightarrow 81 \text{ cm}^2 \\ 7 \text{ units} &\rightarrow \frac{7 \times 81}{4} \\ &= 141.75 \text{ cm}^2 \quad (\text{Al}) \end{aligned}$$

Answer $\dots\dots\dots 141.75 \text{ cm}^2$ [3]

- 7 The box-and-whiskers plot shows the distribution of time taken by a group of 40 girls to complete a Sudoku puzzle.

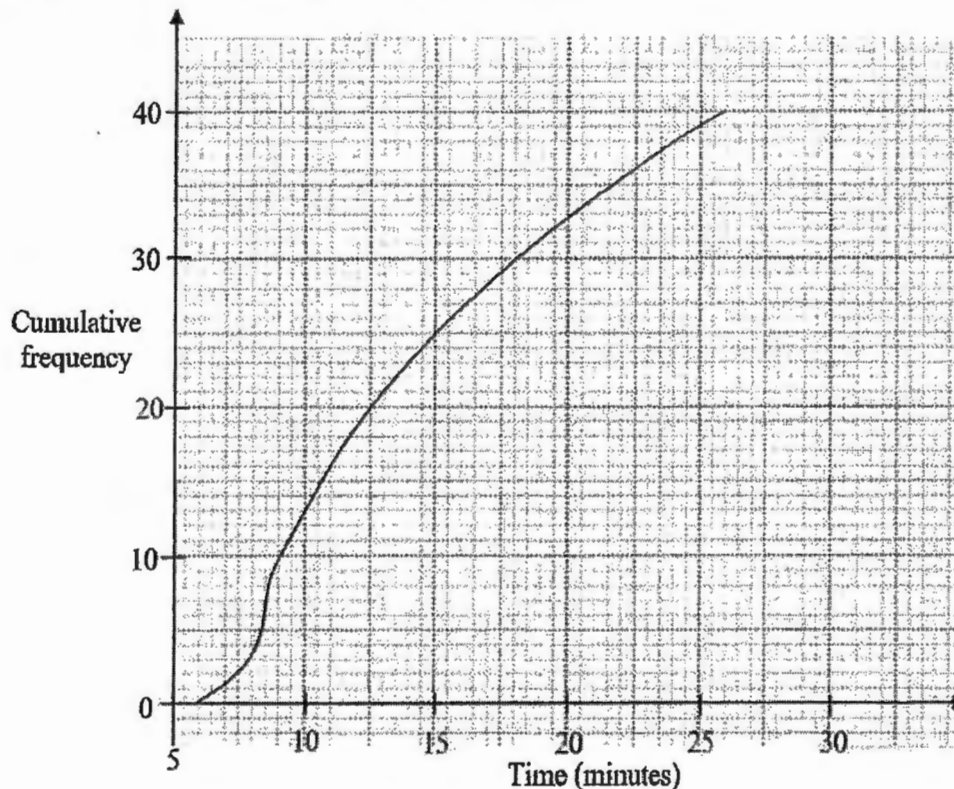


- (a) Find the range of time taken.

Answer 17.5 // min [1]

(B1)

The cumulative frequency graph shows the distribution of the time taken by a group of 40 boys to complete the same Sudoku puzzle.



- (b) Use the curve to estimate
(i) the median time,

Answer 12.5 // (B1) min [1]

- (ii) the inter-quartile range of the time,

18-9
(m)

Answer 9 // (A1) min [2]

(iii) the 40th percentile of the time.

Answer 11 ~~//~~ (B1) min [1]

(c) (i) Dylan claims that the boys were faster than the girls in completing the puzzles.

Do you agree? Explain your answer.

Answer Yes. The median time taken by the girls
is more than the boys.
.....
..... $\therefore$ Boys are faster. (B1)

[1]

(ii) Cash prizes are awarded to those who took 14 minutes or less to complete the puzzle.

James said that a higher percentage of boys than girls received the cash prize.

Do you agree? Explain your answer.

$$\text{Boys} \rightarrow \frac{23}{40} \times 100\% = 57.5\%$$

$$57.5\% > 50\%$$

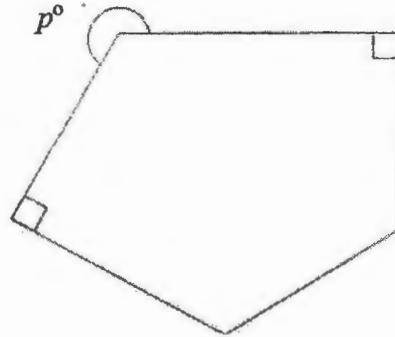
Answer Yes. A higher % of boys than girls
received the cash prize. } (B1)

[1]

- 8 (a) The diagram shows a pentagon.

Two of its interior angles are right angles and the remaining three interior angles are equal.

Find the value of p .



$$\text{Sum of int } \angle = (5-2) \times 180^\circ \\ = 540^\circ \text{ (ml)}$$

$$3x + 90 + 90 = 540$$

$$3x = 360$$

$$x = 120 \text{ (ml)}$$

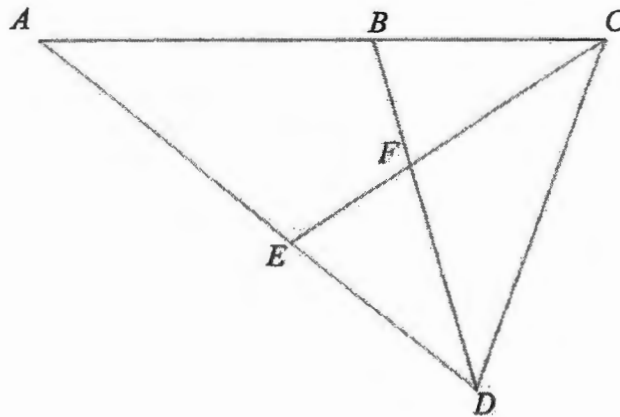
$$p = 360 - 120 -$$

$$= 240^\circ (\angle \text{ at a pt})$$

Answer $p = 240$ (AI) [3]

- (b) In the diagram, $AC = AD$, CE bisects angle ACD and BD bisects angle CDA .

Prove that triangles ACE and ADB are congruent.



Answer In $\triangle ACE$ & $\triangle ADB$,

- * $\angle CAE = \angle DAB$ (Common $\angle$)
- * $AC = AD$ (given)
- * $\angle ACD = \angle ADC$ (Isos $\triangle$)

$\therefore \angle ACE = \angle ECD = \angle BCD = \angle ADB$

$\therefore \angle ACE = \angle ADB$

$\triangle ACE \equiv \triangle ADB$ (AAS) [3]

- 9 The variables x and y are connected by the equation $y = -x^3 + 4x^2 - 3$. Some corresponding values of x and y are given in the table below.

x	-2	-1.5	-1	0	1	2	2.5	3	4
y	21	a	2	-3	0	5	6.4	6	-3

- (a) Find the value of a , correct to 1 decimal place.

Answer $a = \dots\dots\dots 9.4$ // (B1) [1]

- (b) Using the grid on the next page, draw the graph of $y = -x^3 + 4x^2 - 3$ for $-2 \leq x \leq 4$. [2]

- * (c) By drawing a tangent, find the gradient of the curve at the point where $x = -1$.

Gradient = $\frac{y_1 - y_2}{x_1 - x_2}$ (m1)

Answer $\dots\dots\dots -11$ (A1) [2]
accept -8 to -11.5

- (d) The horizontal line $y = k$ touches the curve $y = -x^3 + 4x^2 - 3$ at only one point.

State one of the possible positive integer value of k .

Answer $k = \dots\dots\dots 8$ (B1) [1]
any integer ≥ 7

- * (e) The equation $-2x^3 + 8x^2 - 3x - 4 = 0$ can be solved by drawing a suitable straight line on the grid.

- (i) Find the equation of this straight line.

$-x^3 + 4x^2 - \frac{3}{2}x - 2 = 0$

$-x^3 + 4x^2 - 3 = \frac{3}{2}x + 2 - 3$

$y = \frac{3}{2}x - 1$

Answer $\dots\dots\dots y = \frac{3}{2}x - 1$ (B1) [1]

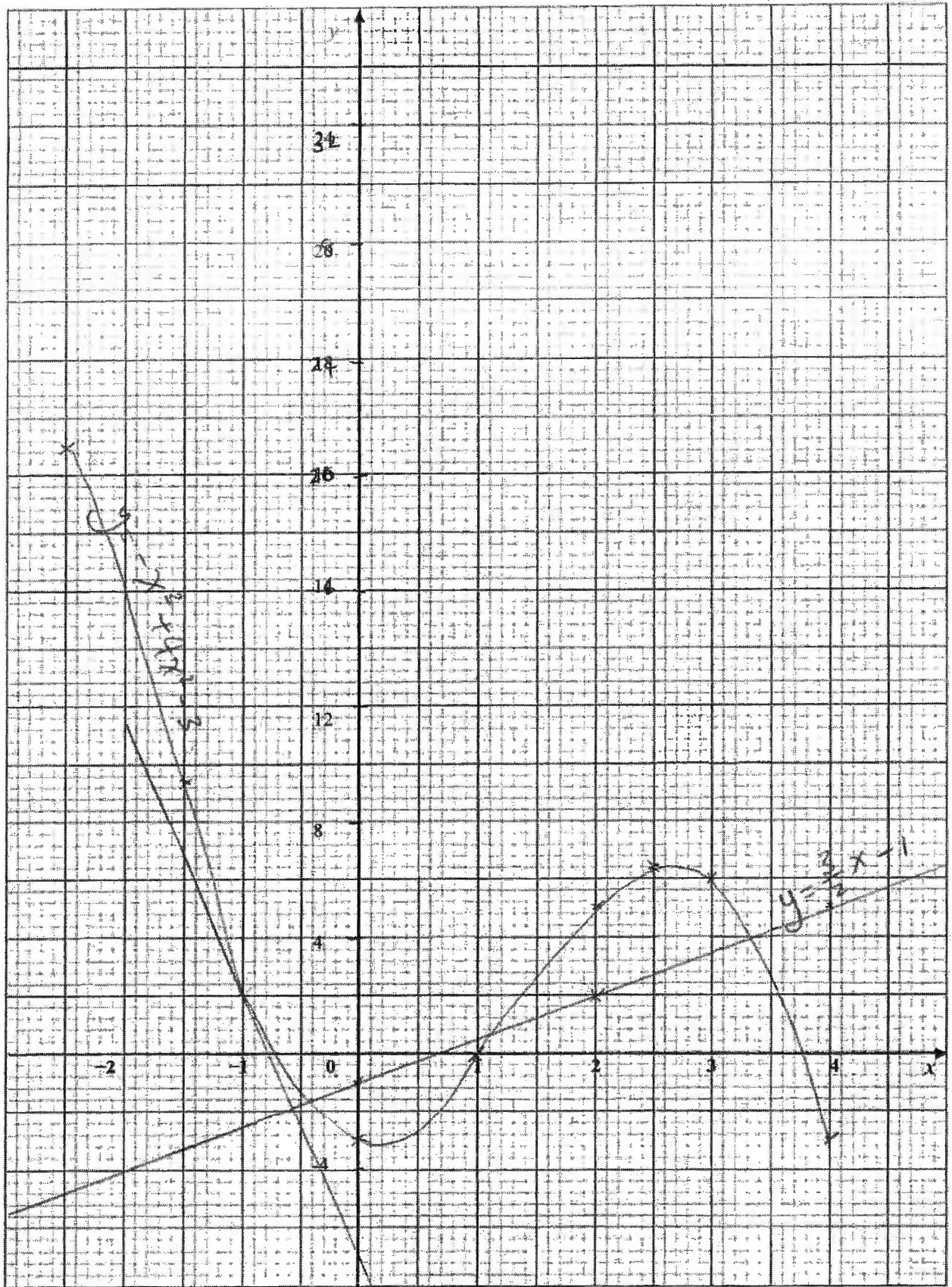
- (ii) By drawing the line in (e)(i), find the three solutions to the equation

$-2x^3 + 8x^2 - 3x - 4 = 0$.

Answer $x = \dots\dots\dots 3.4$ or $\dots\dots\dots 1.1$ or $\dots\dots\dots -0.5$ [3]

B1 for 2 correct ans.

B2 for all correct ans.



10sq. $\rightarrow$ 4 units
 1sq $\rightarrow$ 0.4 units

- 1 Benny, a wholesaler always spends \$300 a month to buy grapes.
 0 In January, the price of grapes was \$ x per kilogram.
 (a) Write down an expression in terms of x for the number of kilograms of grapes he bought.

$$\text{Answer } \frac{300}{x} \text{ kg [1]}$$

(B1)

In February, the price of grapes increased by 30 cents per kilogram.

- (b) Write down an expression in terms of x for the number of kilograms he bought for \$300 in February.

$$\text{Answer } \frac{300}{x+0.3} \text{ kg [1]}$$

(B1)

- (c) Given that he bought 5 kg less grapes in February than January, form an equation in x and show that it reduces to $10x^2 + 3x - 180 = 0$.

Answer

$$\begin{aligned} \frac{300}{x} - \frac{300}{x+0.3} &= 5 \text{ (m1)} \\ 300(x+0.3) - 300x &= 5x(x+0.3) \text{ (m1)} \\ 90 &= 5x^2 + 1.5x \\ 5x^2 + 1.5x - 90 &= 0 \\ \times 2 \hookrightarrow 10x^2 + 3x - 180 &= 0 \text{ (shown)} \end{aligned}$$

[3]

- (d) Solve $10x^2 + 3x - 180 = 0$, giving your answer correct to 2 decimal places.

$$\begin{aligned} x &= \frac{-3 \pm \sqrt{3^2 - 4(10)(-180)}}{2(10)} \\ &= \frac{-3 \pm \sqrt{7209}}{20} \text{ (m1)} \\ &= 4.10 \text{ or } -4.40 \end{aligned}$$

Answer $x = 4.10, -4.40$ [3]
 (A1) (A1)

In March, the price of grapes was \$ k less per kilogram than in February.

Benny was able to buy 4 kg more of grapes in March than in February for \$300.

(e) Show that $k = 0.24$, correct to 2 decimal places.

Answer

$$\frac{300}{x+0.3-k} - \frac{300}{x+0.3} = 4 \quad (\text{m1})$$

$$\frac{300}{4.09529-k} - \frac{300}{4.09529} = 4$$

$$\frac{300}{4.39529-k} - \frac{300}{4.39529} = 4$$

$$\frac{300}{4.39529-k} = 4 + \frac{300}{4.39529}$$

$$\frac{300}{4.39529-k} = 72.25488$$

[3]

$$300 = 72.25488(4.39529-k) \quad (\text{m1})$$

$$300 = 317.58116 - 72.25488k$$

$$72.25488k = 317.58116 - 300$$

$$72.25488k = 17.58116$$

$$k = \frac{17.58116}{72.25488}$$

$$\approx 0.24 \quad (2\text{dp}) \text{ shown} \quad (\text{A1})$$

- 11 Mr Gan wants to start a small tuition centre for secondary school students. The tables provide information on the cost of running the centre. The centre is closed on Mondays.

Start-up Cost	
Renovation	\$40 000
Furniture	\$5100
Computers and ICT Equipment	\$5000

Operational Cost Per Month	
Rent	\$8000
Utilities Bill (Electricity, Water)	\$1000
Advertising	\$300
Printing and Stationery	\$645
Interest of Renovation Loan	(b)

Labour Cost		
	Number of staff	Salary
Administrative Staff	1	\$1500 per month
Teaching Staff	1 per class [#]	\$45 per hour (Weekday)
		\$50 per hour (Weekend)
Cleaner	1	\$40 per day (3 hr)

Mr Gan intends to fix the tuition charges as follows:

Revenue			
	Total Number of Weekday Classes# Per Week	Total Number of Weekend Classes# Per Week	Monthly Tuition Fee Per Student
Lower Secondary	6	4	\$200
Upper Secondary	8	5	\$300

number of students per class: 5 to 8

*1 month has 4.3 weeks or 30 days

duration of each class is 2 hours

- (a) Calculate the total start-up cost.

Answer \$ 50100 [1]

- (b) Mr Gan took a loan for the renovation cost from a finance company that charges 2% simple interest per annum for 3 years.
Calculate the interest he paid on the renovation loan per month.

$$\begin{array}{lcl}
 I = \frac{PRT}{100} & & \frac{2400}{36} = \$66.67 \\
 = \frac{40,000 \times 2 \times 3}{100} & & \\
 = \$2400 & \text{Answer } \$ \dots\dots\dots & \underline{66.67} \quad [2]
 \end{array}$$

- (c) Mr Gan has 2 goals:

First Goal : To recover the total start up cost (including the interest incurred) within 7 months of operating her tuition centre.

Second Goal : To make a profit of at least \$8000 per month after the 6 months.

Will he hit his goals (assuming that the minimum number of students per class is met)?

Justify by showing your calculation clearly.

Answer

You may continue your workings on the next page

[6]

~End of Paper~

(Q11c)

Monthly RevenueLower Secondary

$$(6+4) \times 200 \times 5 = \$10000$$

Upper Secondary

$$(8+5) \times 300 \times 5 = \$19500$$

$$\therefore \text{Total Revenue} = \$10000 + 19500 \\ = \$29500 \quad (\text{mi})$$

Monthly Labour Cost

$$\text{(Weekday) Teaching Staff} = (6+8) \times 45 \times 2 \times 4.3 \\ = \$5418$$

$$\text{(Weekend) Teaching Staff} = (4+5) \times 50 \times 2 \times 4.3 \\ = \$3870$$

$$\text{Teaching} \\ \text{Total Staff Salary} = 5418 + 3870 \\ = \$9288 \quad (\text{mi})$$

Monthly Cleaner Salary

$$40 \times 6 \times 4.3 = \$1032 \quad (\text{mi})$$

$$\therefore \text{Total Labour Cost} = 9288 + 1032 + 1500 = \$11820$$

$$\begin{aligned}\text{Total Operational Cost} &= 8000 + 1000 + 300 + 645 + 66.67 \\ &= \$10011.67. \text{ (m1)}\end{aligned}$$

$$\begin{aligned}\text{Profit made} &= 29500 - 11820 - 10011.67 \\ &= \$7668.33. \text{ (m1)}\end{aligned}$$

$$\begin{aligned}\text{Months taken to recover start-up cost} &= \frac{50100}{7668.33} \\ &\approx 6.5 \text{ months.}\end{aligned}$$

$$6.5 < 7$$

∴ He will recover his start-up cost within 7 months

$$7668.33 < 8000.$$

He will make a profit of less than \$8000.

∴ He will hit his first goal but not his second goal.

(A1)