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GREENRIDGE SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022
Secondary 3 Express

4049/01

28 September 2022
Wednesday

2 h 15 min
1145 - 1400

Additional Materials: No Additional Materials are Required

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READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.

Write your answers and working on the question paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

- 1 A curve has the equation $y = kx^2 + (2k - 4)x + 3k - 2$, where $k > 0$. Find the set of values of k for which the curve lies completely above the x -axis.

[3]

- 2 A rectangular block has a square base. The length of each side of the base is $(\sqrt{3} - \sqrt{2})$ m and the volume of the block is $(4\sqrt{2} - 3\sqrt{3})$ m³. Find, without using a calculator, the height of the block in the form $(a\sqrt{2} + b\sqrt{3})$ m, where a and b are integers. [4]

- 3 Solve the simultaneous equations.

$$9^x (27)^y = 1$$

$$8^y \div (\sqrt{2})^x = 16\sqrt{2}$$

[4]

- 4 (a) Find the values of the constant c for which the line $2y = x + c$ is a tangent to the curve $y = 2x + \frac{6}{x}$. [4]

- (b) If the quadratic equation $m(x^2 + 9) + 2x(x + 1) + (6m - 2)x = -16$ has 2 real and distinct roots, given that m is a constant, determine the range of values of m . [4]

- 5 (a) Find the value of each of the integers p and q for which $\left(\frac{25}{16}\right)^{\frac{3}{2}} = 2^p \times 5^q$.

[2]

- (b) By using the substitution $u = 3^x$, find the values of x such that $3^{2x+1} - 2 = 8 \times 3^{x-1}$.

[5]

- 6 (a) Obtain the first four terms in the expansion of $\left(2 - \frac{x}{4}\right)^8$ in ascending powers of x . [2]

- (b) Hence, find the coefficient of x^3 in the expansion of $(1+x)^2\left(2 - \frac{x}{4}\right)^8$. [3]

- 7 (a) Solve the equation $\lg(x+12) = 1 + \lg(2-x)$.

[3]

- (b) Given that $\log_2 p = a$, $\log_8 q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b .

[4]

- 8 Desmond buys and sells shares in the stock market. The value of the shares he bought is given by the function $y = 3x^2 - 5x + 7$, where y is the value of the shares in thousands of dollars and x is the time in years after it was first bought.

(a) What is the minimum value of the shares and when does it occur?

[4]

- (b) Sketch the graph of $y = -x^2 - 2x + 6$, showing clearly the coordinates of the minimum point and the intersections with the axes.

[3]

- 9 The function $f(x) = x^3 - 6x^2 + ax + b$, where a and b are constants, is exactly divisible by $x - 3$ and leaves a remainder of -55 when divided by $x + 2$.

(a) Find the value of a and of b .

[4]

(b) Solve the equation $f(x) = 0$.

[4]

- 10** The line $4y = 3x + 1$ intersects the curve $xy = 28x - 27y$ at the points $P(1,1)$ and Q . The perpendicular bisector of PQ intersects the line $y = 4x$ at the point R . Calculate the area of triangle PQR .

[9]

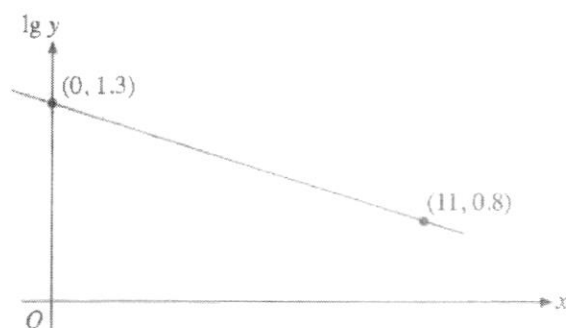
- 11 (a) Express $\frac{2x^2 + x - 3}{(x^2 - 2)(x + 1)}$ in partial fractions.

[4]

- 11 (b) Express $\frac{x^4+9}{x^3+3x}$ into the form $x + \frac{A}{x} + \frac{Bx+C}{x^2+3}$, where A , B and C are constants to be determined.

[6]

12



The variables x and y are connected by the equation $y = kb^x$, where k and b are constants. Experimental values of x and y were obtained. The diagram above shows the straight line graph, passing through the points $(0, 1.3)$ and $(11, 0.8)$, obtained by plotting $\lg y$ against x . Estimate

- (a) the value of k and of b , corrected to 2 significant figures,

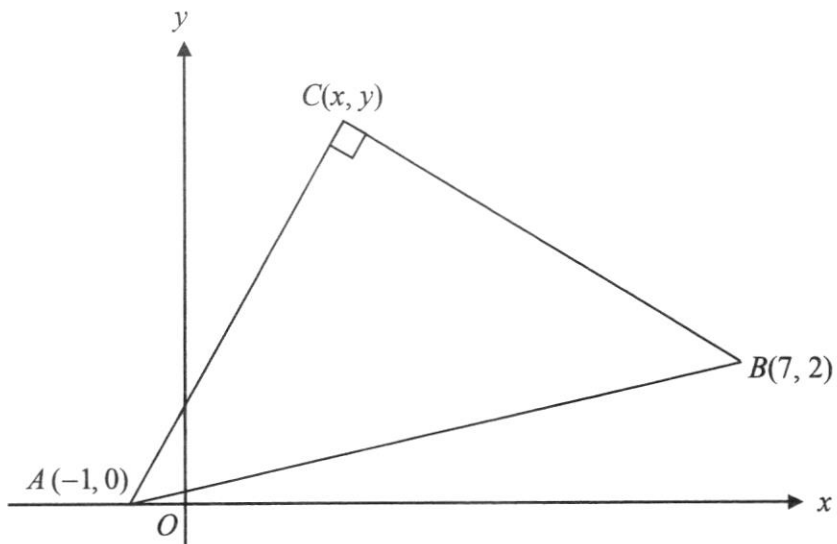
[5]

- (b) the value of y when $x = 8$.

[2]

13 Solutions to this question by accurate drawing will not be accepted.

The diagram shows $\triangle ABC$ with coordinates $A(-1, 0)$, $B(7, 2)$ and $C(x, y)$ and $\angle ACB = 90^\circ$. The point $C(x, y)$ lies on the perpendicular bisector of AB .



- (a) (i) Find the equation of the perpendicular bisector of AB .

[3]

- (ii) Show that the coordinates of C is $(2, 5)$.

[4]

13 (b) The point D is the reflection of point C in the line AB . Find the coordinates of D . [3]

(c) Write down the specific name given to the shape of the quadrilateral $ABCD$. [1]

End of Paper



GREENRIDGE SECONDARY SCHOOL
END-OF-YEAR EXAMINATION 2022
Secondary 3 Express

4049/02

2 h 15 min
1110 – 1325

Additional Materials: No Additional Materials are Required

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You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

- 1 Find the values of k for which the line $x + 3y = k$ and the curve $y^2 = 2x + 3$ do not intersect.

[4]

2 (a) Simplify $3\sqrt{180} + \sqrt{245} - 2\sqrt{125}$, leaving your answer in surd form. [4]

(b) Given that $\sqrt{a+b\sqrt{3}} = \frac{13}{4+\sqrt{3}}$, where a and b are integers, find, without using a calculator, the value of a and of b . [4]

3 In the expansion of $\left(3x - \frac{1}{2x}\right)^{10}$, evaluate

(a) the term independent of x , [4]

(b) the term in x^6 , [2]

- 4 (a) On the same graph, sketch the curves $y = e^x$ and $x + y = -2$. [2]

- (b) Write down the coordinates of the point where the curve $y = e^x$ cuts the y -axis. [1]

- (c) Hence, determine the number of solutions of the equation $e^x + x + 2 = 0$. [2]

5 Given that $\log_2 x = p$ and $\log_4 y = q$, express the following in terms of p and/or q .

(a) $\log_2 \sqrt{x}$, [2]

(b) $\log_2 xy^2$, [3]

(c) $\log_4 \frac{4x}{y}$. [3]

6 (a) Solve the equation $\frac{27^{2+x}}{9} = 3^x \times 81^{2x-1}$. [4]

(b) Given that $25^{x+1} \times 2^{4x-1} = 32^x \times 5^{3x}$, evaluate 10^x . [4]

- 7 A man buys a new car. After t months, its value $\$C$ is given by $C = 100000e^{-at}$, where a is a constant.

(a) Find the value of the car when the man bought it. [2]

(b) The value of the car after 24 months is expected to be \$65000.

(i) Calculate the expected value of the car after 3 years, [3]

(ii) Calculate the age of the car, to the nearest month when its expected value will be \$30000, [2]

(iii) After 5 years, a car dealer offers to pay the man \$35000 for your car. Based on the equation above, should the man agree to sell it? Explain your answer. [3]

8 Solve.

(a) $\log_7(17y+15) = 2 + \log_7(2y-3),$ [3]

(b) $\log_p 8 \times \log_{16} p,$ [3]

(c) $3 \log_5 y - \log_y 5 = 2.$ [5]

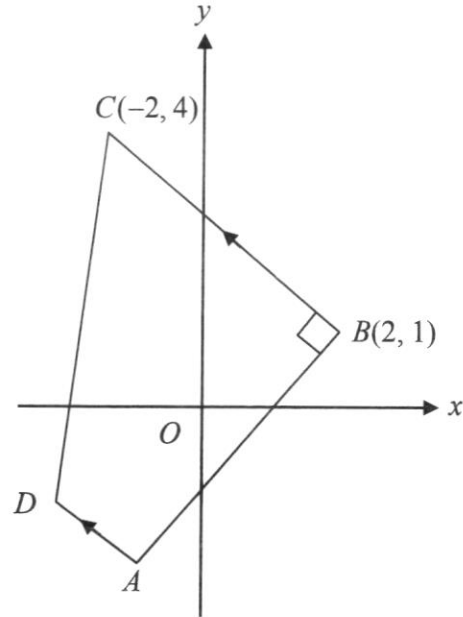
9 Given $f(x) = 2x^3 - 5x^2 - 4x + 12$,

(a) show that $(x - 2)$ is a factor of $f(x)$, [1]

(b) factorise $f(x)$ completely, [3]

(c) hence, solve the equation $2(2^{3y}) - 5(2^{2y}) = 4(2^y - 3)$. [4]

10



In the trapezium $ABCD$, AD and BC are parallel and angle ABC is a right angle. The coordinates of the points B and C are $(2, 1)$ and $(-2, 4)$ respectively.

(a) Find the equation of the line AB .

[3]

(b) The y -intercept of the line DA produced is $-\frac{7}{2}$, show that the coordinates of A is $(-\frac{22}{25}, -\frac{71}{25})$.

[3]

- 10 (c) Given that the midpoint of the line segment BD is $\left(0, -\frac{1}{2}\right)$, find the coordinates of D . [4]

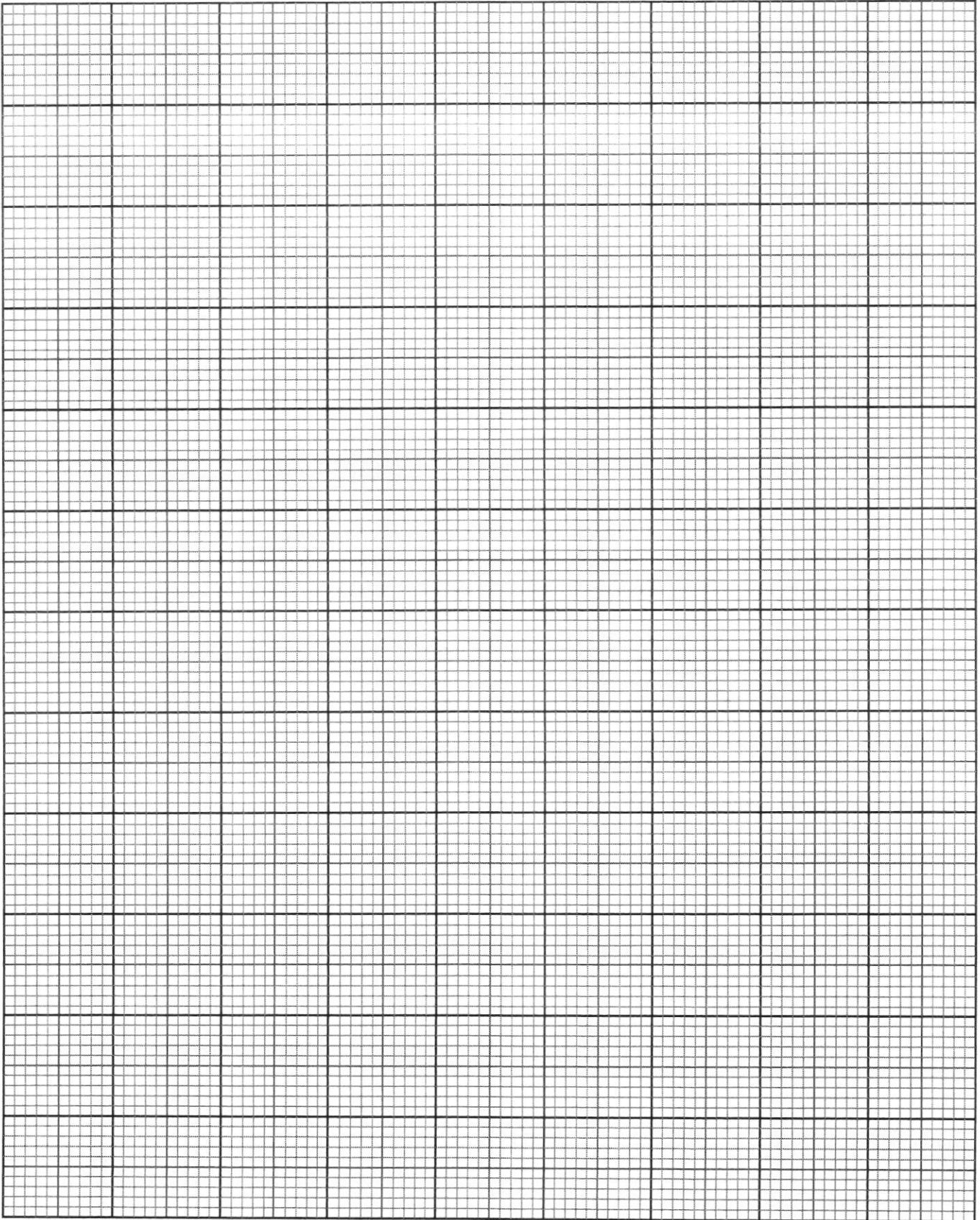
- (d) Find the area of the trapezium $ABCD$. [2]

- 11** The table below shows some experimental values of 2 variables x and y . It is known that one value of y has been recorded incorrectly.

x	0.5	1.0	1.5	2.0	2.5
y	1.20	1.00	0.92	0.75	0.66

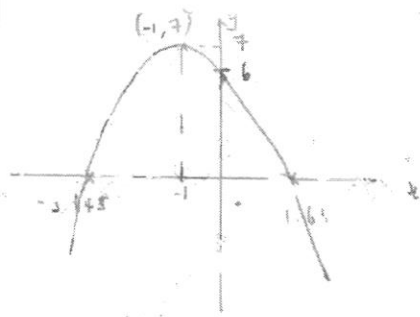
It is known that x and y are related by the equation $y = \frac{a}{x+b}$, where a and b are constants.

- (a) Using 2 cm to represent 0.5 units on the horizontal axis and 2 cm to represent 0.2 units on the vertical axis, plot $\frac{1}{y}$ against x for the given data and draw a straight line graph. [4]
- (b) Using your graph,
- (i) estimate the value of a and of b , [4]
- (ii) identify the value of x which has an incorrect value of y and estimate the correct value of y , correct to 2 decimal places. [2]

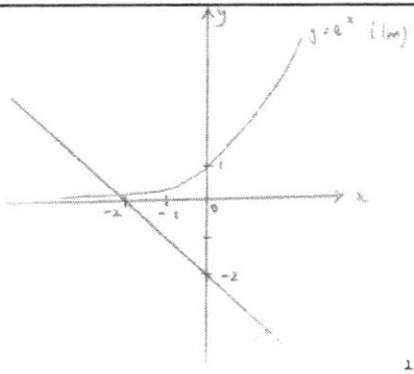


End of Paper

Greenridge Secondary School
2022 Sec 3Exp EYE P1 A Math

1		$k < -2$ or $k > 1$		9	(a)	$a = 10, b = -3$
2		$h = 2\sqrt{2} + \sqrt{3}$ m			(b)	$x = 3, 2.62, 0.382$
3		$x = -\frac{9}{5}$ and $y = \frac{6}{5}$		10		$Q(9, 7)$ Perpendicular bisector $y = -\frac{4}{3}x + \frac{32}{3}$ $R(2, 8)$ Area = 25 units ²
4	(a)	$c = \pm 12$		11	(a)	$\frac{1}{x^2 - 2} + \frac{2}{x + 1}$
	(b)	$m < -\frac{16}{17}$			(b)	$x + \frac{3}{x} - \frac{6}{x^2 + 3}$
5	(a)	$p = 6$ and $q = -3$		12	(a)	$b = 0.90$ $k = 20$
	(b)	$x = 0.290$			(b)	$y = 8.59$
6	(a)	$256 - 256x + 112x^2 - 28x^3 + \dots$		13	(a)(i)	$y = -4x + 13$
	(b)	-60			(a)(ii)	$C(2, 5)$
7	(a)	$x = \frac{8}{11}$			(b)	$D(4, -3)$
	(b)	$c = a - 3b$			(e)	Square
8	(a)	Min value = \$4916.67 after $\frac{5}{6}$ years or 10 months				
	(b)					

Greenridge Secondary School
2022 Sec 3Exp EYE P2 A Math

1		Consider $b^2 - 4ac < 0$ $k < -6$		7	(a)	\$100000
2	(a)	$15\sqrt{5}$			(b)(i)	\$52404.67
	(b)	$a = 19, b = -8$			(b)(ii)	67 months
3	(a)	$1913\frac{5}{8}$			(b)(iii)	\$34063.04 Yes, because dealer is offering more
	(b)	$73811\frac{1}{4}x^6$		8	(a)	$y = 2$
4	(a)				(b)	$\frac{3}{4}$
	(b)	(0, 1)			(c)	$y = 0.585$ and $y = 5$
	(c)	1		9	(a)	Show $f(2) = 0$
5	(a)	$\frac{1}{2}p$			(b)	$f(x) = (x-2)^2(2x+3)$
	(b)	$p + 4q$			(c)	$2^y = 2, -\frac{3}{2}$ $y = 1$
	(c)	$1 + \frac{p}{2} - q$		10	(a)	$y = \frac{4}{3}x - \frac{5}{3}$
6	(a)	$\frac{4}{3}$			(b)	Equation of DA $y = -\frac{3}{4}x - \frac{7}{2}$
	(b)	$\frac{25}{2}$			(c)	$D(-2, -2)$
					(d)	$15\frac{9}{25}$
				11	(b)(i)	Accept $a = 2.5$ to 3.0 , $b = 1.5$ to 2
					(b)(II)	$y = 0.85$

- 1 A curve has the equation $y = kx^2 + (2k-4)x + 3k-2$, where $k > 0$. Find the set of values of k for which the curve lies completely above the x -axis.

[3]

$$y = kx^2 + (2k-4)x + 3k-2$$

Completely above x axis $\rightarrow b^2 - 4ac < 0$ (Im)

$$(2k-4)^2 - 4k(3k-2) < 0$$

$$4k^2 - 16k + 16 - 12k^2 + 8k < 0$$

$$-8k^2 - 8k + 16 < 0$$

$$k^2 + k - 2 > 0 \quad (\text{Im})$$

Roots: $k^2 + k - 2 = 0$

$$(k+2)(k-1) = 0$$

$$k = -2 \text{ or } k = 1$$



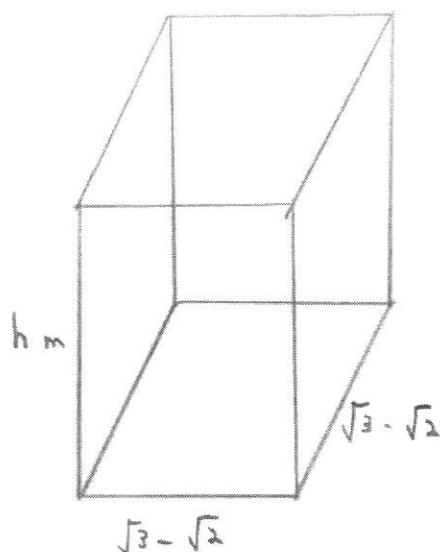
$$\therefore k < -2 \quad \text{or} \quad k > 1 \quad (\text{Im})$$

(NA)

$$\therefore k > 0$$

Im if $k > -2$ is not rejected

- 2 A rectangular block has a square base. The length of each side of the base is $(\sqrt{3} - \sqrt{2})$ m and the volume of the block is $(4\sqrt{2} - 3\sqrt{3})$ m³. Find, without using a calculator, the height of the block in the form $(a\sqrt{2} + b\sqrt{3})$ m, where a and b are integers.



$$Vol = 4\sqrt{2} - 3\sqrt{3} \text{ m}^3$$

Let h m be the height of the block

$$(\sqrt{3} - \sqrt{2})^2 h = 4\sqrt{2} - 3\sqrt{3} \quad (1m)$$

$$(3 - 2\sqrt{6} + 2)h = 4\sqrt{2} - 3\sqrt{3}$$

$$h = \frac{4\sqrt{2} - 3\sqrt{3}}{5 - 2\sqrt{6}} \quad (1m)$$

$$\frac{4\sqrt{2} - 3\sqrt{3}}{5 - 2\sqrt{6}} \times \frac{5 + 2\sqrt{6}}{5 + 2\sqrt{6}} \quad (1m)$$

$$\frac{20\sqrt{2} + 8\sqrt{2} - 15\sqrt{3} - 6\sqrt{18}}{25 - (4 \times 6)}$$

$$= \frac{26\sqrt{2} + 16\sqrt{3} - 15\sqrt{3} - 18\sqrt{2}}{1}$$

$$= 2\sqrt{2} + \sqrt{3} \text{ m} \quad (1m)$$

- 3 Solve the simultaneous equations.

$$9^x (2^y)^y = 1$$

$$8^y \div (\sqrt{2})^x = 16\sqrt{2}$$

[4]
[8]

$$3^{2x} (3^3)^y = 3^0$$

Consider power of 3

$$2x + 3y = 0 \quad (1) \quad (1m)$$

$$(2^3)^y \div (2^{\frac{1}{2}})^x = 2^4 2^{\frac{1}{2}}$$

Consider power of 2

$$3y - \frac{1}{2}x = 4\frac{1}{2}$$

$$\therefore 6y - x = 9 \quad (2) \quad (1m)$$

$$(1) \times 2$$

$$4x + 6y = 0 \quad (3)$$

$$(3) - (2)$$

$$5x = -9$$

$$x = -\frac{9}{5} \quad (1m)$$

$$5x + 3y = -\frac{9}{5}$$

$$2\left(-\frac{9}{5}\right) + 3y = 0$$

$$3y = \frac{18}{5}$$

$$y = \frac{6}{5} \quad (1m)$$

- 4 (a) Find the values of the constant c for which the line $2y = x + c$ is a tangent to the curve $y = 2x + \frac{6}{x}$. [4]

$$\begin{aligned}
 2y &= x + c & \text{--- (1)} \\
 y &= 2x + \frac{6}{x} & \text{--- (2)} \\
 \text{Subst (2) into (1)} & \\
 2\left(2x + \frac{6}{x}\right) &= x + c & (1m) \\
 4x + \frac{12}{x} &= x + c \Rightarrow 0 \\
 3x^2 - cx + 12 &= 0 & (1m) \\
 \text{Tangent} \rightarrow b^2 - 4ac &= 0 & (1m) \\
 (-c)^2 - 4(3)(12) &= 0 \\
 c^2 &= 144 \\
 c &= \pm 12 & (1m)
 \end{aligned}$$

- (b) If the quadratic equation $m(x^2 + 9) + 2x(x + 1) + (6m - 2)x = -16$ has 2 real and distinct roots, given that m is a constant, determine the range of values of m . [4]

$$\begin{aligned}
 mx^2 + 9m + 2x^2 + 2x + 6mx - 2x + 16 &= 0 \\
 (m+2)x^2 + 6mx + (9m+16) &= 0 & (1m) \\
 2 \text{ Real \& distinct roots} \rightarrow b^2 - 4ac &> 0 & (1m) \\
 (6m)^2 - 4(m+2)(9m+16) &> 0 \\
 36m^2 - 36m^2 - 136m - 128 &> 0 & (1m) \\
 -136m - 128 &> 0 \\
 -136m + 128 &< 0 \\
 m &< -\frac{128}{136} \\
 m &< -\frac{16}{17} & (1m)
 \end{aligned}$$

- 5 (a) Find the value of each of the integers p and q for which $\left(\frac{25}{16}\right)^{\frac{3}{2}} = 2^p \times 5^q$. [2]

$$\left(\frac{16}{25}\right)^{\frac{3}{2}} = 2^p \times 5^q$$

$$\left(\frac{4}{5}\right)^3 = 2^p \times 5^q \quad (1m)$$

$$2^6 \times 5^{-3} = 2^p \times 5^q$$

$$\therefore p = 6, \quad q = -3 \quad (1m)$$

- (b) By using the substitution $u = 3^x$, find the values of x such that $3^{2x+1} - 2 = 8 \times 3^{x-1}$. [5]

$$3^{2x+1} - 2 = 8 \times \frac{3^x}{3} \quad (1m)$$

$$\text{Let } u = 3^x$$

$$3u^2 - 2 = \frac{8}{3}u \quad (1m)$$

$$9u^2 - 6 = 8u$$

$$9u^2 - 8u - 6 = 0$$

$$u = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(9)(-6)}}{2(9)} \quad (1m)$$

$$= \frac{8 \pm \sqrt{64 + 216}}{18}$$

$$= \frac{8 \pm \sqrt{280}}{18}$$

$$3^x = 1.37407 \quad \text{or} \quad -0.48518 \quad (1m)$$

$$x \lg 3 = \lg 1.37407 \quad \text{or} \quad x \lg 3 = \lg -0.48518$$

$$x = 0.28917$$

$$= 0.290 \quad (3sf)$$

NA

(1m)

(-1m if solution is
not rejected)

- 6 (a) Obtain the first four terms in the expansion of $\left(2 - \frac{x}{4}\right)^8$ in ascending powers of x . [2]

$$\begin{aligned} \left(2 - \frac{x}{4}\right)^8 &= 2^8 + {}^8C_1 \left(-\frac{x}{4}\right)^1 (2)^7 + {}^8C_2 \left(-\frac{x}{4}\right)^2 (2)^6 + {}^8C_3 \left(-\frac{x}{4}\right)^3 (2)^5 + \dots \quad (1\text{m}) \\ &= 256 + 8 \left(-\frac{x}{4}\right) (128) + 28 \left(\frac{x^2}{16}\right) (64) + 56 \left(-\frac{x^3}{64}\right) (32) + \dots \\ &= 256 - 256x + 112x^2 - 28x^3 + \dots \quad (1\text{m}) \end{aligned}$$

- (b) Hence, find the coefficient of x^3 in the expansion of $(1+x)^2 \left(2 - \frac{x}{4}\right)^8$. [3]

$$\begin{aligned} (1+2x+x^2) & \left(256 - 256x + 112x^2 - 28x^3 + \dots \right) \\ \text{Consider } x^3 & \\ + 1(-28x^3) & + 2(112x^2) + 1(256x) \\ \dots + (-28) & + 224 + 256 \quad (1\text{m}) \\ \dots & \dots \dots \dots \\ \dots & \dots \dots \dots \quad (1\text{m}) \end{aligned}$$

- 7 (a) Solve the equation $\lg(x+12) = 1 + \lg(2-x)$.

[3]

$$\lg(x+12) = \lg 10 + \lg(2-x) \quad (1m)$$

$$\lg(x+12) = \lg 10(2-x) \quad (1m)$$

$$\therefore x+12 = 20-10x$$

$$11x = 8$$

$$x = \frac{8}{11} \quad (1m)$$

- (b) Given that $\log_2 p = a$, $\log_3 q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b .

[4]

$$p = 2^a, \quad q = 3^b$$

$$(1m) \quad (1m)$$

$$\frac{p}{q} = \frac{2^a}{3^b}$$

$$= \frac{2^a}{2^{3b}} \quad (1m)$$

$$= 2^{a-3b}$$

$$c = a - 3b$$

- 8 Desmond buys and sells shares in the stock market. The value of the shares he bought is given by the function $y = 3x^2 - 5x + 7$, where y is the value of the shares in thousands of dollars and x is the time in years after it was first bought.

(a) What is the minimum value of the shares and when does it occur?

4
[2]

$$\begin{aligned} y &= 3\left(x^2 - \frac{5}{3}x + \frac{7}{3}\right) \\ &= 3\left[x^2 - \frac{5}{3}x + \left(\frac{-5}{6}\right)^2 - \left(\frac{-5}{6}\right)^2 + \frac{7}{3}\right] \\ &= 3\left[\left(x - \frac{5}{6}\right)^2 - \frac{25}{36} + \frac{7}{3}\right] \quad (1m) \\ &= 3\left[\left(x - \frac{5}{6}\right)^2 + \frac{59}{36}\right] = 3\left(x - \frac{5}{6}\right)^2 + \frac{59}{12} \end{aligned}$$

$$\text{Min pt} = \left(\frac{5}{6}, \frac{59}{12}\right) \quad (1m)$$

$$\therefore \text{min value} = \$4916.67 \quad \text{after } \frac{5}{6} \text{ yrs or } 10 \text{ mths} \quad (1m)$$

(b) Sketch the graph of $y = -x^2 - 2x + 6$, showing clearly the coordinates of the minimum point and the intersections with the axes. [3]

$$y = -x^2 - 2x + 6$$

$$a = -2 < 0 \rightarrow \text{max pt}$$

$$x \text{ intercept} \rightarrow y = 0$$

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-6)}}{2(-1)} \\ &= \frac{2 \pm \sqrt{4 + 24}}{-2} \\ &= \frac{2 \pm \sqrt{28}}{-2} \end{aligned}$$

$$\begin{aligned} &= -3.145 \quad \text{or } 1.646 \quad (1m) \\ y \text{ intercept} &\rightarrow x = 0 \\ &y = 6 \quad (1m) \end{aligned}$$

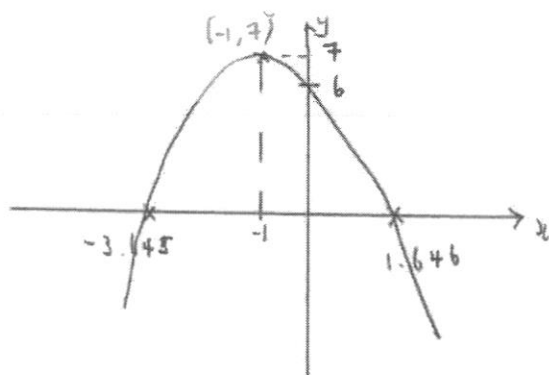
$$y = -(x^2 + 2x - 6)$$

$$= -(x^2 + 2x + 1 - 1 - 6)$$

$$= -(x+1)^2 - 7$$

$$= -(x+1)^2 + 7$$

$$\therefore \text{Turning pt} = (-1, 7) \quad (1m)$$



- 9 The function $f(x) = x^3 - 6x^2 + ax + b$, where a and b are constants, is exactly divisible by $x-3$ and leaves a remainder of -55 when divided by $x+2$.

(a) Find the value of a and of b .

[4]

$$f(3) = 3^3 - 6(3)^2 + a(3) + b = 0$$

$$27 - 54 + 3a + b = 0$$

$$3a + b = 27 \quad \text{--- (1)} \quad (1m)$$

$$f(-2) = (-2)^3 - 6(-2)^2 + a(-2) + b = -55$$

$$-8 - 24 - 2a + b = -55$$

$$-2a + b = -23 \quad \text{--- (2)} \quad (1m)$$

$$(1) - (2)$$

$$5a = 50$$

$$a = 10$$

Subst $a = 10$ into (1)

$$3(10) + b = 27$$

$$b = -3$$

(b) Solve the equation $f(x) = 0$.

[4]

$$\begin{array}{r} x^3 - 3x^2 + 1 \\ x-3 \overline{) x^3 - 6x^2 + 10x - 3} \\ \underline{x^3 - 3x^2} \\ -3x^2 + 10x \\ \underline{-3x^2 + 9x} \\ x - 3 \\ \underline{x - 3} \\ 0 \end{array} \quad (1m)$$

$$\therefore f(x) = (x-3)(x^2 - 3x + 1) = 0 \quad (1m)$$

$$\text{Consider } x^2 - 3x + 1 = 0$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{5}}{2} \quad (1m)$$

$$\therefore x = 3, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2} \quad (1m)$$

$$\text{OR } x = 3, 2.62, 0.382$$

- 10 The line $4y = 3x + 1$ intersects the curve $xy = 28x - 27y$ at the point $P(1, 1)$ and at the point Q . The perpendicular bisector of PQ intersects the line $y = 4x$ at the point R . Calculate the area of triangle PQR .

9
[8]

$$4y = 3x + 1$$

$$y = \frac{3}{4}x + \frac{1}{4}$$

$$\text{Subst } y = \frac{3}{4}x + \frac{1}{4} \text{ into } xy = 28x - 27y$$

$$x\left(\frac{3}{4}x + \frac{1}{4}\right) = 28x - 27\left(\frac{3}{4}x + \frac{1}{4}\right) = 0 \quad (1m)$$

$$\frac{3}{4}x^2 + \frac{1}{4}x - 28x + \frac{27}{4}x + \frac{27}{4} = 0$$

$$3x^2 - 30x + 27 = 0$$

$$x^2 - 10x + 9 = 0$$

$$(x - 9)(x - 1) = 0$$

$$x = 9 \text{ or } x = 1 \quad (pt \ 1) \quad (1m)$$

$$\text{When } x = 9, y = \frac{3(9)}{4} + \frac{1}{4} = 7$$

$$\therefore Q(9, 7) \quad (1m)$$

$$\text{Mid pt } QP = \left(\frac{1+9}{2}, \frac{1+7}{2}\right) = (5, 4) \quad (1m)$$

$$m_{PQ} = \frac{7-1}{9-1} = \frac{3}{4}$$

$$\therefore m \text{ of } \perp \text{ bisector of } PQ = -\frac{4}{3} \quad (1m)$$

$$\text{Consider mid pt } (5, 4)$$

$$\frac{y-4}{x-5} = -\frac{4}{3}$$

$$y = -\frac{4}{3}(x-5) + 4$$

$$y = -\frac{4}{3}x + \frac{32}{3} \quad (1m)$$

$$\text{Subst } y = -\frac{4}{3}x + \frac{32}{3} \text{ into } y = 4x$$

$$-\frac{4}{3}x + \frac{32}{3} = 4x$$

$$x = 2$$

$$y = 4(2) = 8$$

$$\therefore R(2, 8)$$

$$\text{Area } \triangle PQR$$

$$= \frac{1}{2} \begin{vmatrix} 1 & 9 & 2 \\ 1 & 7 & 8 \\ 1 & 1 & 1 \end{vmatrix} \quad (1m)$$

$$= \frac{1}{2}(50)$$

$$= 25 \text{ units}^2 \quad (1m)$$

- 11 (a) Express $\frac{2x^2+x-3}{(x^2-2)(x+1)}$ in partial fractions. [4]

$$= \frac{Ax+B}{x^2-2} + \frac{C}{x+1} \quad (1m)$$

$$2x^2+x-3 = (Ax+B)(x+1) + C(x^2-2)$$

$$\text{When } x = -1, \quad 2(-1)^2 + (-1) - 3 = C(1-2)$$

$$-C = -2$$

$$C = 2$$

$$\text{When } x = 0, \quad -3 = B - 2C$$

$$B = -3 + 4$$

$$B = 1$$

Comparing coefficient of x^2

$$2 = A + C$$

$$\therefore A = 0$$

$$\therefore \frac{2x^2+x-3}{(x^2-2)(x+1)} = \frac{1}{x^2-2} + \frac{2}{x+1} \quad (1m)$$

- (b) Express $\frac{x^4+9}{x^3+3x}$ into the form $x + \frac{A}{x} + \frac{Bx+C}{x^2+3}$, where A , B and C are constants to be determined. [6]

$$\begin{array}{r} x \\ x^3+3x \overline{) x^4+0x^3+0x^2+9} \\ \underline{2x^2+0x^2+3x^2} \quad (1m) \\ -3x^2+9 \end{array}$$

$$\therefore \frac{x^4+9}{x^3+3x} = x + \frac{-3x^2+9}{x^2+3}$$

$$\text{Consider } \frac{-3x^2+9}{x^3+3x} = \frac{-3x^2+9}{x(x^2+3)} \quad (1m)$$

$$= \frac{A}{x} + \frac{Bx+C}{x^2+3} \quad (1m)$$

$$\therefore -3x^2+9 = A(x^2+3) + (Bx+C)x$$

When $x = 0$

$$9 = 3A$$

$$A = 3$$

When $x = 1$

$$6 = 3(4) + B + C$$

$$B + C = -6 \quad (1)$$

Comparing coefficient of x^2

$$-3 = A + B \quad (2)$$

$$(1) - (2)$$

$$C - 3 = -6 - (-3)$$

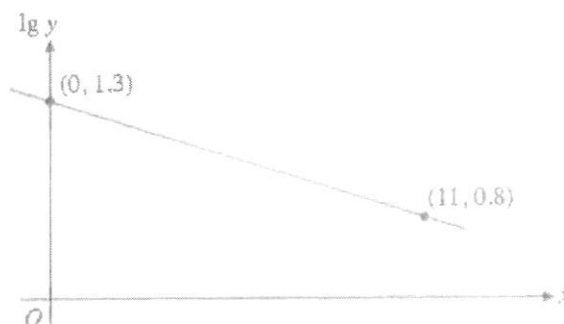
$$C = 0$$

Subst $C = 0$ into (1)

$$B = -6$$

$$\therefore \frac{x^4+9}{x^3+3x} = x + \frac{3}{x} - \frac{6x}{x^2+3} \quad (1m)$$

12



The variables x and y are connected by the equation $y = kb^x$, where k and b are constants. Experimental values of x and y were obtained. The diagram above shows the straight line graph, passing through the points $(0, 1.3)$ and $(11, 0.8)$, obtained by plotting $\lg y$ against x . Estimate

- (a) the value of k and of b , corrected to 2 significant figures,

5
[6]

$$\begin{aligned}
 \lg y &= \lg kb^x \\
 \lg y &= \lg k + x \lg b \quad (1m) \\
 m = \lg b &= \frac{1.3 - 0.8}{0 - 11} \quad (1m) \\
 &= -\frac{1}{22} \\
 b &= 0.90 \quad (1m) \\
 \text{Y-intercept, } \lg k &= 1.3 \quad (2sf) \quad (1m) \\
 k &= 19.950 \quad (2sf) \quad (1m)
 \end{aligned}$$

- (b) the value of y when $x = 8$.

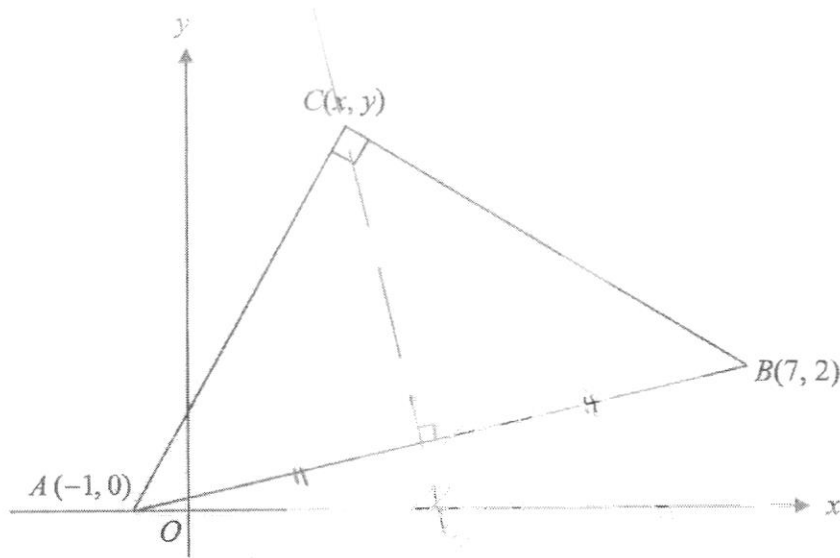
[2]

$$\begin{aligned}
 \text{When } x = 8, \lg y &= \lg 0.90 + \lg 19.952 \\
 &= 0.9339 \quad (1m) \\
 y &= 8.5887 \\
 &= 8.59 \quad (3sf) \quad (1m)
 \end{aligned}$$

or

$$\begin{aligned}
 y &= 19.952 (0.9)^8 \\
 &= 8.58868 \\
 &= 8.59 \quad (3sf)
 \end{aligned}$$

- 13 Solutions to this question by accurate drawing will not be accepted.
The diagram shows $\triangle ABC$ with coordinates $A(-1, 0)$, $B(7, 2)$ and $C(x, y)$ and $\angle ACB = 90^\circ$. The point $C(x, y)$ lies on the perpendicular bisector of AB .



(a) Find

(i) the equation of the perpendicular bisector of AB ,

[3]

$$m_{AB} = \frac{2-0}{7-(-1)} \\ = \frac{1}{4}$$

$$m \text{ of } \perp \text{ bisector of } AB = -4 \quad (1m)$$

$$\text{mid pt } AB = \left(\frac{-1+7}{2}, \frac{0+2}{2} \right) \\ = (3, 1)$$

$$y - 1 = -4(x - 3) + 13 \quad (1m)$$

→ (ii) the coordinates of C . $y = -4x + 13$ (1m)
that co-ordinates of C is (2, 5) [4]

$$m_{AC} = \frac{y}{x+1} \quad (1m)$$

$$m_{BC} = \frac{y-2}{x-7}$$

$$(m_{AC})(m_{BC}) = \left(\frac{y}{x+1} \right) \left(\frac{y-2}{x-7} \right) = -1 \quad (1m)$$

$$y^2 - 2y = -x^2 + 6x + 7$$

$$(-4x+13)^2 - 2(-4x+13) = -x^2 + 6x + 7$$

$$16x^2 - 104x + 169 + 8x - 26 + x^2 - 6x - 7 = 0$$

$$17x^2 - 102x + 136 = 0$$

$$x^2 - 6x + 8 = 0 \quad (1m)$$

$$(x-4)(x-2) = 0$$

$$x = 4 \text{ or } 2$$

(NA) beyond mid pt (3, 1)

- 13 (b) The point D is the reflection of point C in the line AB . Find the coordinates of D . [3]

$$\text{Mid pt } CD = \text{Mid pt } AB$$

$$\text{Let } D \text{ be } (p, q)$$

$$\left(\frac{2+p}{2}, \frac{5+q}{2} \right) = (3, 1) \quad (1m)$$

$$\therefore \frac{2+p}{2} = 3, \quad \frac{5+q}{2} = 1$$

$$p = 3(2) - 2$$

$$= 4$$

$$q = 1(2) - 5$$

$$= -3 \quad (1m)$$

$$\therefore D(4, -3) \quad (1m)$$

- (c) Write down the specific name given to the shape of the quadrilateral $ABCD$. [1]

$$ABCD \text{ is a square/} \quad (1m)$$



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[illegible]

The total number of marks for this paper is **90**.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$1 - 2 \sin^2 A$$

$$\cos \frac{1}{2}(A - B)$$

$$+ \sin \frac{1}{2}(A - B)$$

$$\cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \triangle ABC = \frac{1}{2}ab \sin C$$

- 1 Find the values of k for which the line $x+3y=k$ and the curve $y^2=2x+3$ do not intersect. [4]

$$x+3y=k$$

$$y = -\frac{x}{3} + \frac{k}{3} \quad (1) \quad (1m)$$

$$y^2 = 2x+3 \quad (2)$$

Subst (1) into (2)

$$\left(-\frac{x}{3} + \frac{k}{3}\right)^2 = 2x+3$$

$$\frac{x^2}{9} - \frac{2xk}{9} + \frac{k^2}{9} = 2x+3$$

$$x^2 - 2xk + k^2 - 18x - 27 = 0$$

$$x^2 + (-2k-18)x + (k^2-27) = 0 \quad (1m)$$

No intersection $\rightarrow b^2 - 4ac < 0 \quad (1m)$

$$(-2k-18)^2 - 4(1)(k^2-27) < 0$$

$$4k^2 + 72k + 324 - 4k^2 + 108 < 0$$

$$72k < -432$$

$$k < -6 \quad (1m)$$

OR

$$x+3y=k$$

$$x = k-3y \quad (1m)$$

Subst $x = k-3y$ into $y^2 = 2x+3$

$$y^2 = 2(k-3y)+3$$

$$= 2k-6y+3$$

$$y^2 + 6y + (-2k-3) = 0 \quad (1m)$$

$$b^2 - 4ac < 0 \quad (1m)$$

$$6^2 - 4(1)(-2k-3) < 0$$

$$36 + 8k + 12 < 0$$

$$8k < -48$$

$$k < -6 \quad (1m)$$

- 2 (a) Simplify $3\sqrt{180} + \sqrt{245} - 2\sqrt{125}$, leaving your answer in surd form.

[4]

$$\begin{array}{r} 2 \overline{) 180} \\ 2 \quad 90 \\ 3 \quad 45 \\ 3 \quad 15 \\ 5 \quad 5 \\ 5 \quad 0 \end{array}$$

$$\begin{array}{r} 5 \overline{) 125} \\ 5 \quad 25 \\ 5 \quad 5 \\ 5 \quad 0 \end{array}$$

$$\begin{aligned} & 3\sqrt{180} + \sqrt{245} - 2\sqrt{125} \\ &= 3\sqrt{2^2 \times 3^2 \times 5} + \sqrt{5 \times 7^2} - 2\sqrt{5^2 \times 5} \\ &= 3(6)\sqrt{5} + 7\sqrt{5} - 10\sqrt{5} \quad (1m) \\ &= 15\sqrt{5} \quad (1m) \end{aligned}$$

$$\begin{array}{r} 5 \overline{) 245} \\ 5 \quad 49 \\ 5 \quad 49 \\ 5 \quad 0 \end{array}$$

- (b) Given that $\sqrt{a+b\sqrt{3}} = \frac{13}{4+\sqrt{3}}$, where a and b are integers, find, without using a calculator, the value of a and of b .

[4]

$$\sqrt{a+b\sqrt{3}} = \frac{13}{4+\sqrt{3}}$$

$$a+b\sqrt{3} = \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} \quad (1m)$$

$$= \frac{169}{16-8\sqrt{3}+3}$$

$$\frac{169}{19-8\sqrt{3}} \times \frac{19+8\sqrt{3}}{19+8\sqrt{3}} \quad (1m)$$

$$= \frac{169(19+8\sqrt{3})}{361-192}$$

$$= \frac{169(19+8\sqrt{3})}{169}$$

$$= 19+8\sqrt{3}$$

$$a+b\sqrt{3} = 19+8\sqrt{3} \quad (1m)$$

$$\therefore a = 19, \quad b = 8 \quad (1m)$$

$$\sqrt{a+b\sqrt{3}} = \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} \quad (1m)$$

$$= \frac{13(4-\sqrt{3})}{16-3}$$

$$= \frac{13(4-\sqrt{3})}{13}$$

$$= 4-\sqrt{3} \quad (1m)$$

$$a+b\sqrt{3} = (4-\sqrt{3})^2 \quad (1m)$$

$$= 16-8\sqrt{3}+3$$

$$= 19-8\sqrt{3} \quad (1m)$$

3 In the expansion of $\left(3x - \frac{1}{2x}\right)^{10}$, evaluate

(a) the term independent of x ,

[4]

$$T_{r+1} = {}^{10}C_r \left(-\frac{1}{2x}\right)^r (3x)^{10-r} \quad (1m)$$

Consider x term

$$(x^{-1})^r x^{10-r} = x^0 \quad (1m)$$

Compare power of x

$$-r + 10 - r = 0$$

$$-2r = -10$$

$$r = 5 \quad (1m)$$

$$\therefore T_6 = {}^{10}C_5 \left(-\frac{1}{2x}\right)^5 (3x)^{10-5}$$

$$= 252 \left(-\frac{1}{2}\right)^5 3^5$$

$$= 252 \left(-\frac{1}{32}\right) (243)$$

$$= -1913 \frac{5}{8} \quad (1m)$$

(b) the middle term,

[2]

$$(x^{-1})^r x^{10-r} = x^0$$

$$-2r = -10$$

$$r = 5$$

$$= 2 \quad (1m)$$

$$\therefore T_4 = {}^{10}C_2 \left(-\frac{1}{2x}\right)^2 (3x)^{10-2}$$

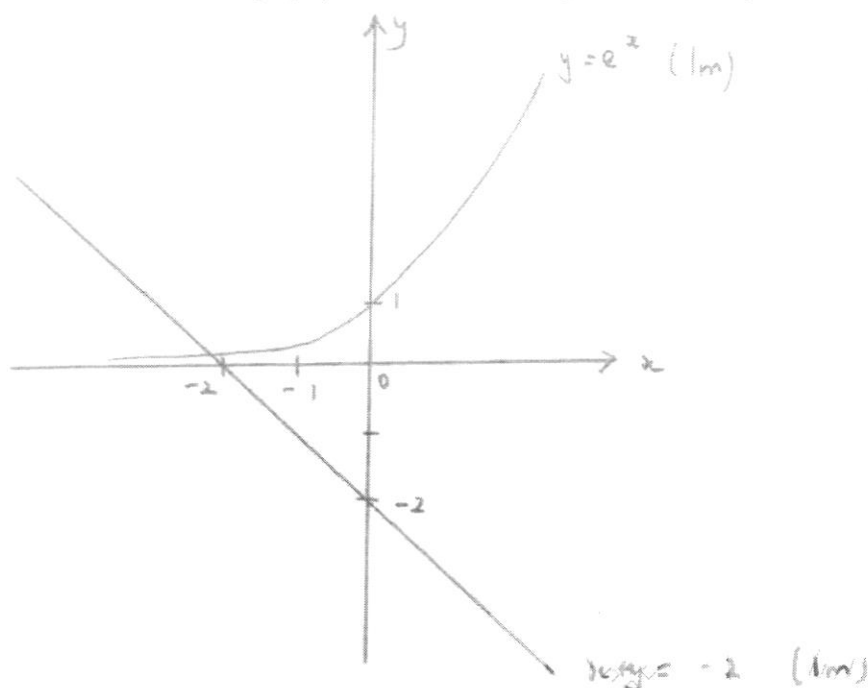
$$= 45 \left(\frac{1}{4x^2}\right) (6561x^8)$$

$$= 7381 \frac{1}{4} x^6 \quad (1m)$$

87

- 4 (a) On the same graph, sketch the curves $y = e^x$ and $x + y = -2$.

[2]



$$\begin{aligned}
 x + y &= -2 \\
 x \text{ intercept} &\rightarrow y = 0 \\
 x &= -2 \\
 y \text{ intercept} &\rightarrow x = 0 \\
 y &= -2
 \end{aligned}$$

- (b) Write down the coordinates of the point where the curve $y = e^x$ cuts the y-axis.

[1]

$$\text{Pt is } (0, 1)$$

- (c) Hence, determine the number of solutions of the equation $e^x + x + 2 = 0$.

[2]

$$\begin{aligned}
 x + y &= -2 \\
 y &= -x - 2 \\
 e^x + x + 2 &= 0 \\
 e^x &= -x - 2 \quad (1m)
 \end{aligned}$$

$$\begin{aligned}
 \text{No. of solutions} &= \text{no. of intersections between} \\
 &\quad y = e^x \text{ and } x + y = -2 \\
 &= 1 \quad (1m)
 \end{aligned}$$

Given that $\log_2 x = p$ and $\log_4 y = q$, express the following in terms of p and/or q .

(a) $\log_2 \sqrt{x}$,

[2]

$$\begin{aligned}\log_2 x^{\frac{1}{2}} &= \frac{1}{2} \log_2 x \quad (1\text{m}) \\ &= \frac{1}{2} p \quad (1\text{m})\end{aligned}$$

(b) $\log_2 xy^2$,

[3]
[2]

$$\begin{aligned}&= \log_2 x + \log_2 y^2 \\ &= \log_2 x + 2 \log_2 y \quad (1\text{m}) \\ &= p + 2 \left(\frac{\log_4 y}{\log_4 2} \right) \quad (1\text{m}) \\ &= p + 2 \left(\frac{q}{\frac{1}{2}} \right) = p + 4q\end{aligned}$$

(c) $\log_4 \frac{4x}{y}$,

[3]
[2]

$$\begin{aligned}&= \log_4 4x - \log_4 y \\ &= \log_4 4 + \log_4 x - \log_4 y \\ &= 1 + \frac{\log_2 x}{\log_2 4} - q \\ &= 1 + \frac{p}{2 \log_2 2} - q \\ &= 1 + \frac{p}{2} - q \quad (1\text{m})\end{aligned}$$

6 ✓ (a) Solve the equation $\frac{27^{2+x}}{9} = 3^x \times 81^{2x-1}$.

[4]

$$\frac{3^{3(2+x)}}{3^2} = 3^x \times 3^{4(2x-1)} \quad (1m)$$

$$3^{6+3x-2} = 3^{x+8x-4}$$

Comparing power of 3

$$4+3x = 9x-4 \quad (1m)$$

$$6x = 8$$

$$x = \frac{4}{3} \quad (1m)$$

(b) Given that $25^{x+1} \times 2^{4x-1} = 32^x \times 5^{3x}$, evaluate 10^x .

[4]

$$5^{2(x+1)} \times 2^{4x-1} = 2^{5x} \times 5^{3x} \quad (1m)$$

$$\frac{5^{2x+2}}{5^{2x}} = \frac{2^{5x}}{2^{4x-1}}$$

$$5^{2x+2} = 2^{x+1}$$

$$\frac{5^2}{5^{2x}} = 2^x \times 2 \quad (1m)$$

$$\frac{5^2}{5^{2x}} = (2^x)(5^x)$$

$$\frac{25}{5^{2x}} = 10^x \quad (1m)$$