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**AHMAD IBRAHIM SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2023**

SECONDARY 3 EXPRESS

Name:	Class:	Register No.:
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MATHEMATICS
Paper 1

4052/01
3 October 2023
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, gives the answer to 3 significant figures. Gives answers in degrees to one decimal place.
For π , use either your calculator value or 3.142 unless the question requires the answer in terms of π .

For Examiner's Use
 /90

This document consists of **22** printed pages.

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

3

Answer all the questions.

- 1 (a) Calculate $\frac{154^2}{\sqrt[3]{135} - 2.67}$, giving your answer correct to 3 significant figures.

Answer[1]

- (b) The mass of a cone is 25.0 g, rounded off to the nearest 1 decimal place.
Write down the minimum mass of the cone.

Answerg [1]

-
- 2 The length of each side of a cube is decreased by 20%.
Find the percentage change in the volume of the cube.

Answer% [2]

4

- 3 In a regular polygon, the ratio interior angle : exterior angle = 7 : 1.
Calculate the number of sides of the polygon.

Answer [3]

- 4 (a) Factorise completely $p^2 + p - 6$.

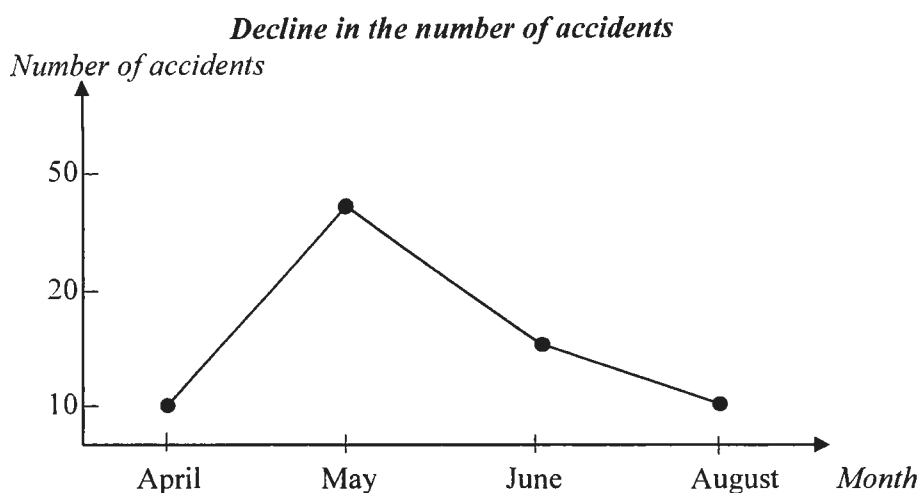
Answer [1]

- (b) Hence express $\frac{1}{2-p} + \frac{5p+15}{p^2+p-6}$ as a single fraction in its simplest terms.

Answer [2]

5

- 5 The data below shows the number of accidents that happened from April to August in district Z.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

-
- 6 Xander deposited \$17 000 in an account that pays 5.5% compound interest per year.
Find the total amount in the account after 6 years if the interest is compounded half-yearly.

Answer \$ [3]

7 (a) Express 540 as a product of its prime factors.

Answer [1]

(b) Explain why 540 is not a cubic number.

Answer
.....
.....
..... [1]

(c) Find the smallest integer k such that $540k$ is a perfect square.

Answer [1]

(d) Written as the product of its prime factors, $84 = 2^2 \times 3 \times 7$.

Find the highest common factor (HCF) and lowest common multiple (LCM) of 84 and 540, giving your answer in index notation.

Answer [2]

7

- 8 (a) Factorise completely $3p^4 - 4p^3q - 20q^2 + 15pq$.

Answer [2]

- (b) Rearrange the formula $a = k\sqrt{\frac{a+1}{b}}$ to make b the subject.

Answer $b =$ [3]

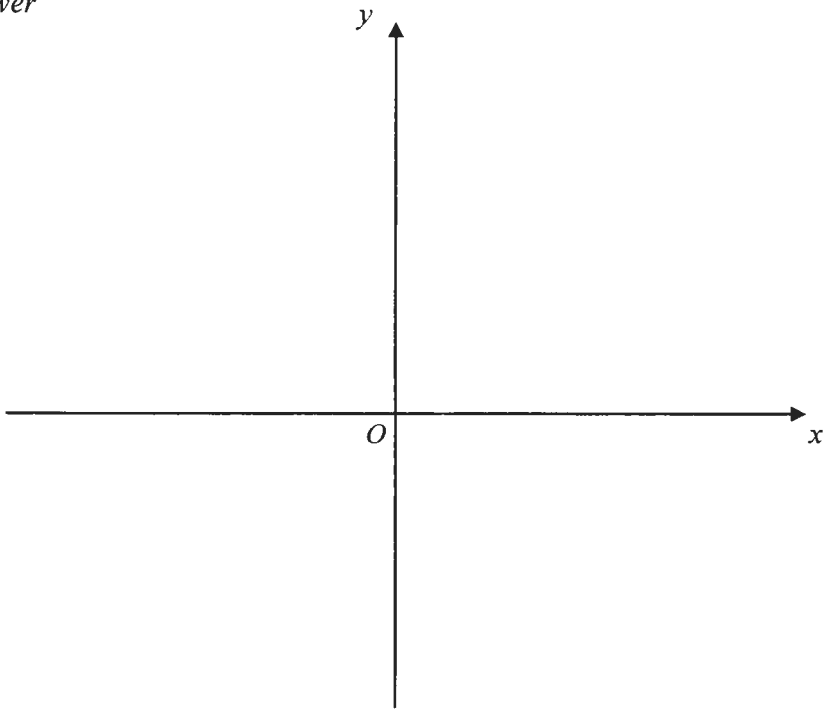
9 (a) Express $x^2 + 2x - 2$ in the form $(x + a)^2 + b$.

Answer [2]

(b) Sketch the graph of $y = x^2 + 2x - 2$ on the axes below.

Indicate clearly the coordinates of the points where the graph crosses the axes and the minimum point on the curve.

Answer



[3]

(c) Explain how your graph shows that there are more than one solution of the equation $x^2 + 2x - 2 = 0$.

Answer
.....
.....[1]

- 10 If $3x = 5y$, find the ratio $x^2 : y^2$.

Answer : [2]

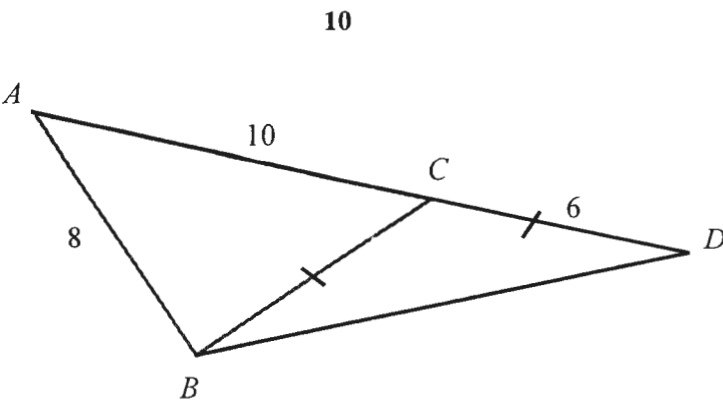
- 11 (a) Solve the inequality $\frac{1}{3}(x-1) \leq 3+2x < 10$.

Answer [3]

- (b) Represent your answer on the number line below. [1]



12



In the diagram above. $AB = 8$ cm, $CD = 6$ cm, $BC = CD$ and ACD is a straight line.

(a) Show that angle ABC is a right angle.

Answer

(b) Find the value of

(i) $\sin \angle BCD$,

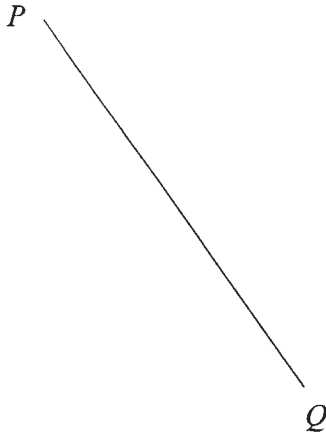
Answer [1]

(ii) $\cos \angle BCD$.

Answer [1]

- 13 In parallelogram $PQRS$, $QR = 7$ cm and $\angle QPS = 56^\circ$.

PQ is drawn below.



- (a) Construct parallelogram $PQRS$. [2]
- (b) Construct the perpendicular bisector of PQ . [1]
- (c) Construct the bisector of angle QPS . [1]
- (d) The two bisectors meet at X .

Measure and write down the length of XR .

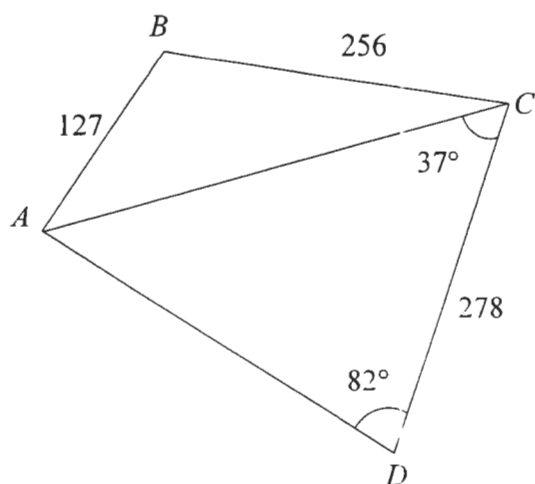
Answer $XR = \dots\dots\dots$ cm [1]

- (e) The point Z is inside parallelogram $PQRS$, nearer to P than to Q and nearer to PS than to PQ .

Mark and label a possible position of Z . [1]

12

14



In the diagram, $AB = 127$ km, $BC = 256$ km and $CD = 278$ km.

Angle $ACD = 37^\circ$ and angle $ADC = 82^\circ$.

Find angle ABC .

Answer Angle $ABC = \dots\dots\dots^\circ$ [3]

15 Solve these simultaneous equations.

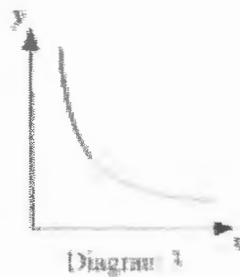
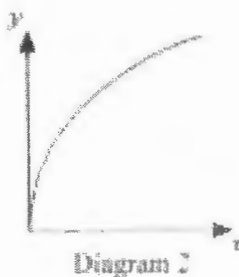
$$a + 129 = 18b$$

$$3b - 4a = 33$$

Answer $a = \dots\dots\dots$, $b = \dots\dots\dots$ [3]

- 16 (a) y is inversely proportional to x^2 .

Which of these diagrams represents the graph of y against x ?



Answer Diagram3..... [1]

- (b) P is directly proportional to Q^3 .

(i) When $Q = \frac{1}{3}$, $P = 2$.

Find P when $Q = \frac{1}{6}$.

Answer $P =$ [2]

- (ii) A change in Q produces a change in P .

When the value of Q is multiplied to $\frac{1}{2}$, the value of P changes by a factor of m .

Find m .

Answer $m =$ [1]

- 17 (a) Prove that $(x^2 + 1)^2 - (x^2 - 1)^2$ is a perfect square.

Answer

[2]

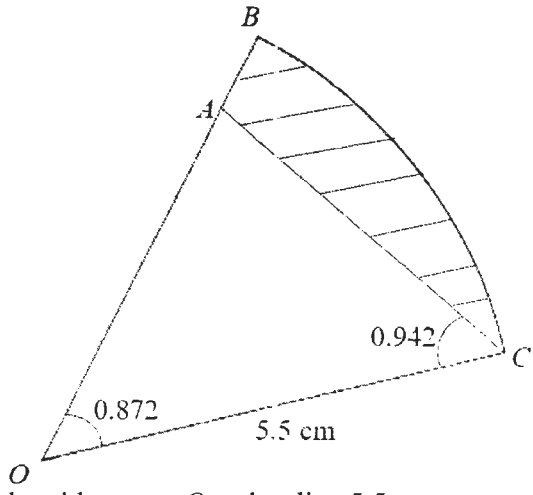
- (b) Alex says that $(x^2 + 1)^2 - (x^2 - 1)^2 = -14$ has no solution.

Explain his reasoning.

Answer

[1]

18



OBC is a sector of a circle with centre O and radius 5.5 cm.

A is a point on radius OB .

Angle $AOC = 0.872$ radians and angle $ACO = 0.942$ radians.

Calculate

(a) OA ,

Answercm [2]

(b) the area of the shaded region ABC .

Answercm² [3]

- 19 The table shows information about the mass of a hydrogen and oxygen atom.

Atom	Mass (in grams)
hydrogen	$1673.5575 \times 10^{-27}$
oxygen	$2.6566962 \times 10^{-23}$

- (a) Express the mass of a hydrogen atom, in grams, in standard form.
Give your answer to 4 significant figures.

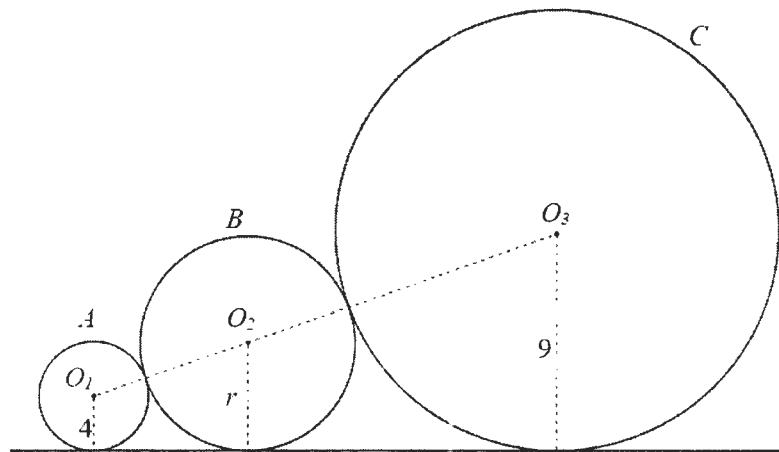
Answerg [1]

- (b) Express the mass of a hydrogen atom as a percentage of the mass of an oxygen atom.

Answer% [2]

- (c) A water molecule consists of two hydrogen atoms and one oxygen atom.
Calculate the number of water molecules in 10 grams of water.
Give your answer to 4 significant figures, in standard form

Answer [2]



Three spheres A , B and C with centres O_1 , O_2 and O_3 respectively are placed side by side on a flat surface such that O_1 , O_2 and O_3 can be connected by a straight line.

The radii of A , B and C are 4 cm, r cm and 9 cm respectively.

(a) Find the value of r .

Answer : cm [2]

(b) The mass of sphere A is 8 kg.

Find the mass of sphere C

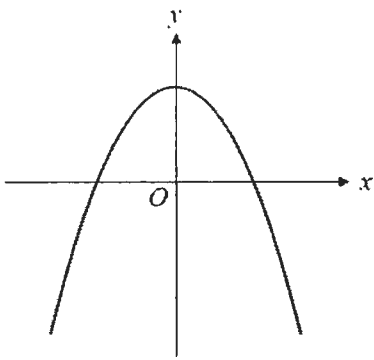
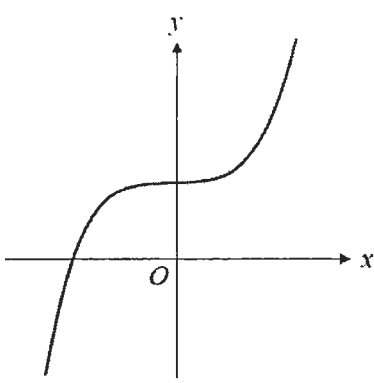
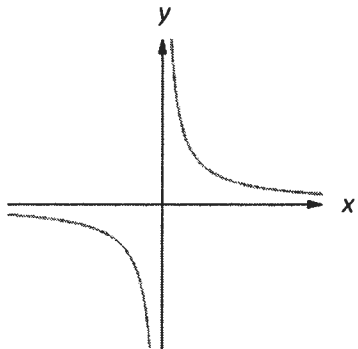
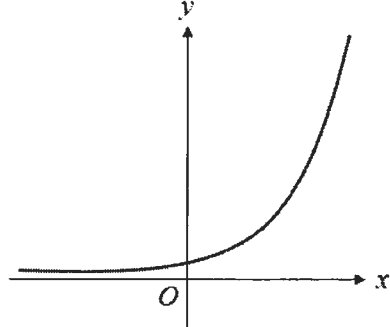
Answer : kg [2]

21

$y = \frac{1}{x}$	$y = 2 - x^3$	$y = 2 - x^2$	$y = -\frac{2}{x}$
$y = 2^x$	$y = x^3 + 2$	$y = x^2 + 2$	$y = x^3 - 2$

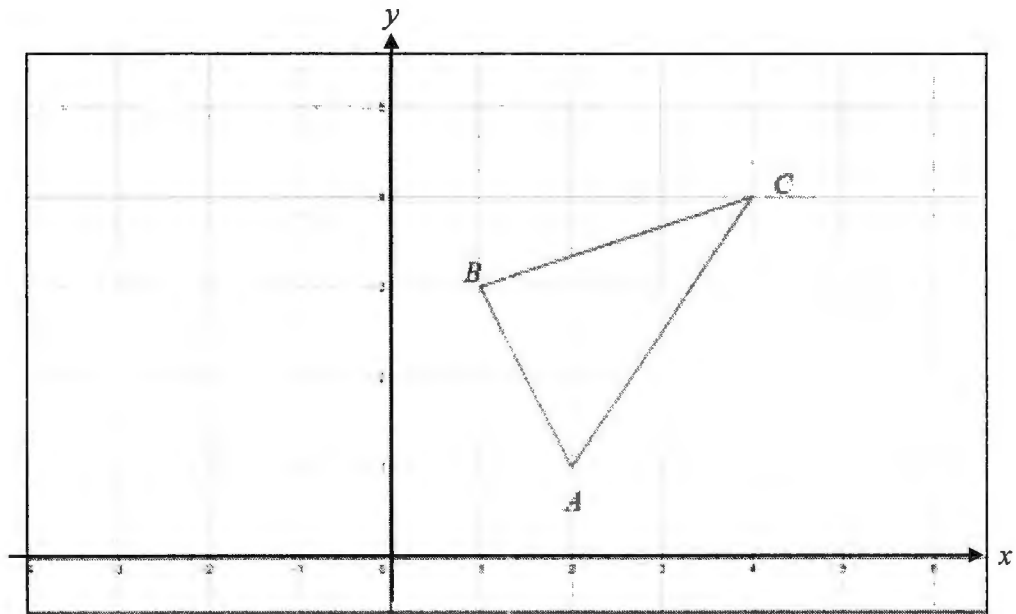
Write down a possible equation for each of the sketch graphs below.

In each case select one of the equations from the box above.

(a)  Answer [1]	(b)  Answer [1]
(c)  Answer [1]	(d)  Answer [1]

20

22



The diagram shows a triangle ABC with vertices $A(2, 1)$, $B(1, 3)$ and $C(4, 4)$.

(a) Find the length of AC .

Answerunits [2]

(b) Find the equation of the line BC .

Answer [2]

- (c) Write down two possible coordinates of the point D , where A , B , C and D are the four vertices of a parallelogram.

Answer (.....,) and (.....,) [2]

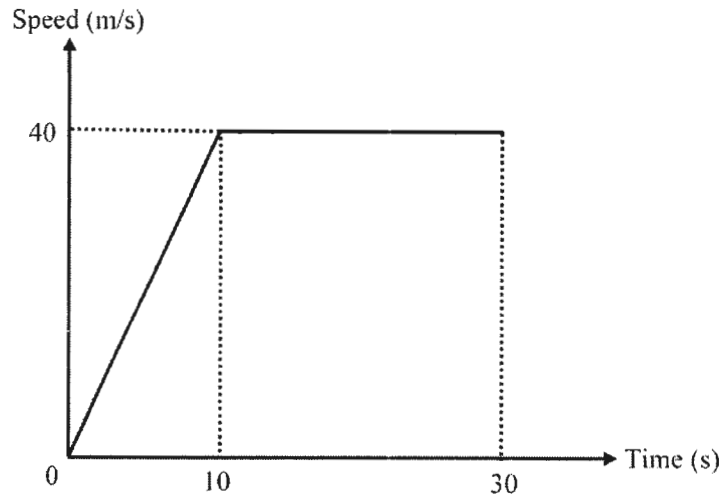
- (d) E is the point (m, n) where m and n are both positive integers less than 10.

The gradient of the line AE is $\frac{2}{3}$.

Find the coordinates of one of the possible positions for E .

Answer (.....,) [2]

- 23 The diagram shows the speed-time graph for a motorist's journey.



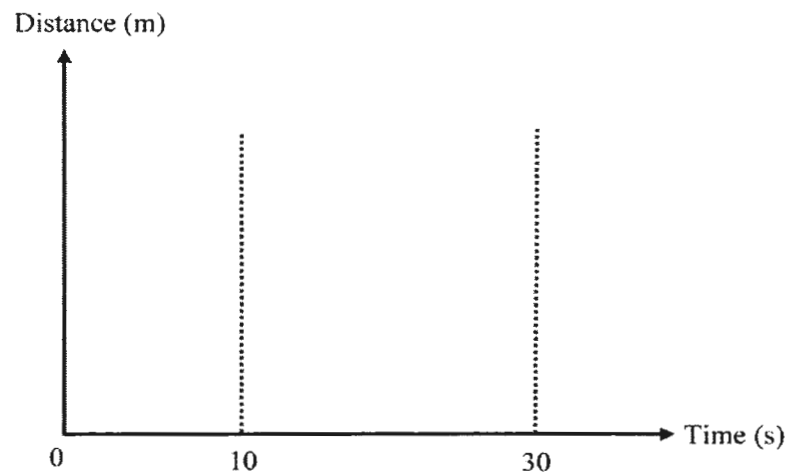
- (a) Find the acceleration of the motorist during the first 10 seconds.

Answer m/s [1]

- (b) Find the speed of the motorist at the 6th second.

Answer m/s [1]

- (c) Sketch the distance-time graph of the motorist for the journey on the axes below.



End of paper



**AHMAD IBRAHIM SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2023**

SECONDARY 3 EXPRESS

Name: MARKING SCHEME	Class:	Register No.:
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MATHEMATICS

Paper 1

4052/01

3 October 2023

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For Examiner's Use

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$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

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$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1 (a) Calculate $\frac{154^2}{\sqrt[3]{135-2.67}}$, giving your answer correct to 3 significant figures.

$$\begin{aligned}\frac{154^2}{\sqrt[3]{135-2.67}} &= 9640.933207 \\ &= 9640 \text{ (3 s.f.)}\end{aligned}$$

Answer[1]

- (b) The mass of a cone is 25.0 g, rounded off to the nearest 1 decimal place.

Write down the minimum mass of the cone.

Answer24.95.....g [1]

- 2 The length of each side of a cube is decreased by 20%.

Find the percentage change in the volume of the cube.

Let side length be x

Old volume = x^3

New volume = $(0.8x)^3$ [B1]

$$= 0.512 x^3$$

$$\% \text{ change} = \left(\frac{0.512x^3 - x^3}{x^3} \right) \times 100\%$$

$$= -48.8\% \quad \text{[B1]}$$

Answer% [2]

- 3 In a regular polygon, the ratio interior angle : exterior angle = 7 : 1.

Calculate the number of sides of the polygon.

Let x be the exterior angle.

$$7x + x = 180^\circ \quad \text{M1}$$

$$8x = 180^\circ$$

$$x = 22.5 \quad \text{M1}$$

$$\begin{aligned} \text{No of sides} &= \frac{360^\circ}{22.5^\circ} \\ &= 16 \quad \text{A1} \end{aligned}$$

Answer [3]

- 4 (a) Factorise completely $p^2 + p - 6$.

$$p^2 + p - 6 = (p+3)(p-2)$$

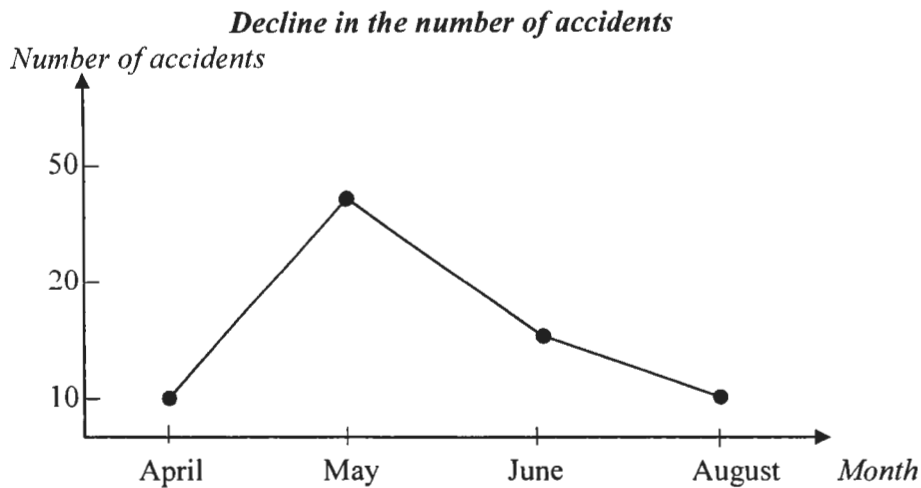
Answer [1]

- (b) Hence express $\frac{1}{2-p} + \frac{5p+15}{p^2+p-6}$ as a single fraction in its simplest terms.

$$\begin{aligned} &\frac{1}{2-p} + \frac{5p+15}{p^2+p-6} \\ &= -\frac{1}{p-2} + \frac{5p+15}{(p-2)(p+3)} \\ &= \frac{-(p+3) + 5p+15}{(p-2)(p+3)} \quad \text{[M1]} \\ &= \frac{4p+12}{(p-2)(p+3)} \\ &= \frac{4(p+3)}{(p-2)(p+3)} \\ &= \frac{4}{p-2} \quad \text{[A1]} \end{aligned}$$

Answer [2]

- 5 The data below shows the number of accidents that happened from April to August in district Z.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

Biased title [B1]

The word 'decline' may let readers assume that there is a decreasing trend. It does not allow readers to interpret on their own/ However, there is an increase from April to May. [B1]

OR

The scale of horizontal axis is not proportional. [B1]

For the horizontal axis, the data for July is missing. Readers will not be able to analyse properly. [B1]

OR

The scale of vertical axis is not proportional. [B1]

For the vertical axis, the interval between 10 to 20 and 20 to 50 are approximately the same. Readers may analyse the data wrongly. [B1]

6

- 6 Xander deposited \$17 000 in an account that pays 5.5% compound interest per year.
Find the total amount in the account after 6 years if the interest is compounded half-yearly.

Since interest is compounded half-yearly,

$$\left. \begin{aligned} r &= \frac{5.5}{2} = 2.75 \\ n &= 6 \times 2 = 12 \end{aligned} \right\} \text{ [M1 for either]}$$

$$\begin{aligned} A &= 17000 \left(1 + \frac{2.75}{100} \right)^{12} \quad \text{[M1]} \\ &= \$23541.32 \text{ (2 d.p.)} \quad \text{[A1]} \end{aligned}$$

Answer \$ [3]

- 7 (a) Express 540 as a product of its prime factors.

$$540 = 2^2 \times 3^3 \times 5$$

Answer [1]

- (b) Explain why 540 is not a cubic number.

Answer

Since the powers of the prime factors 2 and 5 are not multiples of 3,
540 is not a cubic number. [B1]

- (c) Find the smallest integer k such that $540k$ is a perfect square.

$$\begin{aligned} \text{Minimum value of } k &= 3 \times 5 \\ &= 15 \quad \text{[B1]} \end{aligned}$$

Answer [1]

- (d) Written as the product of its prime factors, $84 = 2^2 \times 3 \times 7$.

Find the highest common factor (HCF) and lowest common multiple (LCM) of 84
and 540, giving your answer in index notation.

$$84 = 2^2 \times 3 \times 7$$

$$540 = 2^2 \times 3^3 \times 5$$

$$\text{HCF} = 2^2 \times 3 \quad [\text{B1}]$$

$$\text{LCM} = 2^2 \times 3^3 \times 5 \times 7 \quad [\text{B1}]$$

Answer [2]

- 8 (a) Factorise completely $3p^4 - 4p^3q - 20q^2 + 15pq$.

$$\begin{aligned} & 3p^4 - 4p^3q - 20q^2 + 15pq \\ &= p^3(3p - 4q) - 5q(4q - 3p) \quad [\text{M1 for both factorisation}] \\ &= p^3(3p - 4q) + 5q(3p - 4q) \\ &= (p^3 + 5q)(3p - 4q) \quad [\text{A1}] \end{aligned}$$

Answer [2]

- (b) Rearrange the formula $a = k\sqrt{\frac{a+1}{b}}$ to make b the subject.

$$\begin{aligned} a &= k\sqrt{\frac{a+1}{b}} \\ \frac{a}{k} &= \sqrt{\frac{a+1}{b}} \\ \left(\frac{a}{k}\right)^2 &= \frac{a+1}{b} \quad [\text{M1 for squaring both sides}] \quad \text{or} \\ a^2b &= k^2(a+1) \quad [\text{M1}] \\ b &= \frac{k^2(a+1)}{a^2} \quad [\text{A1}] \end{aligned}$$

$$\begin{aligned} a &= k\sqrt{\frac{a+1}{b}} \\ a^2 &= k^2\left(\frac{a+1}{b}\right) \quad [\text{M1}] \\ \frac{b}{a+1} &= \frac{k^2}{a^2} \quad [\text{M1}] \\ b &= \frac{k^2}{a^2}(a+1) \quad [\text{A1}] \end{aligned}$$

Answer $b =$ [3]

- 9 (a) Express $x^2 + 2x - 2$ in the form $(x + a)^2 + b$.

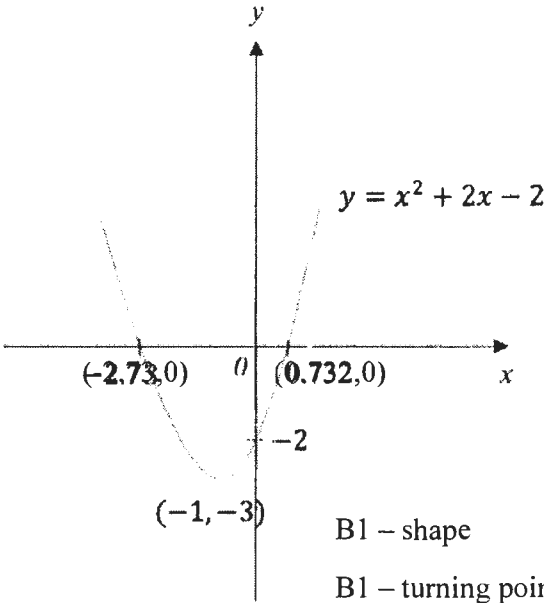
$$\begin{aligned} x^2 + 2x - 2 &= x^2 + 2x + \left(\frac{2x}{2}\right)^2 - \left(\frac{2x}{2}\right)^2 - 2 \\ &= (x + 1)^2 - 3 \quad \text{[B2]} \end{aligned}$$

Answer [2]

- (b) Sketch the graph of $y = x^2 + 2x - 2$ on the axes below.

Indicate clearly the coordinates of the points where the graph crosses the axes and the minimum point on the curve.

Answer



B1 – shape
B1 – turning point
B1 – coordinates on x and y axes

[3]

- (c) Explain how your graph shows that there are more than one solution of the equation $x^2 + 2x - 2 = 0$.

Answer The curve intersects at two points on the x – axis. Thus there are two solutions.

[1]

- 10 If $3x = 5y$, find the ratio $x^2 : y^2$.

$$3x = 5y$$

$$\frac{x}{y} = \frac{5}{3} \quad [\text{M1}]$$

$$\left(\frac{x}{y}\right)^2 = \left(\frac{5}{3}\right)^2$$

$$\frac{x^2}{y^2} = \frac{25}{9}$$

$$x^2 : y^2 = 25 : 9 \quad [\text{A1}]$$

Answer : [2]

- 11 (a) Solve the inequality $\frac{1}{3}(x-1) \leq 3+2x < 10$.

$$\frac{1}{3}(x-1) \leq 3+2x \quad \text{or} \quad 3+2x < 10$$

$$x-1 \leq 9+6x \quad x < \frac{10-3}{2}$$

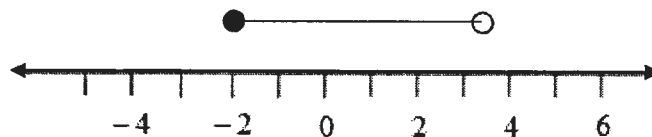
$$5x \geq -10 \quad x < 3.5 \quad [\text{M1}]$$

$$x \geq -2 \quad [\text{M1}]$$

$$\therefore -2 \leq x < 3.5 \quad [\text{A1}]$$

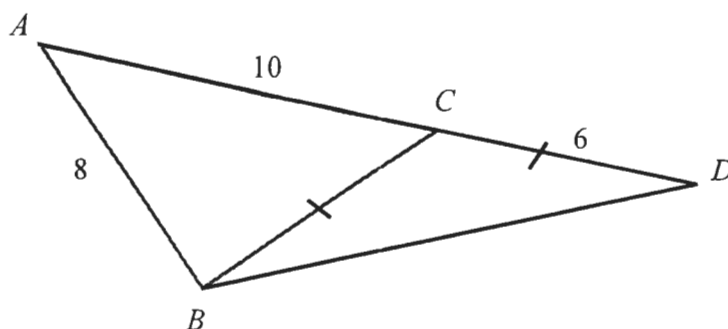
Answer [3]

- (b) Represent your answer on the number line below. [1]



10

12



In the diagram above. $AB = 8$ cm, $CD = 6$ cm. $BC = CD$ and ACD is a straight line.

(a) Show that angle ABC is a right angle.

Answer

$$AB^2 + BC^2 = 8^2 + 6^2 = 100$$

$$AC^2 = 10^2 = 100 \quad \text{M1}$$

Since $AB^2 + BC^2 = AC^2$, by the converse of Pythagoras Theory,

Triangle ABC is a right-angled triangle and Angle $ABC = 90^\circ$. (shown) [A1]

(b) Find the value of

(i) $\sin \angle BCD$,

$$\sin \angle BCD = \sin \angle BCA$$

$$= \frac{8}{10}$$

$$= \frac{4}{5} \quad \text{[B1]}$$

Answer [1]

(ii) $\cos \angle BCD$.

$$\cos \angle BCD = -\cos \angle BCA$$

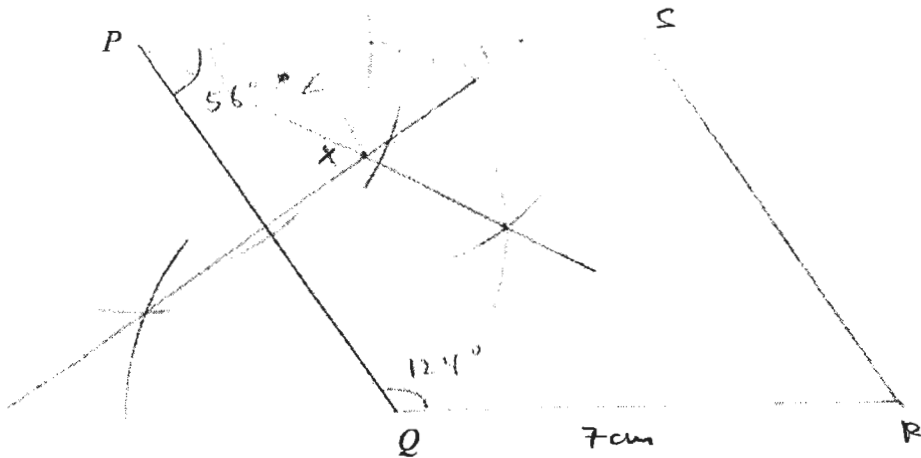
$$= -\frac{6}{10}$$

$$= -\frac{3}{5} \quad \text{[B1]}$$

Answer [1]

- 13 In parallelogram $PQRS$, $QR = 7$ cm and $\angle QPS = 56^\circ$.

PQ is drawn below.



- (a) Construct parallelogram $PQRS$. [2]
- (b) Construct the perpendicular bisector of PQ . [1]
- (c) Construct the bisector of angle QPS . [1]
- (d) The two bisectors meet at X .

Measure and write down the length of XR .

Answer $XR = 8.0 - 8.2$ cm [1]

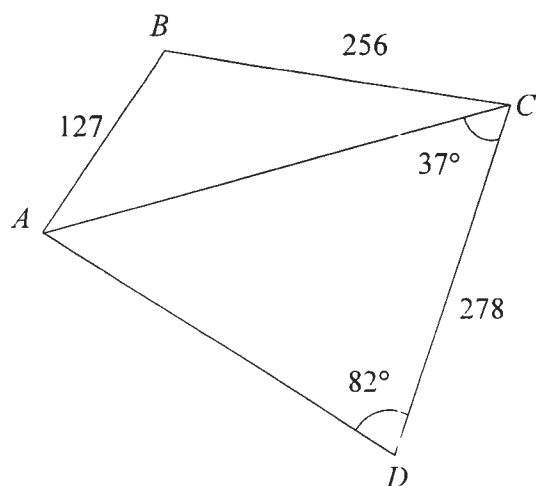
- (e) The point Z is inside parallelogram $PQRS$, nearer to P than to Q and nearer to PS than to PQ .

Mark and label a possible position of Z . [1]

Any point in the shaded region above but not on PS or on the bisectors.

12

14



In the diagram, $AB = 127$ km, $BC = 256$ km and $CD = 278$ km.

Angle $ACD = 37^\circ$ and angle $ADC = 82^\circ$.

Find angle ABC .

$$\begin{aligned}\angle CAD &= 180^\circ - 37^\circ - 82^\circ \\ &= 61^\circ \text{ (}\angle \text{ sum of } \Delta\text{)}\end{aligned}$$

$$\frac{AC}{\sin 82^\circ} = \frac{278}{\sin 61^\circ} \quad [\text{M1}]$$

$$\begin{aligned}AC &= \frac{278}{\sin 61^\circ} \times \sin 82^\circ \\ &= 314.7591129 \text{ km}\end{aligned}$$

$$\begin{aligned}\angle ABC &= \cos^{-1} \left(\frac{127^2 + 256^2 - 314.7591129^2}{2(127)(256)} \right) \quad [\text{M1}] \\ &= 105.5^\circ (1 \text{ d.p}) \quad [\text{A1}]\end{aligned}$$

Answer Angle $ABC = \dots\dots\dots^\circ [3]$

15 Solve these simultaneous equations.

$$a + 129 = 18b$$

$$3b - 4a = 33$$

$$a + 129 = 18b$$

$$a = 18b - 129 \text{ ----- (1)}$$

$$3b - 4a = 33 \text{ ----- (2)}$$

Sub (1) into (2):

$$3b - 4(18b - 129) = 33 \quad [\text{M1}]$$

$$3b - 72b + 516 = 33$$

$$69b = 483$$

$$b = 7 \quad [\text{A1}]$$

Sub $b = 7$ into (1):

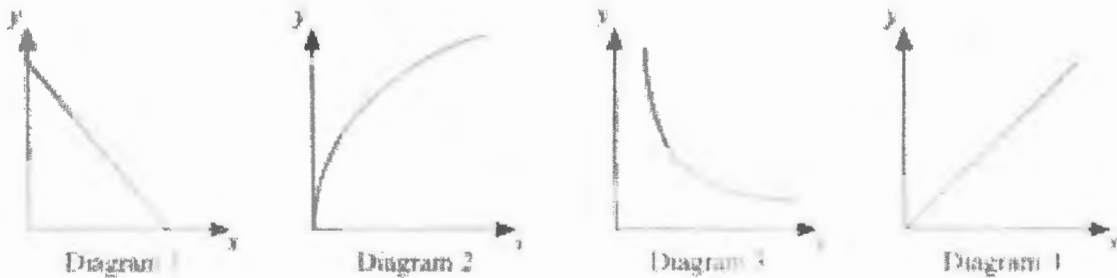
$$a = 18(7) - 129$$

$$a = -3 \quad [\text{A1}]$$

Answer $a = \dots\dots\dots$, $b = \dots\dots\dots$ [3]

- 16 (a) y is inversely proportional to x^2 .

Which of these diagrams represents the graph of y against x ?



Answer Diagram3..... [1]

- (b) P is directly proportional to Q^3 .

- (i) When $Q = \frac{1}{3}$, $P = 2$.

Find P when $Q = \frac{1}{6}$.

$$P = kQ^3$$

$$2 = k\left(\frac{1}{3}\right)^3$$

$$k = 54 \quad [\text{M1}]$$

$$P = 54\left(\frac{1}{6}\right)^3$$

$$= \frac{1}{4} \quad [\text{A1}]$$

$$Q^3 = kP$$

$$\left(\frac{1}{3}\right)^3 = k(2)$$

$$k = \frac{1}{54} \quad [\text{M1}]$$

$$Q^3 = \frac{1}{54}P$$

$$\left(\frac{1}{6}\right)^3 = \frac{1}{54}P$$

$$P = \frac{1}{4} \quad [\text{A1}]$$

Answer $P =$ [2]

- (ii) A change in Q produces a change in P .

When the value of Q is multiplied to $\frac{1}{2}$, the value of P changes by a factor of m .

Find m .

15

$$P = 54Q^3$$

$$P = 54\left(\frac{Q}{2}\right)^3$$

$$= \frac{1}{8}(54Q^3)$$

$$m = \frac{1}{8} \quad [\text{B1}]$$

Answer $m = \dots\dots\dots$ [1]

- 17 (a) Prove that $(x^2 + 1)^2 - (x^2 - 1)^2$ is a perfect square.

Answer

$$\begin{aligned} & (x^2 + 1)^2 - (x^2 - 1)^2 \\ &= [(x^2 + 1) + (x^2 - 1)][(x^2 + 1) - (x^2 - 1)] \quad [\text{M1}] \\ &= (2x^2)(2) \\ &= 4x^2 \\ &= (2x)^2 \quad [\text{A1}] \end{aligned}$$

- (b) Alex says that $(x^2 + 1)^2 - (x^2 - 1)^2 = -14$ has no solution.

Explain his reasoning.

Answer

$$(x^2 + 1)^2 - (x^2 - 1)^2 = -14$$

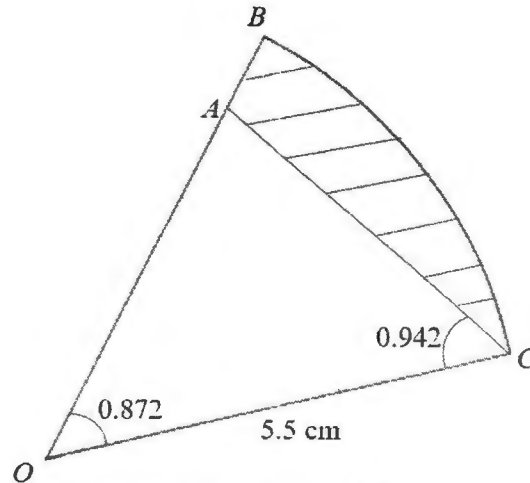
$$(2x)^2 = -14$$

Since $(2x)^2 \geq 0$ for all real values of x , $(x^2 + 1)^2 - (x^2 - 1)^2 = -14$ has no solution. [B1]

or $(x^2 + 1)^2 - (x^2 - 1)^2 = -14$ has no solution as -14 is a negative number and thus not a perfect square [B1]

16

18



OBC is a sector of a circle with centre O and radius 5.5 cm.

A is a point on radius OB .

Angle $AOC = 0.872$ radians and angle $ACO = 0.942$ radians.

Calculate

(a) OA ,

$$\begin{aligned}\angle OAC &= \pi - 0.872 - 0.942 \\ &= 1.3276 \text{ rad } (\angle \text{ sum of } \Delta)\end{aligned}$$

$$\frac{OA}{\sin 0.942} = \frac{5.5}{\sin 1.3276} \quad [\text{M1}]$$

$$\begin{aligned}OA &= \frac{5.5}{\sin 1.3276} \times \sin 0.942 \\ &= 4.5829 \\ &= 4.58 \text{ cm (3 s.f.)} \quad [\text{A1}]\end{aligned}$$

Answercm [2]

(b) the area of the shaded region ABC .

$$\begin{aligned}\text{Area of } \triangle OAC &= \frac{1}{2}(5.5)(4.5829)\sin 0.872 \quad [\text{M1}] \text{ ecf} \\ &= 9.6491 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Shaded area} &= \left[\frac{1}{2} \times (5.5)^2 \times 0.872 \right] - 9.6491 \quad [\text{M1}] \text{ ecf} \\ &= 13.189 - 9.6491 \\ &= 3.5399 \\ &= 3.54 \text{ cm}^2 \quad [\text{A1}]\end{aligned}$$

Answercm² [3]

- 19 The table shows information about the mass of a hydrogen and oxygen atom.

Atom	Mass (in grams)
hydrogen	$1673.5575 \times 10^{-27}$
oxygen	$2.6566962 \times 10^{-23}$

- (a) Express the mass of a hydrogen atom, in grams, in standard form.
Give your answer to 4 significant figures.

Answer 1.674×10^{-24} g [1]

- (b) Express the mass of a hydrogen atom as a percentage of the mass of an oxygen atom.

$$\frac{1673.5575 \times 10^{-27}}{2.6566962 \times 10^{-23}} \times 100 \quad [\text{M1}]$$

$$= 6.30\% \text{ (3 s.f.)} \quad [\text{A1}]$$

Answer% [2]

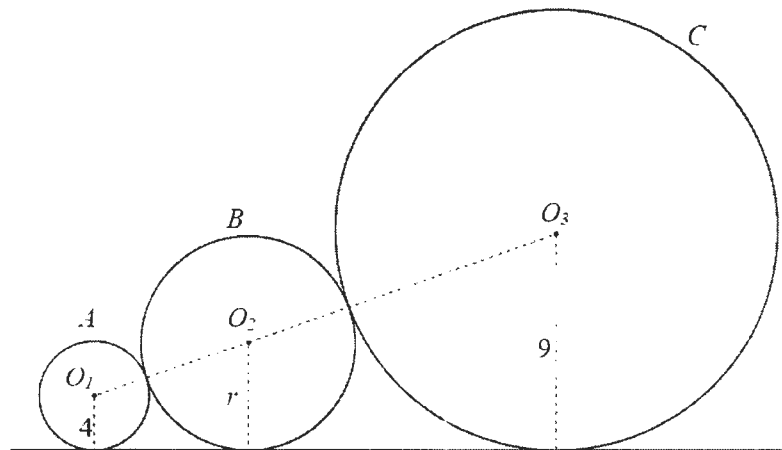
- (c) A water molecule consists of two hydrogen atoms and one oxygen atom.

Calculate the number of water molecules in 10 grams of water.

Give your answer to 4 significant figures, in standard form

$$\begin{aligned} & \text{1 molecule} \\ &= 1673.5575 \times 10^{-27} \times 2 + 2.6566962 \times 10^{-23} \quad [\text{M1}] \\ &= 2.9914077 \times 10^{-23} \text{ g} \\ &10 \div (2.9914077 \times 10^{-23}) = 3.343 \times 10^{23} \quad [\text{A1}] \end{aligned}$$

Answer [2]



Three spheres A , B and C with centres O_1 , O_2 and O_3 respectively are placed side by side on a flat surface such that O_1 , O_2 and O_3 can be connected by a straight line.

The radii of A , B and C are 4 cm, r cm and 9 cm respectively.

(a) Find the value of r .

$$\frac{r}{4} = \frac{9}{r} \quad [\text{M1}]$$

$$r^2 = 36$$

$$r = 6 \text{ cm} \quad [\text{A1}]$$

OR

$$\frac{r-4}{r+4} = \frac{9-r}{9+r} \quad [\text{M1}]$$

$$(r-4)(9+r) = (9-r)(r+4) \quad \text{OR}$$

$$2r^2 = 72$$

$$r = 6 \text{ cm} \quad [\text{A1}]$$

$$\frac{r+4}{r-4} = \frac{13+2r}{5} \quad [\text{M1}]$$

$$5(r+4) = (13+2r)(r-4)$$

$$2r^2 = 72$$

$$r = 6 \text{ cm} \quad [\text{A1}]$$

$$\frac{r}{4} = \frac{r+9}{r+4} \quad [\text{M1}]$$

OR $r(r+4) = 4(r+9)$

$$r^2 = 36$$

$$r = 6 \text{ cm} \quad [\text{A1}]$$

Answer : cm [2]

19

- (b) The mass of sphere A is 8 kg.

Find the mass of sphere C

$$\frac{m}{8} = \left(\frac{9}{4}\right)^3 \quad [\text{M1}]$$

$$m = \left(\frac{9}{4}\right)^3 \times 8$$

$$= 91\frac{1}{8} \text{ kg or } 91.125 \text{ kg (exact answer only)} \quad [\text{A1}]$$

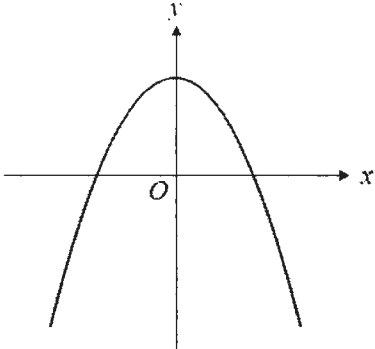
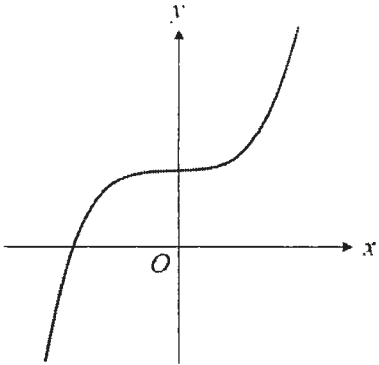
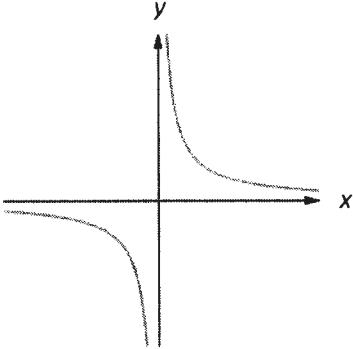
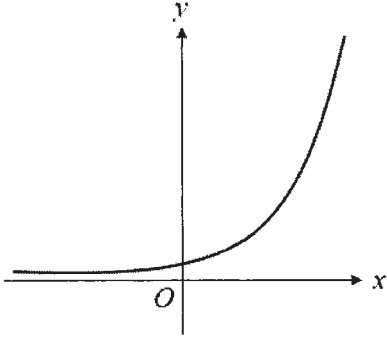
Answer : kg [2]

 21

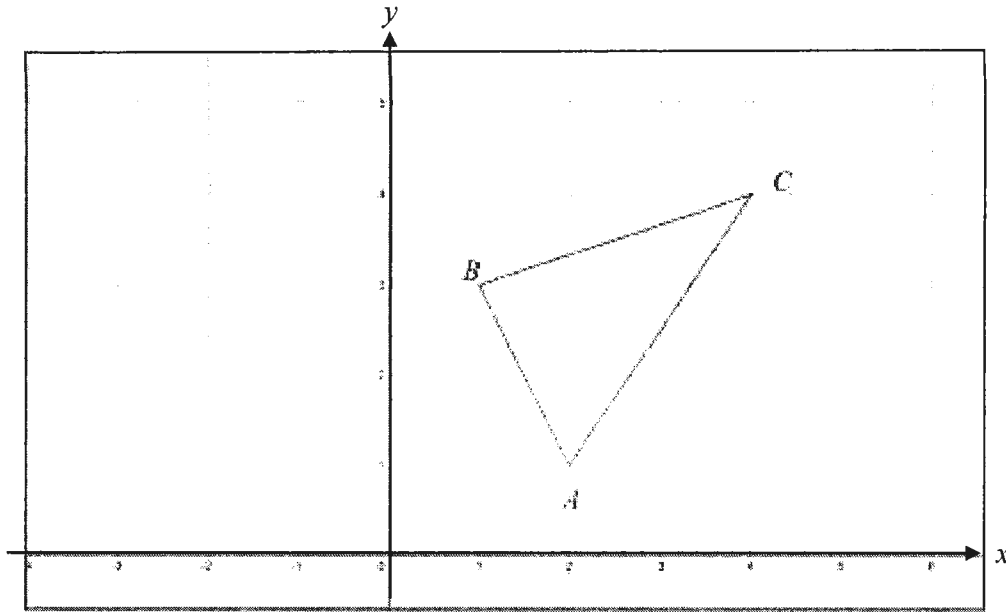
$y = \frac{1}{x}$	$y = 2 - x^3$	$y = 2 - x^2$	$y = -\frac{2}{x}$
$y = 2^x$	$y = x^3 + 2$	$y = x^2 + 2$	$y = x^3 - 2$

Write down a possible equation for each of the sketch graphs below.

In each case select one of the equations from the box above.

(a)  Answer $y = 2 - x^2$ [1]	(b)  Answer $y = x^3 + 2$ [1]
(c)  Answer $y = \frac{1}{x}$ [1]	(d)  Answer $y = 2^x$ [1]

22



The diagram shows a triangle ABC with vertices $A(2, 1)$, $B(1, 3)$ and $C(4, 4)$.

(a) Find the length of AC .

$$AC = \sqrt{(4-2)^2 + (4-1)^2} \quad [\text{M1}]$$

$$= 3.61 \text{ units (3 s.f.)} \quad [\text{A1}]$$

Answerunits [2]

(b) Find the equation of the line BC .

$$y = \frac{1}{3}x + c$$

Subst (1,3)

$$3 = \frac{1}{3}(1) + c \quad [\text{M1}]$$

$$c = 2\frac{2}{3}$$

$$y = \frac{1}{3}x + 2\frac{2}{3} \quad [\text{A1}]$$

Answer [2]

(c) Write down two possible coordinates of the point D , where A , B , C and D are the

four vertices of a parallelogram.

Using the diagram, any two of the following ordered pairs:

$(5, 2), (-1, 0), (3, 6)$ $B1, B1$

Answer (.....,) and (.....,) [2]

(d) E is the point (m, n) where m and n are both positive integers less than 10.

The gradient of the line AE is $\frac{2}{3}$.

Find the coordinates of one of the possible positions for E .

$$\frac{n-1}{m-2} = \frac{2}{3} = \frac{4}{6} \quad [\text{M1}]$$

$$(m, n) = (5, 3) \text{ or } (8, 5) \quad [\text{B1}]$$

OR

$$\text{Sub } A(2,1) \text{ into } n = \frac{2}{3}m + c$$

$$1 = \frac{2}{3}(2) + c$$

$$c = -\frac{1}{3}$$

$$\therefore n = \frac{2}{3}m - \frac{1}{3} \quad [\text{M1}]$$

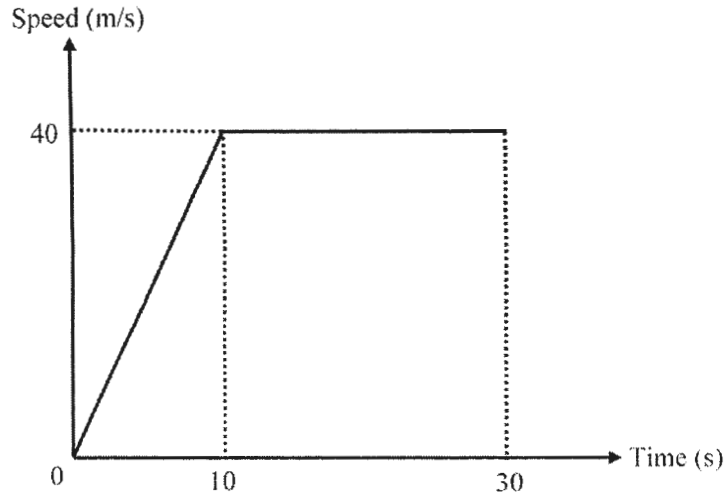
By guess and check :

when sub $m = 8, n = 5$ Or

when sub $m = 5, n = 3$ [A1]

Answer (.....,) [2]

- 23 The diagram shows the speed-time graph for a motorist's journey.



- (a) Find the acceleration of the motorist during the first 10 seconds.

$$\text{Acceleration} = \frac{40}{10} = 4 \text{ m/s}^2$$

Answer m/s [1]

- (b) Find the speed of the motorist at the 6th second.

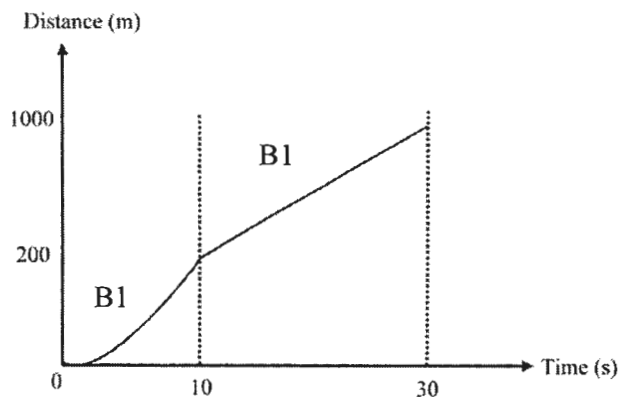
Let V be the speed of motorist at 6 seconds

$$\frac{V}{6} = 4$$

$$V = 24 \text{ m/s}$$

Answer m/s [1]

- (c) Sketch the distance-time graph of the motorist for the journey on the axes below.



[2]

End of paper



AHMAD IBRAHIM SECONDARY SCHOOL END-OF-YEAR EXAMINATION 2023

SECONDARY 3 EXPRESS

Name:	Class:	Register No.:
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MATHEMATICS

Paper 2

4052/02

4 October 2023

2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any questions it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

For Examiner's Use
/ 90

This document consists of **23** printed pages.

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

3

Answer **all** the questions.

- 1** A Mathematics competition scores students in three sections: Algebra, Geometry and Statistics.

The maximum score of each section is 100.

Grades are awarded to students based on their mean score attained from the three sections.

Range of mean scores from 3 sections	Grade
More than 85 but less than or equal to 100	A
More than 75 but less than or equal to 85	B
More than 65 but less than or equal to 75	C

Maddy scored 74 marks for Algebra, 81 marks for Geometry and x marks for Statistics.

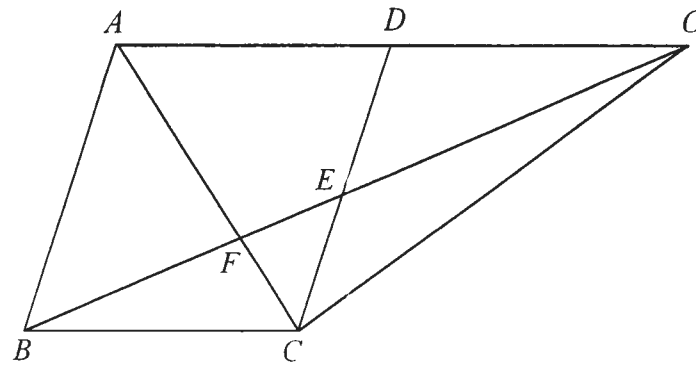
Hariz claimed that Maddy needs to score more than 67 in the Statistics section to obtain a Grade B.

By forming an inequality, justify if Hariz's claim is true.

Answer

[4]

2



The diagram shows a rhombus $ABCD$.
 O is on AD produced such that $AO = 2 AD$.
 BO intersects AC and DC at F and E respectively.

- (a) Show that triangle ODE and BCE are congruent.
Give a reason for each statement you make.

Answer

[3]

5

- (b) Show that triangles ECF and BAF are similar.
Give a reason for each statement you make.

Answer

[2]

- (c) Find area of $\triangle ECF$: area of $\triangle BAF$.

Answer [1]

6

3 (a) Simplify

(i) $\sqrt{\frac{x^5}{y}} \times \frac{y^{-3}}{x^6},$

Answer [2]

(ii) $\left(\frac{343x^3}{y^6}\right)^{\frac{1}{3}} \times 2x^0y^2.$

Answer [2]

(b) Solve $\left(\frac{1}{4}\right)^{3x-1} = 8^x.$

Answer $x =$ [3]

4 (a) Write as a single fraction in its simplest form

(i) $\frac{16x^2 - 1}{y^2} \div \frac{4x - 1}{xy},$

Answer [2]

(ii) $\frac{3}{3x - 2} - \frac{13}{(2 - 3x)^2}.$

Answer [3]

8

(b) Solve $\frac{5}{x+4} - \frac{2}{x-2} = 3$.

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [4]

9

- 5 (a) A die with 6 faces numbered 1 to 6 is tossed and the scores are recorded in the table below.

Score	1	2	3	4	5	6
Frequency	5	9	5	6	x	7

- (i) Calculate the value of x if the mean score is 3.4.

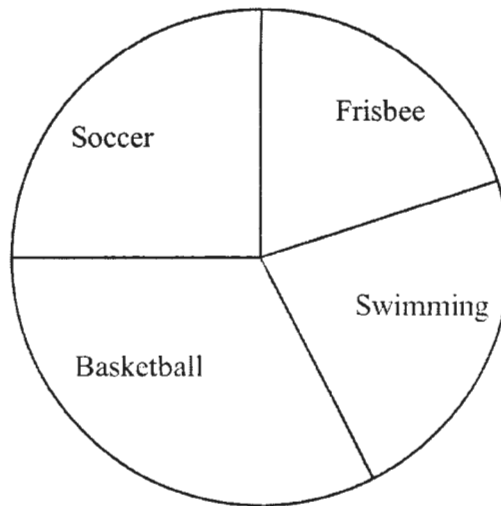
Answer $x = \dots\dots\dots$ [2]

- (ii) If the median score is 4, find the minimum value of x .

Answer $x = \dots\dots\dots$ [2]

10

(b)



The **accurate** pie chart above shows the different sports that a group of students enjoy playing during their free time.

130 of the students enjoy playing basketball during their free time.
Find the number of students that enjoy playing frisbee.

Answer [2]

11

- 6 (a) An area of 40 cm^2 on a map represents an actual area of 360 km^2 .
(i) Find the scale of the map in the form $1 : n$.

Answer $1 : \dots\dots\dots$ [2]

- (ii) The area of Singapore is 81 cm^2 on the map.
Find the actual area, in square metres, of Singapore.

Answer $\dots\dots\dots \text{m}^2$ [2]

- (b) The first four terms in a sequence of numbers $T_1, T_2, T_3, \dots, T_n$ are given below.

$$T_1 = 5^2 - 2$$

$$T_2 = 5^3$$

$$T_3 = 5^4 + 2$$

$$T_4 = 5^5 + 4$$

- (i) Write down an expression for T_5 .

Answer $T_5 = \dots\dots\dots$ [1]

- (ii) Find an expression, in terms of n , for T_n .

Answer T_n
 $= \dots\dots\dots$ [2]

- (iii) T_n and T_{n+1} are consecutive terms in the sequence.

Show that $T_{n+1} - T_n = 4(5^{n+1}) + 2$.

Answer

[3]

- (iv) Explain why $T_{n+1} - T_n$ is always even.

Answer
 [1]

13

- 7 In September 2013, Taylor changed 500 Singapore Dollars (S\$) to British Pound (£) when the exchange rate was $\text{£}1 = \text{S\$ } x$.

- (a) Write down an expression, in terms of x , for the amount of British Pounds she received.

Answer $\text{£} \dots\dots\dots$ [1]

- (b) In September 2023, Taylor changed 500 Singapore Dollars to British Pound. The exchange rate was $\text{£}1 = \text{S\$ } (x - 2)$.

Write down an expression, in terms of x , for the amount of British Pounds she received in September 2023.

Answer $\text{£} \dots\dots\dots$ [1]

- (c) Taylor received £180 more in 2023 than in 2013.
Write down an equation to represent this information and show that it simplifies to $9x^2 - 18x - 50 = 0$.

Answer

[3]

14

- (d) Solve the equation $9x^2 - 18x - 50 = 0$.

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]

- (e) In 2023, Taylor rented a room in London at a cost of £870 per month.
Calculate the total amount of rent, in Singapore dollars, Taylor paid in September.
Give you answer to the nearest dollar.

Answer S\$ = $\dots\dots\dots$ [2]

- 8 The variables x and y are connected by the equation $y = \frac{1}{2x^2} + \frac{1}{10}x^2 - 3$.

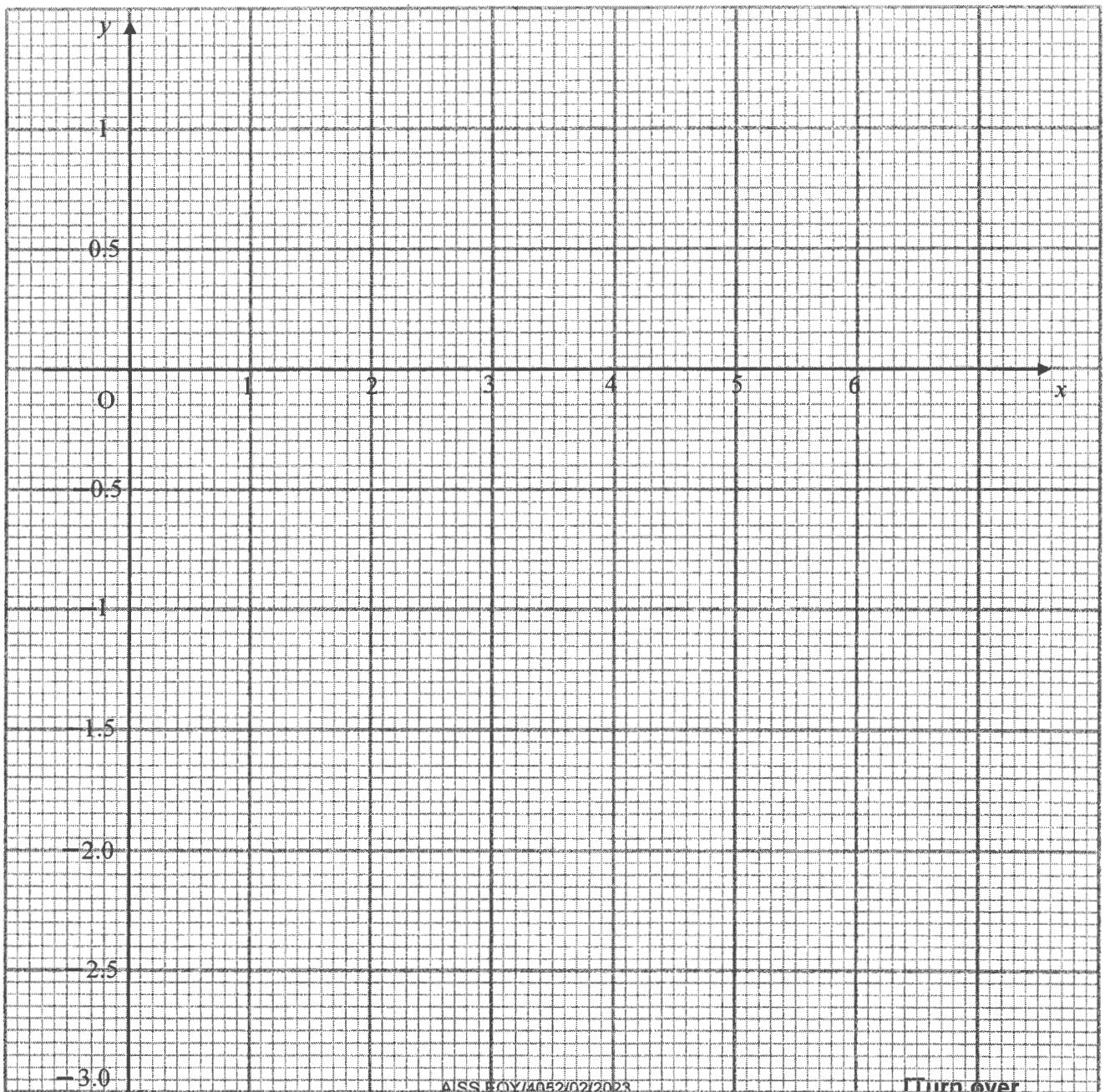
The table below shows some corresponding values of x and y , correct to 2 decimal places.

x	0.5	1	1.5	2	3	4	5	6
y	-0.98	-2.40	-2.55	-2.48	-2.04	p	-0.48	0.61

- (a) Calculate the value of p .

Answer $p = \dots\dots\dots$ [1]

- (b) On the grid below, draw the graph of $y = \frac{1}{2x^2} + \frac{1}{10}x^2 - 3$ for $0.5 \leq x \leq 6$. [3]



- (c) By drawing a tangent, find the gradient of the curve at $x = 2$.

Answer [2]

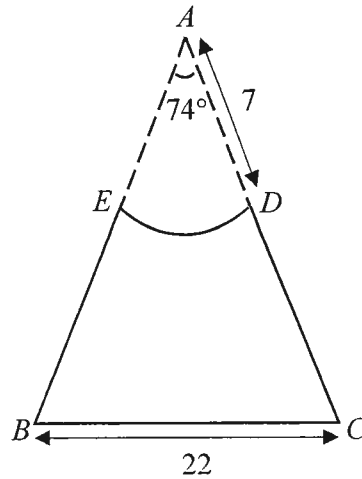
- (d) By drawing a suitable straight line on the same axes, solve the equation

$$\frac{1}{2x^2} + \frac{1}{10}x^2 - x = 0.$$

Answer $x =$ [4]

17

9



A sector of a circle radius 7 cm is removed from an isosceles triangle ABC as shown in the diagram.

Angle EAD is 74° and $BC = 22$ cm.

- (a) Calculate the length of the arc ED .

Answer cm [2]

- (b) (i) Show that the perpendicular height of the triangle ABC is 14.597 cm correct to 5 significant figures.

Answer

[1]

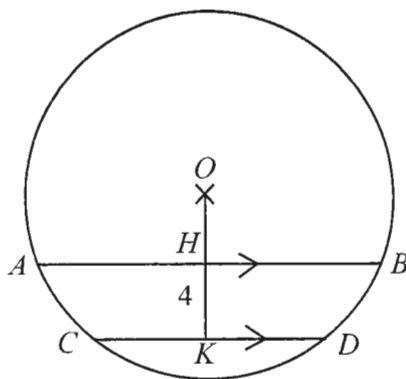
- (ii) Hence find the area of $BCDE$.

Answer cm^2 [3]

- (c) Given that the portion $BCDE$ is cut from a sheet of metal 4 mm thick, calculate, in cm^3 , the volume of the metal used.

Answer cm^3 [2]

10 (a)

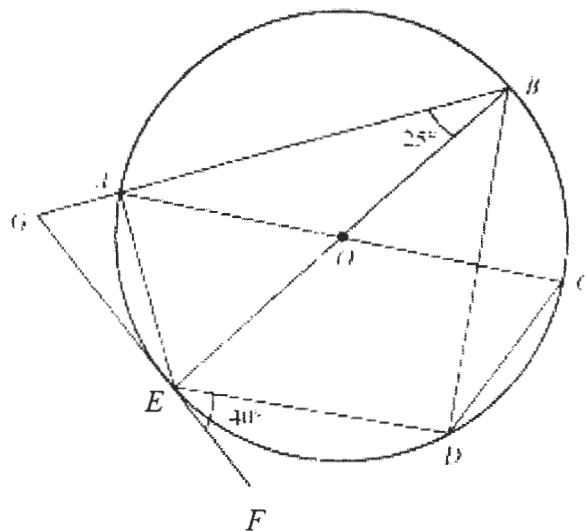


In the figure, AB and CD are parallel chords in a circle with center O .
 H and K are the midpoints of AB and CD respectively.
 Given that $AB = 26$ cm, $CD = 22$ cm and $HK = 4$ cm, find the length of OH .

Answer cm [3]

20

(b)



A, B, C, D and E are points on the circle, centre O .

AC and BE are diameters of the circle.

GF is a tangent to circle at E .

Angle $ABE = 25^\circ$ and angle $DEF = 40^\circ$.

Find, giving reasons for each answer,

(i) angle DBE ,

Answer $^\circ$ [2]

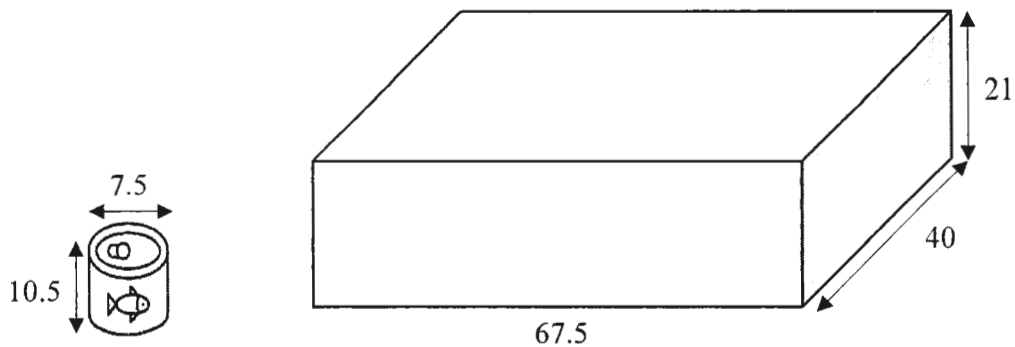
(ii) angle COE ,

Answer $^\circ$ [2]

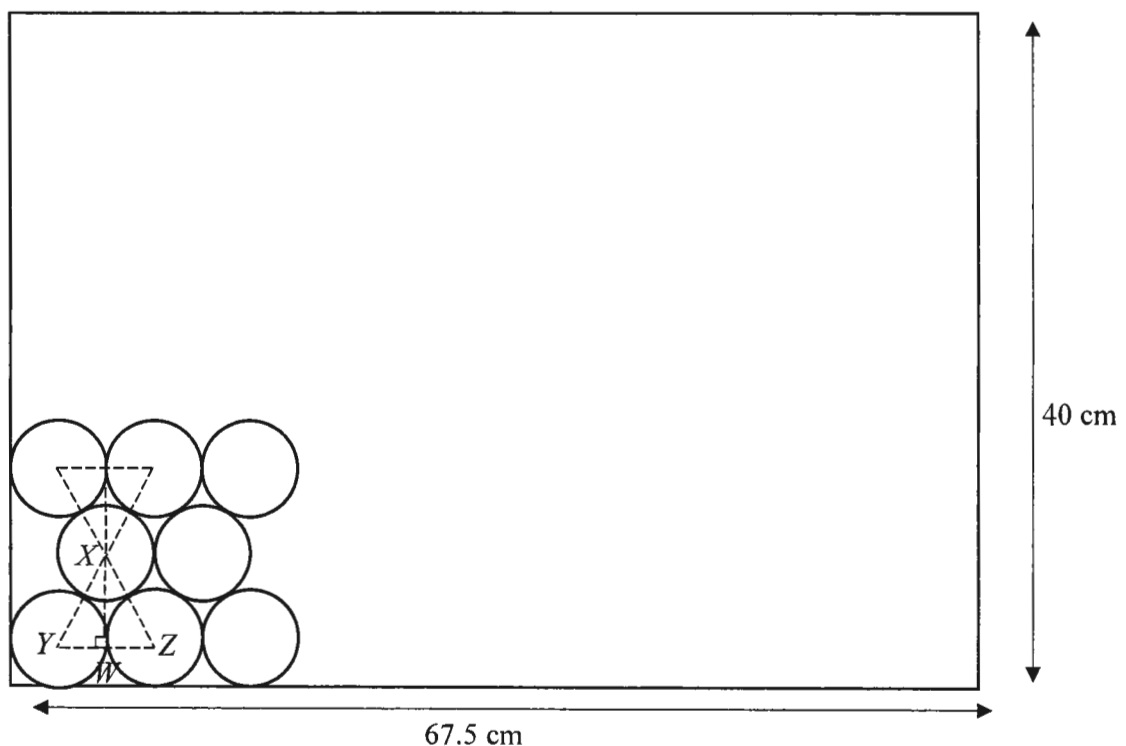
(iii) angle CDE ,

Answer $^\circ$ [2]

- 11 Canned food are packed into boxes for shipping.
 The external diameter and height of each can are 7.5 cm and 10.5 cm respectively.
 The internal length, breadth and height of the box are 67.5 cm, 40 cm and 21 cm respectively.



There are two layers of cans in each box, one directly on top of the other.
 The diagram below shows the arrangement of the cans in the box from the top view.



Each of the vertices of the equilateral triangle XYZ lies at the centre of the respective circles.

W is a point on YZ such that angle $XWY = 90^\circ$.

- (a) Find the length of XW , giving your answer to 4 significant figures.

Answer cm [2]

- (b) Show that the maximum number of cans that can be placed in the box is 102 cans.

[2]

- (c) A company uses corrugated carton boxes to transport the boxes of canned food.

There are four different types of corrugated composition that the company can choose from.

Types of Corrugated Composition	Maximum Load	Cost per unit (\$)
Single Face	Up to 9 kg	2.50
Single Wall	Up to 36 kg	3.00
Double Wall	Up to 67 kg	5.50
Triple Wall	Up to 100 kg	8.00

More information about the content of the canned food is given below:

- Density of the content: 1.09 g/cm^3
- Weight of an empty can: 120 g

Suggest which corrugated composition the company should use.

Justify the decision you make with clear calculations and state an assumption you made.

Answer

[6]

End of Paper



**AHMAD IBRAHIM SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2023**

SECONDARY 3 EXPRESS

Name:	Class:	Register No.:
MARKING SCHEME		

MATHEMATICS
Paper 2

4052/02
4 October 2023
2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any questions it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

For Examiner's Use

/ 90

This document consists of **21** printed pages.

Mathematical Formulae*Compound Interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1 A Mathematics competition scores students in three sections: Algebra, Geometry and Statistics.

The maximum score of each section is 100.

Grades are awarded to students based on their mean score attained from the three sections.

Range of mean scores from 3 sections	Grade
More than 85 but less than or equal to 100	A
More than 75 but less than or equal to 85	B
More than 65 but less than or equal to 75	C

Maddy scored 74 marks for Algebra, 81 marks for Geometry and x marks for Statistics.

Hariz claimed that Maddy needs to score more than 67 in the Statistics section to obtain a Grade B.

By forming an inequality, justify if Hariz's claim is true.

Answer

$$75 < \frac{74 + 81 + x}{3} \leq 85$$

B1 for finding correct mean

M1 (set up inequality with correct sign)

$$225 < 155 + x \leq 255$$

$$70 < x \leq 100$$

M1

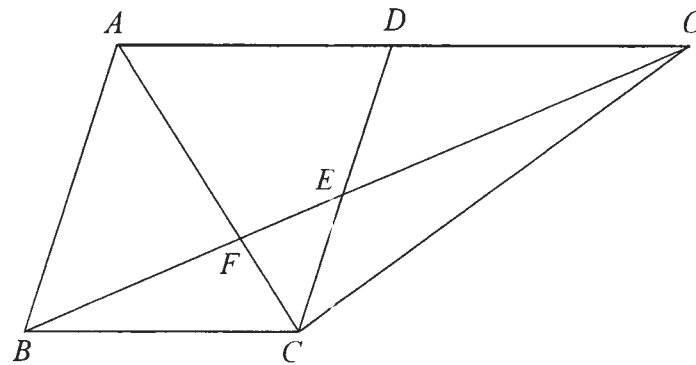
Hariz is not correct as the range of marks for Maddy to obtain a Grade B is to get more than 70 and not more than 67 marks.

A1

[4]

4

2



The diagram shows a rhombus $ABCD$.
 O is on AD produced such that $AO = 2 AD$.
 BO intersects AC and DC at F and E respectively.

- (a) Show that triangle ODE and BCE are congruent.
 Give a reason for each statement you make.

Answer

Answer

Since D is midpoint of AO ,

$AO = 2 AD = AD + OD$

Since $BCDF$ is a rhombus, $AD = BC = OD$

M1

$BC = OD$

$\angle CEB = \angle DEO$ (Vert opp angles)

$\angle BCE = \angle ODE$ (alt angles, $CD \parallel OF$)

M1

triangle $CDE \equiv$ triangle ODE (AAS congruency test)

A1

[3]

5

- (b) Show that triangles ECF and BAF are similar.
Give a reason for each statement you make.

Answer

Answer

$\angle EFC = \angle AFB$ (Vert opp angles)

$\angle ECF = \angle BAF$ (alt angles, $AB \parallel DC$)

or $\angle CEF = \angle ABF$ (alt angles, $AB \parallel DC$)

M1

Since all 3 pairs of corresponding angles are equal, triangle ECF and BAF are similar by AA similarity test.

A1

[2]

- (c) Find area of $\triangle ECF$: area of $\triangle BAF$.

Area of $\triangle ECF$: Area of $\triangle BAF$

1 : 2²

1 : 4

B1

Answer [1]

3 (a) Simplify

(i) $\sqrt{\frac{x^5}{y}} \times \frac{y^{-3}}{x^6},$

$$= \frac{x^{\frac{5}{2}}}{y^{\frac{1}{2}}} \times \frac{1}{x^6 y^3} \quad \text{M1}$$

$$= \frac{1}{x^{\frac{7}{2}} y^{\frac{7}{2}}} \quad \text{A1 (must be in positive index form)}$$

Answer [2]

(ii) $\left(\frac{343x^3}{y^6}\right)^{\frac{1}{3}} \times 2x^0 y^2.$

$$= \frac{7x}{y^2} \times 2y^2 \quad \text{M1}$$

$$= 14x \quad \text{A1}$$

Answer [2]

(b) Solve $\left(\frac{1}{4}\right)^{3x-1} = 8^x.$

$$(2^{-2})^{3x-1} = 2^{3x} \quad \text{M1 (rewrite the equation as base 2)}$$

$$2^{-6x+2} = 2^{3x}$$

$$-6x+2 = 3x \quad \text{M1}$$

$$-9x = -2$$

$$x = \frac{2}{9} \quad \text{A1}$$

Answer $x =$ [3]

- 4 (a) Write as a single fraction in its simplest form

$$\begin{aligned}
 \text{(i)} \quad & \frac{16x^2-1}{y^2} \div \frac{4x-1}{xy} \\
 &= \frac{(4x-1)(4x+1)}{y^2} \div \frac{4x-1}{xy} \quad \text{M1 (for correctly factorising } 16x^2-1\text{)} \\
 &= \frac{(4x-1)(4x+1)}{y^2} \times \frac{xy}{4x-1} \\
 &= \frac{x(4x+1)}{y} \quad \text{A1}
 \end{aligned}$$

Answer [2]

$$\begin{aligned}
 \text{(ii)} \quad & \frac{3}{3x-2} - \frac{13}{(2-3x)^2}, \\
 &= \frac{3}{-(2-3x)} - \frac{13}{(2-3x)^2} \quad \text{M1} \\
 &= \frac{-3(2-3x)-13}{(2-3x)^2} \quad \text{M1} \\
 &= \frac{9x-19}{(2-3x)^2} \quad \text{A1}
 \end{aligned}$$

OR

$$\begin{aligned}
 &= \frac{3}{3x-2} - \frac{13}{(3x-2)^2} \quad \text{Or M1} \\
 &= \frac{3(3x-2)-13}{(3x-2)^2} \quad \text{Or M1} \\
 &= \frac{9x-19}{(3x-2)^2} \quad \text{Or A1}
 \end{aligned}$$

Answer [3]

(b) Solve $\frac{5}{x+4} - \frac{2}{x-2} = 3$.

$$\frac{5}{x+4} - \frac{2}{x-2} = 3.$$

$$\frac{5(x-2) - 2(x+4)}{(x+4)(x-2)} = 3 \quad \text{M1}$$

$$\frac{3x-18}{(x+4)(x-2)} = 3$$

$$3(x^2 + 2x - 8) = 3x - 18$$

$$3x^2 + 3x - 6 = 0 \quad \text{M1}$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0 \quad \text{M1 (factorise/ multiplication frame)}$$

$$x = -2 \text{ or } x = 1 \quad \text{A1 (both correct answers)}$$

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [4]

- 5 (a) A die with 6 faces numbered 1 to 6 is tossed and the scores are recorded in the table below.

Score	1	2	3	4	5	6
Frequency	5	9	5	6	x	7

- (i) Calculate the value of x if the mean score is 3.4.

$$\frac{5+18+15+24+5x+42}{5+9+5+6+x+7} = 3.4 \quad \text{M1}$$

$$\frac{104+5x}{32+x} = 3.4$$

$$104+5x = 108.8 + 3.4x$$

$$1.6x = 4.8$$

$$x = 3 \quad \text{A1}$$

Answer $x = \dots\dots\dots$ [2]

- (ii) If the median score is 4, find the minimum value of x .

$$5+9+5 = 5+x+7 \quad \text{B1}$$

$$19 = 12+x$$

$$x = 7 \quad \text{B1}$$

Answer $x = \dots\dots\dots$ [2]

10

(b)



The **accurate** pie chart above shows the different sports that a group of students enjoy playing during their free time.

130 of the students enjoy playing basketball during their free time.
Find the number of students that enjoy playing frisbee.

117° represent 130 students

M1

1° represent $\frac{130}{117}$

(Frisbee) 72° represent $\frac{130}{117} \times 72 = 80$ students

A1

Answer [2]

- 6 (a) An area of 40 cm^2 on a map represents an actual area of 360 km^2 .
 (i) Find the scale of the map in the form $1 : n$.

$$40 \text{ cm}^2 : 360 \text{ km}^2$$

$$1 \text{ cm}^2 : 9 \text{ km}^2$$

M1

$$1 \text{ cm} : 3 \text{ km}$$

$$1 : 300000$$

A1

Answer 1 : [2]

- (ii) The area of Singapore is 81 cm^2 on the map.
 Find the actual area, in square metres, of Singapore.

$$1 \text{ cm}^2 : 9 \text{ km}^2$$

$$81 \text{ cm}^2 : 729 \text{ km}^2$$

M1

$$729 \text{ km}^2$$

$$= 729 \times 10^6 \text{ m}^2$$

$$= 7.29 \times 10^8 \text{ m}^2$$

A1

Answer m^2 [2]

- (b) The first four terms in a sequence of numbers $T_1, T_2, T_3, \dots, T_n$ are given below.

$$T_1 = 5^2 - 2$$

$$T_2 = 5^3$$

$$T_3 = 5^4 + 2$$

$$T_4 = 5^5 + 4$$

- (i) Write down an expression for T_5 .

$$T_5 = 5^6 + 6 \quad \text{B1}$$

Answer $T_5 = \dots\dots\dots$ [1]

- (ii) Find an expression, in terms of n , for T_n .

$$T_n = 5^{n+1} + 2n - 4$$

$$\text{or } T_n = 5^{n+1} + 2(n-2)$$

B1 (if student gets $2n - 4$)

B1 (if students gets 5^{n+1})

Answer T_n [2]

$= \dots\dots\dots$

- (iii) T_n and T_{n+1} are consecutive terms in the sequence.

Show that $T_{n+1} - T_n = 4(5^{n+1}) + 2$.

Answer

$$T_{n+1} - T_n$$

M1 for finding T_{n+1}

$$= [5^{n+2} + 2(n+1) - 4] - [5^{n+1} + 2n - 4]$$

M1 for subtracting

$$= 5^{n+1}(5-1) + 2n - 2n + 2 - 4 + 4$$

$$= 4(5^{n+1}) + 2$$

A1

[3]

- (iv) Explain why $T_{n+1} - T_n$ is always even.

$$T_{n+1} - T_n$$

$$= 4(5^{n+1}) + 2$$

$$= 2[2(5^{n+1}) + 1]$$

[1]

Since $T_{n+1} - T_n$ is a multiple of 2 where $2(5^{n+1}) + 1$ is an integer for positive integers of n , $T_{n+1} - T_n$ will always be even. B1

- 7 In September 2013, Taylor changed 500 Singapore Dollars (S\$) to British Pound (£) when the exchange rate was £1 = S\$ x .

- (a) Write down an expression, in terms of x , for the amount of British Pounds she received.

$$\frac{500}{x} \quad \text{—————} \quad \boxed{\text{B1}}$$

Answer £..... [1]

- (b) In September 2023, Taylor changed 500 Singapore Dollars to British Pound again. The exchange rate was £1 = S\$ $(x - 2)$.

Write down an expression, in terms of x , for the amount of British Pounds she received in September 2023.

$$\frac{500}{x-2} \quad \text{—————} \quad \boxed{\text{B1}}$$

Answer £..... [1]

- (c) Taylor received £180 more in 2023 than in 2013. Write down an equation to represent this information and show that it simplifies to

$$9x^2 - 18x - 50 = 0.$$

Answer

$$\frac{500}{x-2} - \frac{500}{x} = 180 \quad \text{—————} \quad \boxed{\text{M1 (allow ECF)}}$$

$$\frac{500x - 500(x-2)}{x(x-2)} = 180$$

$$\frac{1000}{x^2 - 2x} = 180 \quad \text{—————} \quad \boxed{\text{M1}}$$

$$180x^2 - 360x - 1000 = 0$$

$$9x^2 - 18x - 50 = 0 \quad \text{—————} \quad \boxed{\text{A1}}$$

[3]

- (d) Solve the equation $9x^2 - 18x - 50 = 0$.

$$9x^2 - 18x - 50 = 0$$

$$x = \frac{18 \pm \sqrt{(-18)^2 - 4(9)(-50)}}{2(9)}$$

M1 (correct formula and substitution of values)

$$x = \frac{18 \pm \sqrt{2124}}{18}$$

$$x = 3.5604 \text{ or } x = -1.5604$$

$$x = 3.56 \text{ (3 sf) or } x = -1.56 \text{ (3 sf)}$$

A2 (A1 each)

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]

- (e) In 2023, Taylor rented a room in London at a cost of £870 per month. Calculate the total amount of rent, in Singapore dollars, Taylor paid in September. Give your answer to the nearest dollar.

Reject $x = -1.56$

Exchange rate in September 2023

£1 = S\$ 1.5604

M1

Rent in SGD

$$= 870 \times 1.5604$$

= S\$1358

A1

(nearest dollars)

Answer S\$ = [2]

- 8 The variables x and y are connected by the equation $y = \frac{1}{2x^2} + \frac{1}{10}x^2 - 3$.

The table below shows some corresponding values of x and y , correct to 2 decimal places.

x	0.5	1	1.5	2	3	4	5	6
y	-0.98	-2.40	-2.55	-2.48	-2.04	p	-0.48	0.61

- (a) Calculate the value of p .

$$p = -1.37$$

B1

Answer $p = \dots\dots\dots$ [1]

- (b) On the grid below, draw the graph of $y = \frac{1}{2x^2} + \frac{1}{10}x^2 - 3$ for $0.5 \leq x \leq 6$. [3]

(Refer to Graph Paper)

- (c) By drawing a tangent, find the gradient of the curve at $x = 2$.

$$\begin{aligned} &\text{Gradient} \\ &\frac{-2.7+2.1}{1-4} \\ &= 0.2 \end{aligned}$$

Draw Tangent Correct **-B1**

Calculate Tangent (Ans: 0.275 ± 0.1) **-B1**

Answer [2]

(d) By drawing a suitable straight line on the same axes, solve the equation

$$\frac{1}{2x^2} + \frac{1}{10}x^2 - x = 0.$$

$$\frac{1}{2x^2} + \frac{1}{10}x^2 - x = 0$$

$$\frac{1}{2x^2} + \frac{1}{10}x^2 - 3 + 3 - x = 0$$

$$y = x - 3 \quad \text{--M1}$$

Draw the line $y = x - 3$ **-M2** (M1 for correct points, M1 for line drawn. Award M1 if student draw based on their graph)

$$x = 0.8 \pm 0.1 \quad \text{--A1}$$

Answer $x =$ [4]

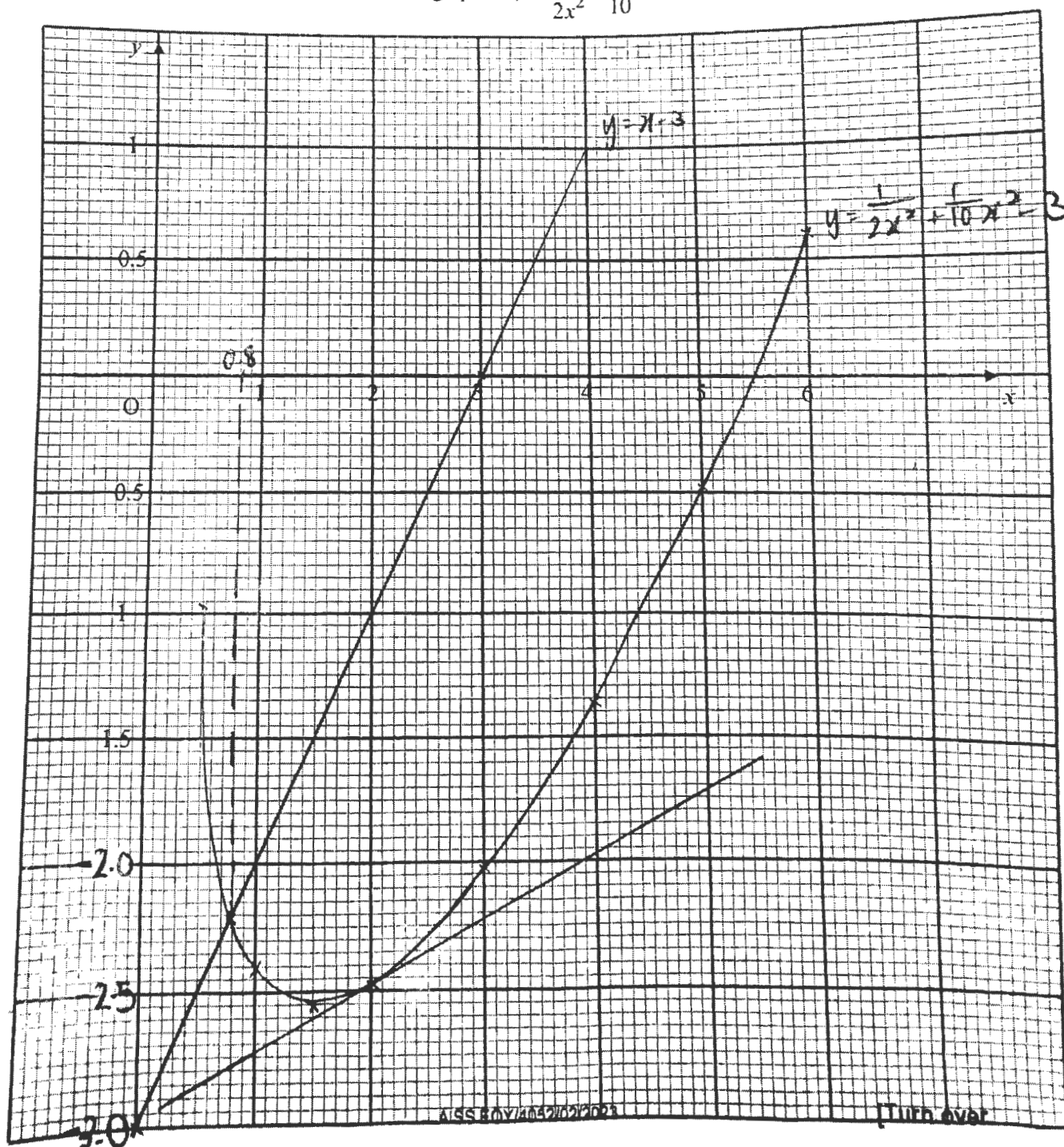
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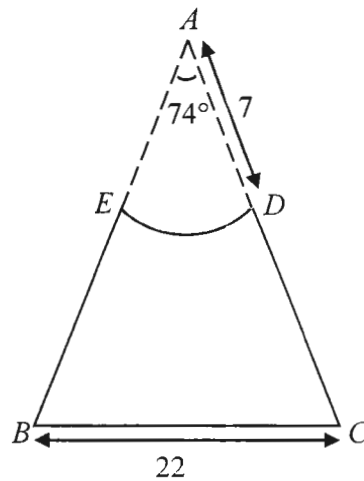
x	0.5	1	1.5	2	3	4	5	6
y	-0.98	-2.40	-2.55	-2.48	-2.04	p	-0.48	0.61

- (a) Calculate the value of p .

Answer $p = \dots\dots\dots -1.37 \dots\dots\dots$ [1]

- (b) On the grid below, draw the graph of $y = \frac{1}{2x^2} + \frac{1}{10}x^2 - 3$ for $0.5 \leq x \leq 6$. [3]





A sector of a circle radius 7 cm is removed from an isosceles triangle ABC as shown in the diagram.

Angle EAD is 74° and $BC = 22$ cm.

- (a) Calculate the length of the arc ED .

$$\frac{74^\circ}{360^\circ} \times 2\pi(7) \quad \text{--- M1}$$

$$= 9.0408$$

$$= 9.04 \text{ cm (3 sf)} \quad \text{--- A1}$$

Answer cm [2]

- (b) (i) Show that the perpendicular height of the triangle ABC is 14.597 cm in 5 significant figures.

Answer

Perpendicular height of triangle, h

$$\tan 37^\circ = \frac{11}{h}$$

$$h = \frac{11}{\tan 37^\circ}$$

$$h = 14.597 \text{ cm} \quad \text{--- B1}$$

[1]

(ii) Hence find the area of $BCDE$.

Area of sector EAD

$$= \frac{74^\circ}{360^\circ} \times \pi(7^2)$$

$$= \frac{1813}{180} \pi \text{ or } 31.6428 \text{ cm}^2 \quad \text{M1}$$

Area of $BCDE$

$$= \left(\frac{1}{2} \times 22 \times 14.5975\right) - \frac{1813}{180} \pi \quad \text{M1}$$

$$= 128.9297 \text{ cm}^2$$

$$= 129 \text{ cm}^2 \quad \text{A1}$$

Answer cm^2 [3]

(c) Given that the portion $BCDE$ is cut from a sheet of metal 4 mm thick, calculate, in cm^3 , the volume of the metal used.

Volume

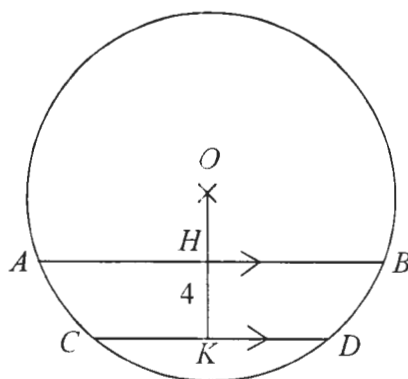
$$= 128.9297 \times 0.4 \quad \text{M1}$$

$$= 51.57188 \text{ cm}^3$$

$$= 51.6 \text{ cm}^3 \quad \text{A1}$$

Answer cm^3 [2]

10 (a)



In the figure, AB and CD are parallel chords in a circle with center O .
 H and K are the midpoints of AB and CD respectively.
 Given that $AB = 26$ cm, $CD = 22$ cm and $HK = 4$ cm, find the length of OH .

the length of OH ,

$$AH = 13\text{cm}$$

$$CK = 11\text{cm}$$

B1

$$\angle OHA = \angle OKC = 90^\circ (\perp \text{ bisector of chord})$$

$$OA = OC$$

$$OA^2 = OC^2$$

$$13^2 + OH^2 = 11^2 + (OH + 4)^2$$

M1

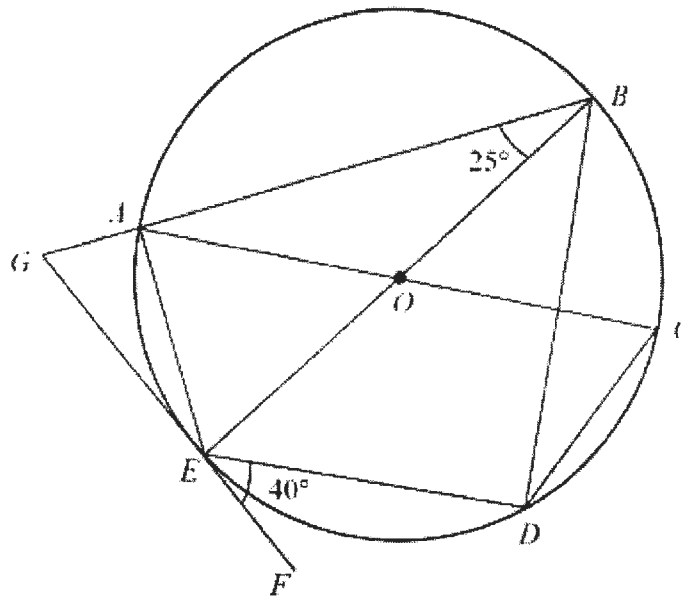
$$169 + OH^2 = 121 + OH^2 + 8OH + 16$$

$$8OH = 32$$

$$OH = 4\text{cm}$$

A1

Answer cm [3]



A, B, C, D and E are points on the circle, centre O .
 AC and BE are diameters of the circle.
 GF is a tangent to circle at E .
 Angle $ABE = 25^\circ$ and angle $DEF = 40^\circ$.

- (b) Find, giving reasons for each answer,
 (i) angle DBE ,

$$\angle OEF = 90^\circ \text{ (tangent } \perp \text{ radius)}$$

$$\angle BED = 90^\circ - 40^\circ = 50^\circ$$

M1

$$\angle BDE = 90^\circ \text{ (rt } \angle \text{ in semicircle)}$$

$$\angle DBE$$

$$= 180^\circ - 50^\circ - 90^\circ \text{ (} \angle \text{ sum of } \Delta \text{)}$$

$$= 40^\circ$$

A1

Answer $^\circ$ [2]

- (ii) angle COE ,

$$\angle OAB = 25^\circ \text{ (base } \angle \text{ of isoc triangle)}$$

$$\angle AOB = 180^\circ - 50^\circ = 130^\circ \text{ (} \angle \text{ sum of } \Delta \text{)}$$

$$\angle AOB = \angle COF = 130^\circ \text{ (vert opp angles)}$$

OR

$$\angle OAB = 25^\circ \text{ (base } \angle \text{ of isoc triangle)}$$

$$\angle CAE = 90^\circ - 25^\circ \text{ (rt. } \angle \text{ in semicircle)}$$

$$\angle COE = 2 \times 65^\circ \text{ (} \angle \text{ at centre} = 2 \angle \text{ at circumference)}$$

$$= 130^\circ$$

Or A1

M1

A1

Or M1

21

Answer° [2]

(iii) angle CDE ,

$$\text{Reflex } \angle COE = 360^\circ - 130^\circ = 230^\circ (\angle \text{ at the point})$$

M1

$$\angle CDE = 230 \div 2 = 115^\circ (\angle \text{ at centre} = 2\angle \text{ at circumference})$$

A1

OR

$\angle CDE$

$$= 180^\circ - \angle CAE (\angle \text{ in opp segments})$$

$$= 180^\circ - 65^\circ$$

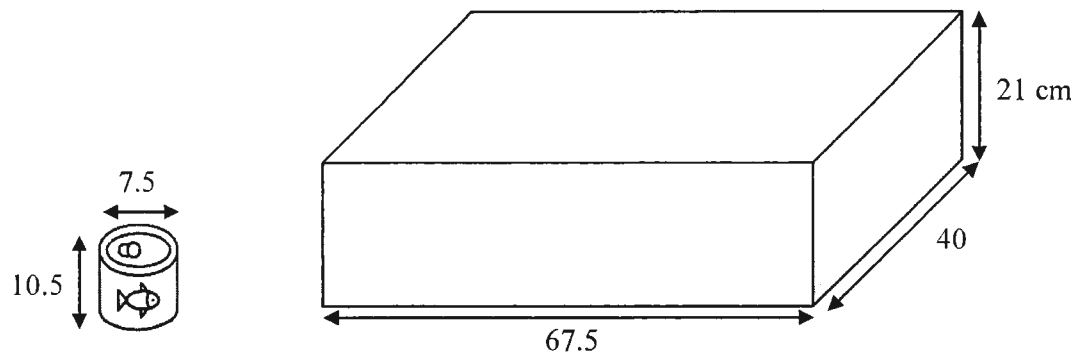
Or M1

$$= 115^\circ$$

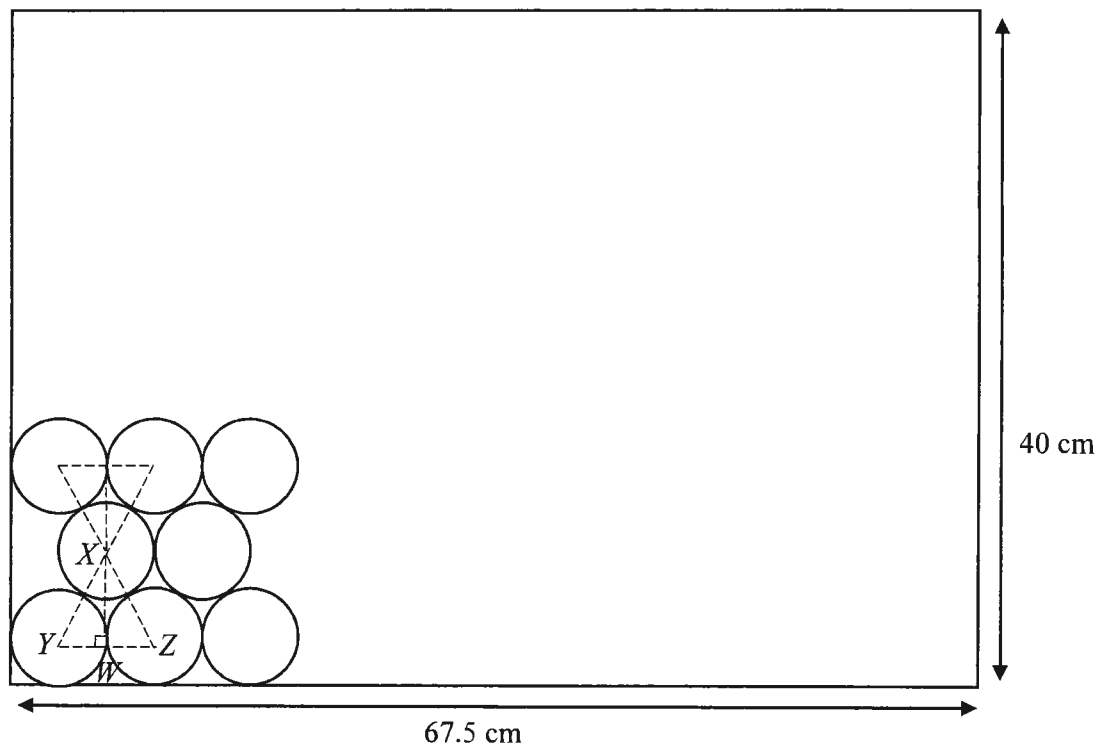
Or A1

Answer° [2]

- 11** Canned food are packed into boxes for shipping.
 The external diameter and height of each can are 7.5 cm and 10.5 cm respectively.
 The internal length, breadth and height of the box are 67.5 cm, 40 cm and 21 cm respectively.



There are two layers of cans in each box, one directly on top of the other.
 The diagram below shows the arrangement of the cans in the box from the top view.



Each of the vertices of the equilateral triangle XYZ lies at the centre of the respective circles.

W is a point on YZ such that angle $XWY = 90^\circ$.

- (a) Find the length of XW , giving your answer to 4 significant figures.

$$YW = 3.75 \text{ cm}$$

$$\tan 60^\circ = \frac{XW}{3.75}$$

$$XW = 6.495 \text{ cm}$$

M1

A1

Answer cm [2]

- (b) Show that the maximum number of cans that can be placed in the box is 102 cans.

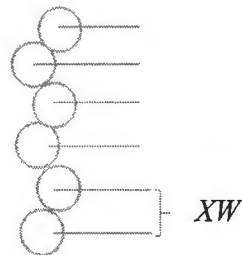
Max number of XW based on breadth

$$= \frac{40 - 3.75 - 3.75}{6.4952}$$

$$\approx 5$$

Max number of rows of cans based on breadth

$$= 6$$



Odd rows:

$$= \frac{67.5}{7.5}$$

$$= 9$$

Even rows

$$= 8$$

B1

Total cans in box

$$= 2(3 \times 9 + 3 \times 8)$$

$$= 102$$

B1

Answer [2]

- (c) A company uses corrugated carton boxes to transport the boxes of canned food.

There are four different types of corrugated composition that the company can choose from.

Types of Corrugated Composition	Maximum Load	Cost per unit (\$)
Single Face	Up to 9 kg	2.50
Single Wall	Up to 36 kg	3.00
Double Wall	Up to 78 kg	5.50
Triple Wall	Up to 115 kg	8.00

More information about the content of the canned food is given below:

- Density of the content: 1.09 g/cm³
- Weight of an empty can: 120 g

Suggest which corrugated composition the company should use.

Justify the decision you make with clear calculations and state an assumption you made.

Answer

Volume of 1 canned food

$$= \pi(3.75^2)(10.5)$$

$$= 463.8758 \text{ cm}^3$$

M1

Weight of 1 canned food + weight can

$$= 463.8758 \times 1.09 + 120 \text{ g}$$

$$= 625.6246 \text{ g}$$

$$= 0.6256 \text{ kg}$$

M2 (calculate 1 canned food M1, add weight of can M2). Allow ECF

Total weight of 102 cans

$$= 102 \times 0.6256$$

$$= 63.8112 \text{ kg}$$

M1 (Allow ecf)

The company should use **double wall corrugated composition** as it is the cheapest option and can handle the required load where the weight of carton box with canned 63.8112 kg < the maximum load of the corrugated composition of the box.

A1 (State the option with clear justifications)

Assumption

- Assume that the weight of the box is negligible.
- Assume that all the canned food are consistent in weight.
- Assume that the canned food items are always pack in the same arrangement.

A1 (any 1 acceptable assumption)

End of Paper