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Class	Register Number	Name
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南洋女子中学校
NANYANG GIRLS' HIGH SCHOOL

End-of-Year Examination 2015
Secondary Three

INTEGRATED MATHEMATICS 1

2 hours

Monday

12 October 2015

0845 – 1045

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Answer questions 1 – 10 before attempting the Bonus Question.
2. Answer the whole of question 8 on the graph paper provided.
3. Write your answers and working on the separate writing paper provided.
4. Write your name, register number and class on each separate sheet of paper that you use and fasten the sheets together with the string provided. Do not staple your answer sheets together.
5. Omission of essential workings will result in loss of marks.
6. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

1. The number of marks is given in brackets [] at the end of each question or part question.
2. The total number of marks for this paper is 80.
3. The use of an electronic calculator is expected, where appropriate.
4. You are reminded of the need for clear presentation in your answers.

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Mathematical Formulae

1. TRIGONOMETRY

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1 Given that $\cos A = -\frac{15}{17}$ and $0^\circ < A < 180^\circ$, find, without using a calculator,

the value of

(i) $\sin A$, [1]

(ii) $\tan(180^\circ - A)$, [2]

(iii) $\cos(A - 90^\circ)$. [2]

- 2 The following equations

(i) $y = \frac{a}{\sqrt{x}} - b\sqrt{x}$,

(ii) $y = 2 + ab^x$,

may be represented by a straight line in the form $Y = mX + c$.

a , b , m and c are constants and X and Y are each functions of x and/or y .

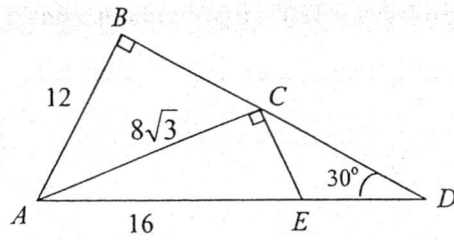
Copy the following table and insert in it an expression each for Y , X , m and c . [4]

	Y	X	m	c
$y = \frac{a}{\sqrt{x}} - b\sqrt{x}$	$y\sqrt{x}$			
$y = 2 + ab^x$				

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In the diagram, $\triangle ABD$ and $\triangle ACE$ are right-angled triangles. BCD and AED are straight lines. Given that $AB = 12$ cm, $AC = 8\sqrt{3}$ cm, $AE = 16$ cm and $\angle ADC = 30^\circ$,

- (a) find the exact value of $\sin \angle CED$, [2]
- (b) show that $\triangle CED$ is isosceles, [2]
- (c) find the radius of the circle through A , B and D . [2]

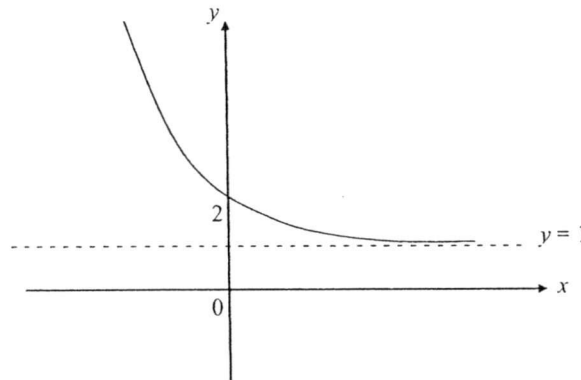
4

- (a) On different axes, sketch the graph of

(i) $y = -\frac{1}{2}x^3$, [2]

(ii) $y = ax^n$, where $a > 0$ and $n = -2$. [2]

- (b) Write down a possible equation for this graph, where y -intercept = 2 and asymptote is $y = 1$. [2]



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The variables x and y , where $x > 0$ and $y \geq 1$, are related in such a way that when $y^2 - x$ is plotted against y , a straight line graph is obtained.

Given that the line passes through the points $(3, 5)$ and $(1, 1)$,

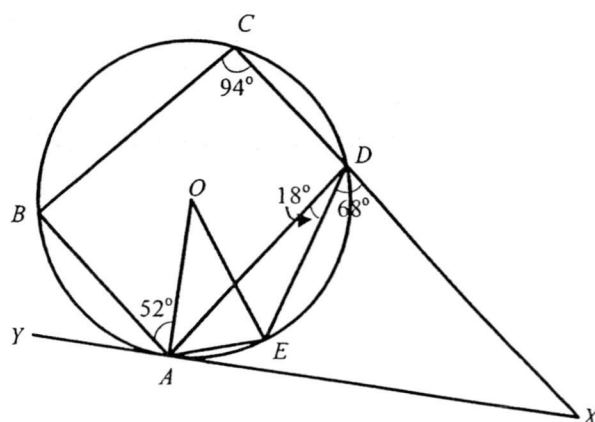
- (i) express y in terms of x , [4]
- (ii) find the value of y when $x = 9$. [1]

- 6 A circle, with centre C , passes through the points $P(-5, 0)$ and $Q(7, -12)$.

The point C lies on the line $y = 3x - 13$.

- (i) Find the equation of the perpendicular bisector of PQ . [4]
 (ii) Find the coordinates of C . [2]
 (iii) Find the equation of the circle. [2]

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The diagram shows a circle $ABCDE$ with centre O . XY is a tangent to the circle at A and CDX is a straight line. $\angle BAO = 52^\circ$, $\angle ADE = 18^\circ$, $\angle BCD = 94^\circ$ and $\angle EDX = 68^\circ$.

- (a) Stating your reasons clearly, find
 (i) $\angle AOE$, [2]
 (ii) $\angle XAE$. [2]
- (b) (i) Show that BA is parallel to CX . [2]
 (ii) Hence, or otherwise, find $\angle AXD$. [2]
- (c) Is $AODX$ a cyclic quadrilateral? Explain your answer clearly. [2]

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8 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = 2x - 5 + \frac{3}{x}.$$

Some corresponding values of x and y are given in the table below.

x	0.25	0.5	1	1.5	2	3	4	5	6
y	7.5	2	0	0	0.5	2	3.75	5.6	7.5

- (a) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x -axis for $0.25 \leq x \leq 6$ and a vertical y -axis for $-1 \leq y \leq 8$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

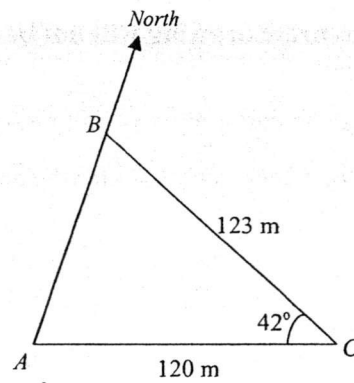
- (b) By drawing a tangent, find the gradient of the curve at $(2, 0.5)$. [2]

- (c) Use your graph to find the solutions to the equation $x - \frac{9}{2} + \frac{3}{2x} = 0$. [2]

- (d) Find the equation, in the form $7x^2 + Bx + C = 0$, which is satisfied by the x -coordinates of the points where the line $y = 4 - \frac{3}{2}x$ intersects the curve $y = 2x - 5 + \frac{3}{x}$. [2]

- (e) (i) On the same axes, draw the line $y = 3 - x$ for $0 \leq x \leq 3$. [1]

- (ii) State the value of c such that $2x - 5 + \frac{3}{x} = -x + c$ has exactly 1 solution for $0.25 \leq x \leq 6$. [1]



Three markers, A , B and C , are placed on a horizontal field, where B is due north of A . $AC = 120$ m, $BC = 123$ m and $\angle ACB = 42^\circ$.

(a) Calculate

(i) AB ,

[2]

(ii) the bearing of C from A .

[3]

(b) A man standing at A is flying a drone. The drone, D , is 65 m vertically above B . Calculate the angle of depression of an object at C from the drone.

[2]

(c) The man starts to walk from A to C . Calculate his distance from B when his angle of elevation of D is the greatest.

[2]

(d) The marker at B is relocated along the path BC to a point E such that $AE = AB$. Calculate the area of $\triangle AEC$, giving your answer to 3 significant figures.

[3]

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[Turn over

10 Solutions to this question by accurate drawing will not be accepted.

The diagram below shows part of the curve $y = (x - 2)(x + 4)$ with its minimum point at T . The curve cuts the x -axis at R and S and intersects the line $y = -x + 10$ at P and Q .

(i) Find the coordinates of S , R and T . [3]

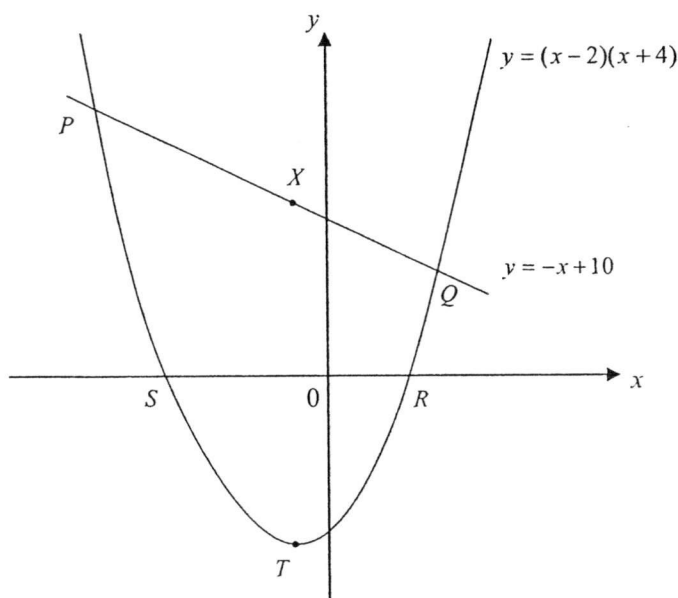
(ii) Find the coordinates of P and Q . [4]

(iii) Find the area of quadrilateral $PQRS$. [2]

The point X is such that $STRX$ is a kite and X lies on the line PQ .

(iv) Find the coordinates of X . [2]

(v) Find the value of $\frac{\text{area of } \triangle RQX}{\text{area of } \triangle RXP}$. [2]

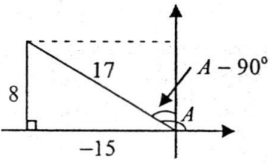


Bonus Question

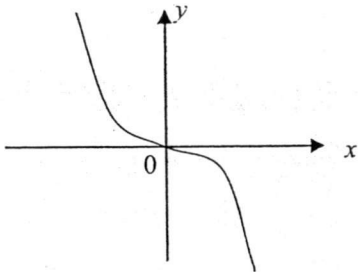
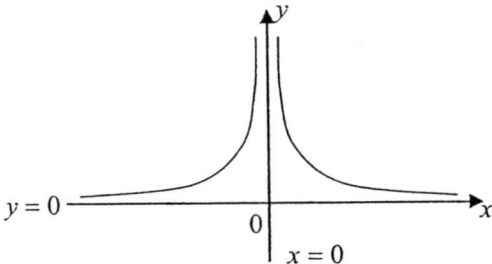
- 11 A region A is bounded by the lines $x = \pm 5$, $y = \pm 5$, $y = 2x + 10$ and $y = 2x - 10$. The largest circle with centre at $(0, 0)$ that can be fitted into region A has an area of $k\pi$ square units. Find the value of k . [3]

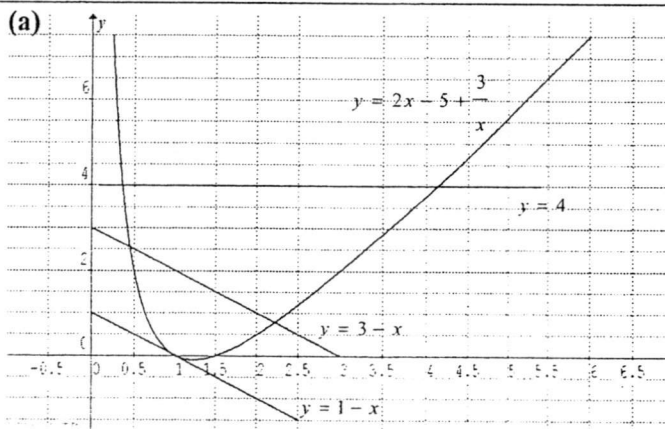
End of paper

2015 S3 EOY IM1 Solutions

No.																
1	(i) $\sin A = \frac{8}{17}$ (ii) $\tan(180^\circ - A) = -\tan A$ $= \frac{8}{15}$ (iii) $\cos(A - 90^\circ) = \sin A$ $= \frac{8}{17}$ 															
2	<table><tr><td></td><td>Y</td><td>X</td><td>m</td><td>c</td></tr><tr><td>$y = \frac{a}{\sqrt{x}} - b\sqrt{x}$</td><td>$y\sqrt{x}$ (given)</td><td>x</td><td>-b</td><td>a</td></tr><tr><td>$y = 2 + ab^x$</td><td>$\lg(y - 2)$ $\ln(y - 2)$</td><td>x</td><td>$\lg b$ $\ln b$</td><td>$\lg a$ $\ln a$</td></tr></table> $y = \frac{a}{\sqrt{x}} - b\sqrt{x}$ $y\sqrt{x} = -bx + a$ $y = 2 + ab^x$ $\lg(y - 2) = \lg ab^x$ $\lg(y - 2) = x \lg b + \lg a$		Y	X	m	c	$y = \frac{a}{\sqrt{x}} - b\sqrt{x}$	$y\sqrt{x}$ (given)	x	-b	a	$y = 2 + ab^x$	$\lg(y - 2)$ $\ln(y - 2)$	x	$\lg b$ $\ln b$	$\lg a$ $\ln a$
	Y	X	m	c												
$y = \frac{a}{\sqrt{x}} - b\sqrt{x}$	$y\sqrt{x}$ (given)	x	-b	a												
$y = 2 + ab^x$	$\lg(y - 2)$ $\ln(y - 2)$	x	$\lg b$ $\ln b$	$\lg a$ $\ln a$												
3 (a)	$\sin \angle AEC = \frac{8\sqrt{3}}{16} = \frac{\sqrt{3}}{2}$ $\sin \angle CED = \frac{\sqrt{3}}{2}$															
3 (b)	$\angle CED = 180^\circ - \sin^{-1}(\frac{\sqrt{3}}{2}) = 120^\circ$ (take obtuse $\angle$) $\Rightarrow \angle ECD = 30^\circ$ ($\angle$ sum of Δ) OR $\angle CEA = \sin^{-1}(\frac{\sqrt{3}}{2}) = 60^\circ$ $\Rightarrow \angle ECD = 30^\circ$ (ext. $\angle$ = sum of int. opp. $\angle$) OR $\angle BCA = \sin^{-1}(\frac{12}{8\sqrt{3}}) = 60^\circ$ $\Rightarrow \angle ECD = 30^\circ$ (adj. $\angle$s on a str. line) Since $\angle ECD = \angle CDE = 30^\circ$, ΔCED is isosceles															
3 (c)	Using the property rt $\angle$ in a semicircle, AD is the diameter of circle through A, B and D $\sin 30^\circ = \frac{12}{AD}$ OR $ED = EC = \sqrt{16^2 - (8\sqrt{3})^2} = 8 \text{ cm}$ $\Rightarrow AD = 24 \text{ cm}$ $\Rightarrow \text{radius} = 12 \text{ cm}$															

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4 (a) (i)	
4 (a) (ii)	
4 (b)	$y = a^{bx} + 1$
5 (i)	$\text{grad} = \frac{5-1}{3-1} = 2$ $Y - 1 = 2(X - 1)$ $Y = 2X - 1$ $y^2 - x = 2y - 1 \quad \text{OR} \quad y = \frac{2 \pm \sqrt{4 - 4(1-x)}}{2}$ $y^2 - 2y + 1 = x$ $(y-1)^2 = x$ $y = \sqrt{x} + 1 \quad (\text{rej } -\sqrt{x} + 1)$ $y = \frac{2 \pm \sqrt{4x}}{2} = 1 \pm \sqrt{x}$
5 (ii)	sub $x = 9$, $y = 4$
6 (i)	Let l be the perpendicular bisector of PQ $\text{grad of } PQ = \frac{-12-0}{7-(-5)} = -1 \Rightarrow \text{grad of } l = 1$ $\text{midpt of } PQ = \left(\frac{-5+7}{2}, \frac{0-12}{2} \right) = (1, -6)$ $y - (-6) = x - 1$ eqn of l is $y = x - 7$
6 (ii)	At pt C, $3x - 13 = x - 7 \Rightarrow 2x = 6$ $x = 3, y = -4$ $C(3, -4)$
6 (iii)	$CP = \sqrt{(-5-3)^2 + (0-(-4))^2} = \sqrt{80} \text{ units}$ eqn of circle is $(x-3)^2 + (y+4)^2 = 80$
7 (a)	(i) $\angle AOE = 36^\circ$ ($\angle$ at centre = $2\angle$ at $\odot^{\text{ce}}$) (ii) $\angle XAE = 18^\circ$ ($\angle$ in alt. seg)

7	$\angle ABC = 68^\circ + 18^\circ$ (ext. $\angle$ of cyclic quad) $= 86^\circ$ (b) (i) $\angle ABC + \angle BCD = 86^\circ + 94^\circ = 180^\circ$ By interior $\angle$ s, // lines, BA is parallel to CX <u>OR</u> $\angle BAD = 180^\circ - 94^\circ$ ($\angle$ s in opp. segment) $= 86^\circ$ $\angle BAD = \angle ADX = 86^\circ$ By alternate $\angle$ s, // lines, BA is parallel to CX
7	$\angle OAX = 90^\circ$ (tan $\perp$ rad) $\angle AXD = 180^\circ - 90^\circ - 52^\circ$ (interior $\angle$ s, $BA \parallel CX$) $= 38^\circ$ (b) (ii) <u>OR</u> $\angle BAY = 90^\circ - 52^\circ$ (tan $\perp$ rad) $= 38^\circ$ $\angle AXD = \angle BAY = 38^\circ$ (corr. $\angle$ s, $BA \parallel CX$)
7	(c) CX is not a tangent $\Rightarrow \angle ODX \neq 90^\circ$ ($\because$ tan $\perp$ rad) $\Rightarrow \angle OAX + \angle ODX \neq 180^\circ$ Since we can't use the reasoning $\angle$ s in opp. segment, $AODX$ is not a cyclic quadrilateral
8	(a)  (b) grad at $(2, 0.5) = 1.0$ to 1.4 (actual 1.25) (c) $x - \frac{9}{2} + \frac{3}{2x} = 0$ $2x - 5 + \frac{3}{x} = 4 \Rightarrow$ add in line $y = 4$ $x = 0.3 - 0.4, 4.1 - 4.2$ (d) $2x - 5 + \frac{3}{x} = 4 - \frac{3}{2}x$ $\frac{7}{2}x - 9 + \frac{3}{x} = 0$ $7x^2 - 18x + 6 = 0$ (e) (i) Draw $y = 3 - x$ (ii) $c = 1$
9	(a) $AB^2 = 123^2 + 120^2 - 2(123)(120) \cos 42^\circ$ (i) $AB = 87.128... = 87.1 \text{ m (3 s.f.)}$

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9 (a) (ii)	$\frac{\sin \angle BAC}{123} = \frac{\sin 42^\circ}{87.128}$ $\angle BAC = \sin^{-1} \left(\frac{123 \sin 42^\circ}{87.128} \right)$ $= 70.843^\circ \text{ (3 d.p.)}$ Bearing of C from $A = 070.8^\circ$ (1 d.p.)
9 (b)	let the angle of depression be θ $\tan \theta = \frac{65}{123}$ $\theta = \tan^{-1} \frac{65}{123}$ $= 27.854...^\circ = 27.9^\circ \text{ (1 d.p.)}$
9 (c)	Let the shortest distance of B from AC be d $\sin 42^\circ = \frac{d}{123} \qquad \frac{1}{2} \times 120 \times d$ $d = 123 \sin 42^\circ \qquad = \frac{1}{2} (123)(120) \sin 42^\circ$ $= 82.303...$ $= 82.3 \text{ m (3 s.f.)}$
9 (d)	$\angle ABC = 180^\circ - 42^\circ - 70.843^\circ \text{ (}\angle \text{ sum of } \Delta \text{)}$ $= 67.157^\circ \text{ (3 d.p.)}$ $\angle EAC = 67.157^\circ - 42^\circ \text{ (ext. } \angle = \text{sum of int. opp. } \angle \text{)}$ $= 25.157^\circ \text{ (3 d.p.)}$ $\text{area of } \triangle AEC = \frac{1}{2} (120)(87.128) \sin 25.157^\circ$ $= 2220 \text{ m}^2 \text{ (3 s.f.)}$
10 (i)	$S(-4, 0), R(2, 0)$ $x\text{-coord of } T = \frac{-4+2}{2} = -1$ sub $x = -1, y = -9$ $T(-1, -9)$
10 (ii)	$(x-2)(x+4) = -x+10$ $x^2 + 2x - 8 + x - 10 = 0$ $x^2 + 3x - 18 = 0$ $(x+6)(x-3) = 0$ $x = -6 \text{ or } x = 3$ $y = 16 \text{ or } y = 7$ $P(-6, 16) \quad Q(3, 7)$
10 (iii)	Area of quadrilateral $PQRS$ $= \frac{1}{2} \begin{vmatrix} 2 & 3 & -6 & -4 & 2 \\ 0 & 7 & 16 & 0 & 0 \end{vmatrix}$ $= \frac{1}{2} [(14 + 48) - (-42 - 64)]$ $= 84 \text{ sq units}$

10 (iv)	x-coord of $X = -1$ sub $x = -1, y = 11$ $X(-1, 11)$
10 (v)	By similar triangles, $QX : XP = 4 : 5$ <u>OR</u> $QX = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ units $XP = \sqrt{5^2 + 5^2} = 5\sqrt{2}$ units ΔRQX and ΔRXP share the same height $\frac{\text{area of } \Delta RQX}{\text{area of } \Delta RXP} = \frac{4}{5}$
11	The circle is tangent to the lines $y = 2x + 10$ and $y = 2x - 10$ at A and B respectively Eqn of diameter is $y = -\frac{1}{2}x$ $2x + 10 = -\frac{1}{2}x$ OR $2x - 10 = -\frac{1}{2}x$ $x = -4$ OR $x = 4$ $A(-4, 2),$ $B(4, -2)$ radius $= \sqrt{4^2 + 2^2} = \sqrt{20}$ units Area of circle $= \pi(\sqrt{20})^2$ $= 20\pi$ sq units $k = 20$ 