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SWISS COTTAGE SECONDARY SCHOOL
SECONDARY THREE EXPRESS
SECOND SEMESTRAL EXAMINATION

Name: _____ () Class: Sec 3E _____

ADDITIONAL MATHEMATICS

Paper 1

4047/01

Tuesday 6 October 2015

2 hours

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Write your answers on the separate Answer Paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

This question paper consists of 5 printed pages.

Setter: Ms Liew Jia Meng

Vetter: Ms Zoe Pow

[Turn over

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}.$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

Answer all questions.

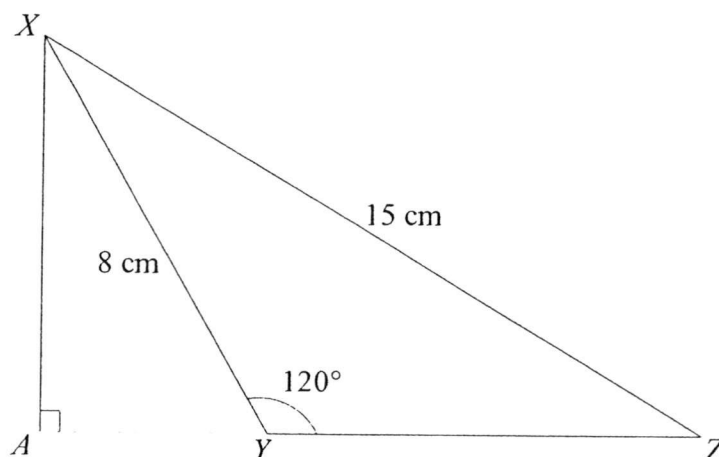
1 Solve $7^x = \frac{10}{3} - 7^{-x}$. [3]

2 Solve the equation $\frac{1}{8}e^x(e^x - 3) = \frac{1}{2}$. [3]

3 Find the value of $\frac{9^{x+2} + 12(9^x)}{27^{x+2} 3^{-x-2}}$. [3]

4 The line $x = y - 2$ intersects the curve $\frac{x^2}{3} + \frac{y^2}{4} = \frac{13}{4}$ at the points A and B .
Find the coordinates of the midpoint of A and B . [5]

- 5 The diagram shows a triangle XYZ in which $XY = 8$ cm, $XZ = 15$ cm and angle $XYZ = 120^\circ$. The line ZY is extended to the point A where angle $XAY = 90^\circ$.



- (i) Find the exact length of AY . [2]

- (ii) Show that the angle $XZY = \sin^{-1}\left(\frac{4\sqrt{3}}{15}\right)$. [3]

- 6 Given that $\tan A = \frac{8}{15}$ and that $\tan A$ and $\sin A$ have different signs, find the exact value of

(i) $\operatorname{cosec}(-A)$, [1]

(ii) $\sin(90^\circ - A)$, [1]

(iii) $\cos A \cot A$. [2]

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- 7 (i) Sketch the curve of $y = e^{2x+1} - 3$. [2]
- (ii) In order to solve the equation $\ln(x+6) - 1 = 2x$, a graph of a suitable straight line is drawn on the same set of axes as the graph of $y = e^{2x+1} - 3$.
- (a) Find the equation of this straight line. [2]
- (b) By sketching this line on the same axes as (i), determine the number of solutions to the equation $\ln(x+6) - 1 = 2x$. [2]
- 8 (i) Prove the identity $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$. [3]
- (ii) Hence, find all angles between 0° and 360° which satisfy the equation
- $$(\sec x - \tan x) = \frac{5}{6(\operatorname{cosec} x + 1)}. \quad [3]$$
- 9 (a) Without using a calculator, express $\left(\frac{\sqrt{48}}{6} + \frac{2}{\sqrt{12}} + \frac{36}{\sqrt{75}}\right) \times \frac{6}{\sqrt{2}}$ in the form of $p\sqrt{6}$. [3]
- (b) Given that $\frac{7\sqrt{2}}{3-\sqrt{2}} - \frac{5}{1+\sqrt{2}} = a + b\sqrt{2}$ such that a and b are integers, find the value of a and of b . [4]
- 10 (a) (i) Sketch, on the same axes, the graphs of $y^2 = -2x$ and $y = -\frac{5}{x^2}$. [2]
- (ii) State the value of k for which the x -coordinate of the points of intersection satisfies the equation $x^5 = k$. [2]
- (b) Given that $\log_m 2 = x$ and $\log_m 3 = y$, express $\log_m \left(\frac{\sqrt{2}}{9m}\right)$ in terms of x and y . [3]
- 11 (a) Find all the angles between 0° and 360° which satisfy the equation
- $$8\sin(2x - 40^\circ) = -5. \quad [3]$$
- (b) Given that $0 \leq x \leq 6$, find the values of x for which $3\sin^2 x + \cos x = \cos^2 x$. [4]

- 12 A certain radioactive substance is known to decay with time such that the amount of substance left after t days is given by $N = 180e^{-kt}$, where k is a constant. It is found that the amount of substance is halved after 3.5 days.

Find

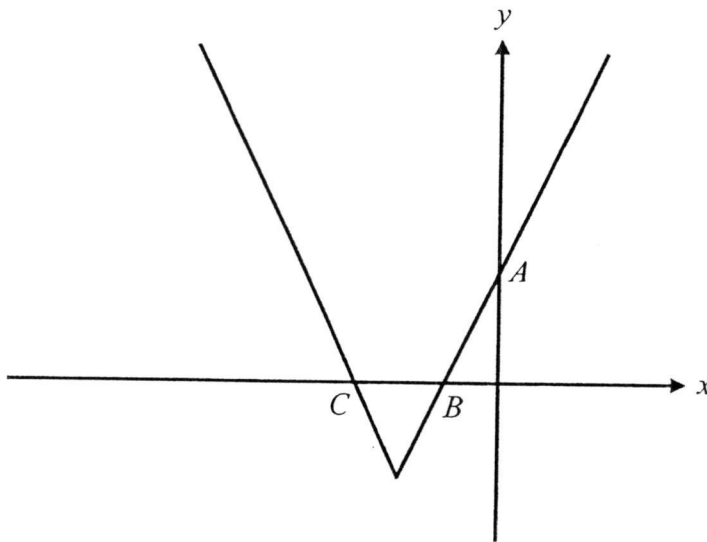
- (i) the initial amount of substance, [1]
- (ii) the value of k , [2]
- (iii) the number of days it takes before the amount of substance is reduced to 6 grams, [2]
- (iv) the amount of substance remaining after 1 week. [2]

- 13 Express the following in partial fractions.

(a) $\frac{3x - 21}{(x - 3)(x^2 + 1)}$ [4]

(b) $\frac{x^2 - x + 1}{x^2 - 5x - 6}$ [4]

- 14 The diagram shows part of the graph of $y = |3x + 5| - 2$.

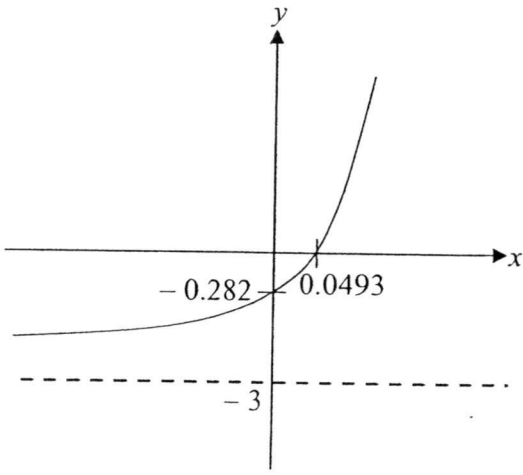


- (i) Find the coordinates of the points A , B and C . [3]
- (ii) Solve the equation $|3x + 5| - 2 = x + 4$. [2]
- (iii) Find the number of solutions of the equation $|3x + 5| - 2 = mx + 4$ when
 - (a) $m = -1$, [2]
 - (b) $m = 3$. [2]

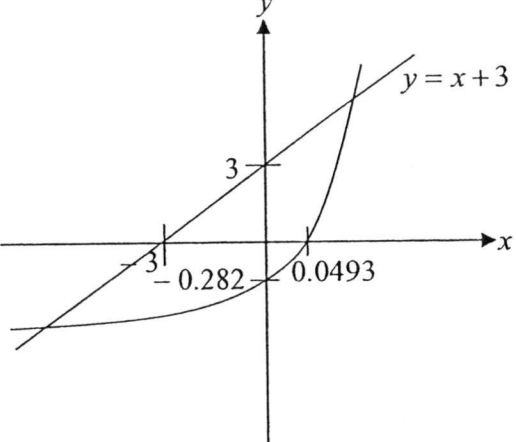
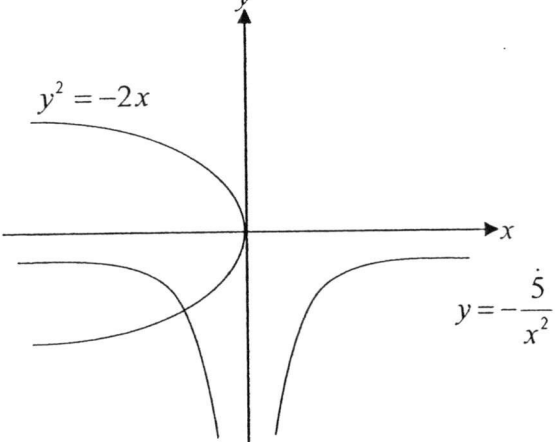
END OF PAPER

SECOND SEMESTRAL EXAMINATION 2015
Sec 3E AM P1 Solution

Paper 1 (80 marks)

Qn.	Answer Key
1.	$x = 0.565$ (3 sig. fig.) or $x = -0.565$ (3 sig. fig.)
2.	$x = 1.39$ (3 sig. fig.)
3.	$\frac{31}{27}$
4.	Coordinates of A and B $\left(1\frac{2}{7}, 3\frac{2}{7}\right)$ and $(-3, -1)$. Midpoint of $AB = \left(-\frac{6}{7}, 1\frac{1}{7}\right)$
5i.	$AY = 4$ units
6i.	$2\frac{1}{8}$
6ii.	$-\frac{15}{17}$
6iii.	$-1\frac{89}{136}$
7i.	 The graph shows a function on a Cartesian coordinate system. The x-axis and y-axis are shown. A curve passes through the points $(-0.282, 0)$ and $(0, 0.0493)$. A horizontal dashed line is drawn at $y = -3$.
7iia.	$y = x + 3$

SECOND SEMESTRAL EXAMINATION 2015
 Sec 3E AM P1 Solution

7iib.	Draw the line $y = x + 3$. 2 solutions 
8ii.	$x = 50.2^\circ, 230.2^\circ$ (to 1 decimal place)
9i.	$10\frac{1}{5}\sqrt{6}$
9ii.	$a = 7, b = -2$
10ai.	
10aii.	$k = -12\frac{1}{2}$
10b.	$\frac{1}{2}x - 2y - 1$
11i.	$x = 0.7^\circ, 129.3^\circ, 180.7^\circ, 309.3^\circ$
11ii.	$x = 0 \text{ rad}, 2.42 \text{ rad}, 3.86 \text{ rad}$
12i.	$N = 180$

SECOND SEMESTRAL EXAMINATION 2015

Sec 3E AM P1 Solution

12ii.	$k = 0.198$ (3 sig. fig.)
12iii.	$t = 18$ days (round up)
12iv.	$N = 45.0$ (3 sig. fig.)
13i.	$\frac{3x-21}{(x-3)(x^2+1)} = -\frac{6}{5(x-3)} + \frac{6x+33}{5(x^2+1)}$
13ii.	$\frac{x^2-x+1}{x^2-5x-6} = 1 + \frac{31}{7(x-6)} - \frac{3}{7(x+1)}$
14i.	Coordinates are $A(0,3)$, $B(-1,0)$ and $C\left(-2\frac{1}{3}, 0\right)$.
14ii.	$x = \frac{1}{2}$ or $x = -2\frac{3}{4}$
14iiia	$ 3x+5 -2 = -x+4$ Gradient of line is gentler than gradient of left arm. Line cuts both left and right arms at two points respectively. 2 solutions
14iiib	$ 3x+5 -2 = 3x+4$ Gradient of line is parallel to right arm. Line cuts left arm at only 1 point. 1 solution

SECOND SEMESTRAL EXAMINATION 2015

Sec 3E AM P1 Solution

Paper 1 (80 marks)

Qn.	Solution	Marker's Remarks
1.	$7^x = \frac{10}{3} - 7^{-x}$ $7^x = \frac{10}{3} - \frac{1}{7^x}$ Let $y = 7^x$ $y = \frac{10}{3} - \frac{1}{y}$ $y^2 - \frac{10}{3}y + 1 = 0$ $3y^2 - 10y + 3 = 0$ $(y-3)(3y-1) = 0$ $y = 3 \text{ or } y = \frac{1}{3}$ $7^x = 3 \text{ or } 7^x = \frac{1}{3}$ $x = \frac{\ln 3}{\ln 7} \text{ or } x = \frac{\ln \frac{1}{3}}{\ln 7}$ $x = 0.565 \text{ (3 sig. fig.) or } x = -0.565 \text{ (3 sig. fig.)}$	M1 - factorise M1 - taking ln A1
2.	$\frac{1}{8}e^x(e^x - 3) = \frac{1}{2}$ Let $y = e^x$ $\frac{1}{8}y(y-3) = \frac{1}{2}$ $y^2 - 3y - 4 = 0$ $(y-4)(y+1) = 0$ $y = 4 \text{ or } y = -1$ $e^x = 4 \quad \text{or} \quad e^x = -1 \text{ (NA)}$ $x = \ln 4$ $x = 1.39 \text{ (3 sig. fig.)}$	M1 - factorise M1 - take 'ln' A1

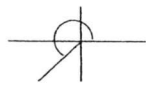
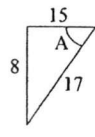
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 Sec 3E AM P1 Solution

3.	$\frac{9^{x+2} + 12(9^x)}{27^{x+2} 3^{-x-2}}$ $= \frac{3^{2(x+2)} + 12(3^{2x})}{3^{3(x+2)} 3^{-x-2}}$ $= \frac{3^{2x}(3^4) + 12(3^{2x})}{3^{2x+4}}$ $= \frac{3^{2x}(81+12)}{3^{2x}(81)}$ $= \frac{31}{27}$	M1 – change to base 3 M1 – factorise 3^{2x} correctly A1
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Sec 3E AM P1 Solution

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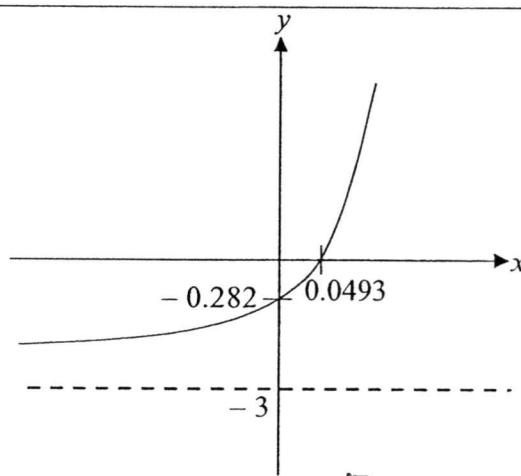
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5ii.	$\sin 60^\circ = \frac{AX}{8}$ $AX = 8 \sin 60^\circ$ $AX = 8 \left(\frac{\sqrt{3}}{2} \right)$ $AX = 4\sqrt{3} \text{ units} \quad \text{M1}$ $\sin \angle XZY = \frac{4\sqrt{3}}{15} \quad \text{M1}$ $\angle XZY = \sin^{-1} \left(\frac{4\sqrt{3}}{15} \right) \quad \text{A1}$	$\frac{\sin \angle XZY}{8} = \frac{\sin 120^\circ}{15} \quad \text{M1}$ $\sin \angle XZY = \frac{8}{15} \sin 60^\circ$ $= \frac{8}{15} \left(\frac{\sqrt{3}}{2} \right) \quad \text{M1}$ $= \frac{4\sqrt{3}}{15}$ $\angle XZY = \sin^{-1} \left(\frac{4\sqrt{3}}{15} \right) \quad \text{A1}$	
6i.	$\operatorname{cosec}(-A)$ $= \frac{1}{\sin(-A)}$ $= \frac{1}{-\sin A}$ $= \frac{1}{-\left(-\frac{8}{17}\right)}$ $= 2\frac{1}{8}$	 	B1
6ii	$\sin(90^\circ - A)$ $= \cos A$ $= -\frac{15}{17}$		B1

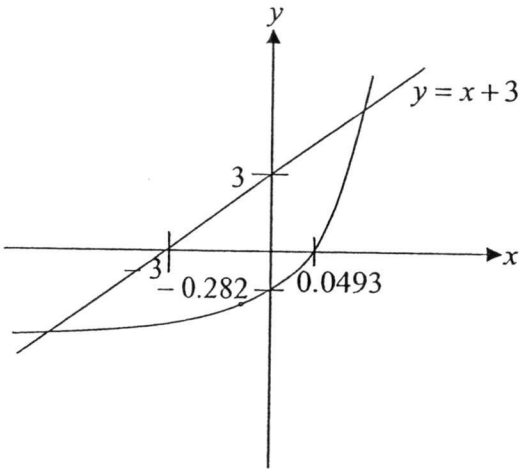
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6iii	$\cos A \cot A$ $= \left(-\frac{15}{17}\right) \left(\frac{\cos A}{\sin A}\right)$ $= \left(-\frac{15}{17}\right) \left(\frac{-\frac{15}{17}}{-\frac{8}{17}}\right)$ $= -1\frac{89}{136}$	M1 A1
7i	At $x = 0$, $y = e - 3$ $y = -0.282 \text{ (3 sig. fig.)}$ At $y = 0$, $e^{2x+1} = 3$ $2x + 1 = \ln 3$ $x = \frac{\ln 3 - 1}{2}$ $x = 0.0493 \text{ (3 sig. fig.)}$	B1 – shape B1 – label intercepts and indicate asymptote
7iia.	$\ln(x + 6) - 1 = 2x$ $\ln(x + 6) = 2x + 1$ $x + 6 = e^{2x+1}$ $x + 3 = e^{2x+1} - 3$ Equation of line: $y = x + 3$	M1 A1



SECOND SEMESTRAL EXAMINATION 2015
 Sec 3E AM P1 Solution

7iib	Draw the line $y = x + 3$. 2 solutions 	B1 – Drawing of line $y = x + 3$ B1 – No of solns
8i	$ \begin{aligned} & (\sec x - \tan x)(\operatorname{cosec} x + 1) \\ &= \sec x \operatorname{cosec} x + \sec x - \tan x \operatorname{cosec} x - \tan x \\ &= \left(\frac{1}{\cos x} \right) \left(\frac{1}{\sin x} \right) + \frac{1}{\cos x} - \left(\frac{\sin x}{\cos x} \right) \left(\frac{1}{\sin x} \right) - \frac{\sin x}{\cos x} \\ &= \frac{1}{\sin x \cos x} + \frac{1}{\cos x} - \frac{1}{\cos x} - \frac{\sin x}{\cos x} \\ &= \frac{1}{\sin x \cos x} - \frac{\sin x}{\cos x} \\ &= \frac{1 - \sin^2 x}{\sin x \cos x} \\ &= \frac{\cos^2 x}{\sin x \cos x} \\ &= \frac{\cos x}{\sin x} \\ &= \cot x \text{ (shown)} \end{aligned} $	M1 M1 A1

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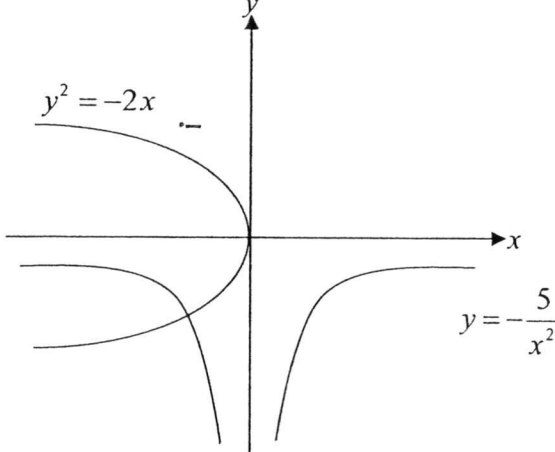
Sec 3E AM P1 Solution

8ii.	$(\sec x - \tan x) = \frac{5}{6(\operatorname{cosec} x + 1)}$ $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \frac{5}{6}$ $\cot x = \frac{5}{6}$ $\tan x = \frac{6}{5}$ $\text{Basic angle} = \tan^{-1} \frac{6}{5} = 50.19442891^\circ$ x lies in the 1st and 3rd quadrants. $x = 50.19442891^\circ, 180^\circ + 50.19442891^\circ$ $x = 50.2^\circ, 230.2^\circ \text{ (to 1 decimal place)}$	M1 – $\tan x$ M1 – basic angle A1
9i.	$\left(\frac{\sqrt{48}}{6} + \frac{2}{\sqrt{12}} + \frac{36}{\sqrt{75}} \right) \times \frac{6}{\sqrt{2}}$ $= \frac{\sqrt{48}}{\sqrt{2}} + \frac{12}{(\sqrt{12})(\sqrt{2})} + \frac{36(6)}{(\sqrt{75})(\sqrt{2})}$ $= \sqrt{24} + \sqrt{6} + \frac{216}{(\sqrt{25 \times 3})(\sqrt{2})}$ $= 2\sqrt{6} + \sqrt{6} + \frac{216}{5\sqrt{6}}$ $= 2\sqrt{6} + \sqrt{6} + \frac{216}{5\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$ $= 2\sqrt{6} + \sqrt{6} + \frac{216\sqrt{6}}{30}$ $= 10\frac{1}{5}\sqrt{6}$	M1 – simplify to square root 6 M1 – rationalize correctly A1

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Sec 3E AM P1 Solution

9ii.	$\frac{7\sqrt{2}}{3-\sqrt{2}} - \frac{5}{1+\sqrt{2}} = a + b\sqrt{2}$ $\frac{7\sqrt{2}(1+\sqrt{2}) - 5(3-\sqrt{2})}{(3-\sqrt{2})(1+\sqrt{2})} = a + b\sqrt{2}$ $\frac{7\sqrt{2} + 14 - 15 + 5\sqrt{2}}{3 + 2\sqrt{2} - 2} = a + b\sqrt{2}$ $\frac{12\sqrt{2} - 1}{1 + 2\sqrt{2}} \times \frac{1 - 2\sqrt{2}}{1 - 2\sqrt{2}} = a + b\sqrt{2}$ $\frac{12\sqrt{2} - 24(2) - 1 + 2\sqrt{2}}{-7} = a + b\sqrt{2}$ $\frac{14\sqrt{2} - 49}{-7} = a + b\sqrt{2}$ $7 - 2\sqrt{2} = a + b\sqrt{2}$ $a = 7, b = -2$	M1 – single M1 M1 A1
10ai		B1 each – Shape
10aii	$-2x = \left(-\frac{5}{x^2}\right)^2$ $-2x = \frac{25}{x^4}$ $x^5 = -12\frac{1}{2}$ $k = -12\frac{1}{2}$	M1 A1

SECOND SEMESTRAL EXAMINATION 2015

Sec 3E AM P1 Solution

10b	Given: $\log_m 2 = x$ and $\log_m 3 = y$ $\log_m \left(\frac{\sqrt{2}}{9m} \right)$ $= \log_m \sqrt{2} - \log_m (9m)$ $= \log_m \sqrt{2} - (\log_m 9 + \log_m m)$ $= \log_m \sqrt{2} - \log_m 9 - 1$ $= \frac{1}{2} \log_m 2 - 2 \log_m 3 - 1$ $= \frac{1}{2} x - 2y - 1$	M1 – pdt rule M1 – power rule A1
11i	$8 \sin(2x - 40^\circ) = -5 \quad 0^\circ \leq x \leq 360^\circ$ $\sin(2x - 40^\circ) = -\frac{5}{8} \quad 0^\circ \leq 2x \leq 720^\circ$ $-40^\circ \leq 2x - 40^\circ \leq 680^\circ$ Basic angle = $\sin^{-1} \left(\frac{5}{8} \right) = 38.68218745^\circ$ $(2x - 40^\circ)$ lies in the 3rd and 4th quadrants. $(2x - 40^\circ) = -38.68218745^\circ, 180^\circ + 38.68218745^\circ, 360^\circ - 38.68218745^\circ, 540^\circ + 38.68218745^\circ$ $(2x - 40^\circ) = -38.68218745^\circ, 218.6821874^\circ, 321.3178126^\circ, 578.6821874^\circ, 681.3178126^\circ$ $x = 0.7^\circ, 129.3^\circ, 180.7^\circ, 309.3^\circ$	M1 – basic angle M1 A1

Sec 3E AM P1 Solution

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Sec 3E AM P1 Solution

13i.	$\frac{3x-21}{(x-3)(x^2+1)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+1}$ $3x-21 = A(x^2+1) + (Bx+C)(x-3)$ Subst $x = 3$, $9-21 = 10A$ $10A = -12$ $A = -\frac{6}{5}$ Subst $x = 1$, $-18 = 2(-1\frac{1}{5}) - 2B - 2C$ $B + C = 7\frac{4}{5}$ $B = \frac{39}{5} - C$ Subst $x = 0$, $-21 = -1\frac{1}{5} - 3C$ $C = \frac{33}{5}$ $B = \frac{6}{5}$ $\frac{3x-21}{(x-3)(x^2+1)} = -\frac{6}{5(x-3)} + \frac{6x+33}{5(x^2+1)}$	M1
		M2
		A1

SECOND SEMESTRAL EXAMINATION 2015

Sec 3E AM P1 Solution

13ii.	$\frac{x^2 - x + 1}{x^2 - 5x - 6} = 1 + \frac{4x + 7}{(x - 6)(x + 1)}$ $\frac{4x + 7}{(x - 6)(x + 1)} = \frac{A}{x - 6} + \frac{B}{x + 1}$ $4x + 7 = A(x + 1) + B(x - 6)$ Subst $x = -1$, $-4 + 7 = -7B$ $B = -\frac{3}{7}$ Subst $x = 6$, $24 + 7 = 7A$ $A = \frac{31}{7}$ $\frac{4x + 7}{(x - 6)(x + 1)} = \frac{31}{7(x - 6)} - \frac{3}{7(x + 1)}$ $\frac{x^2 - x + 1}{x^2 - 5x - 6} = 1 + \frac{31}{7(x - 6)} - \frac{3}{7(x + 1)}$	M1 – Long Division M1 M1 – for both A and B A1
14i	$y = 3x + 5 - 2$ At $x = 0$, $y = 3(0) + 5 - 2$ $y = 3$ At $y = 0$, $0 = 3x + 5 - 2$ $3x + 5 = 2 \quad \text{or} \quad -(3x + 5) = 2$ $x = -1 \quad \text{or} \quad x = -2\frac{1}{3}$ Coordinates are $A(0, 3)$, $B(-1, 0)$ and $C(-2\frac{1}{3}, 0)$.	B1 each – for each coordinate

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Sec 3E AM P1 Solution

14ii.	$ 3x + 5 - 2 = x + 4$ $ 3x + 5 = x + 6$ $3x + 5 = x + 6$ or $-(3x + 5) = x + 6$ $2x = 1$ or $4x = -11$ $x = \frac{1}{2}$ or $x = -2\frac{3}{4}$	 M1 A1
14iiia	$ 3x + 5 - 2 = -x + 4$ Gradient of line is gentler than gradient of left arm. Line cuts both left and right arms at two points respectively. 2 solutions	 B1 B1
14iiib	$ 3x + 5 - 2 = 3x + 4$ Gradient of line is parallel to right arm. Line cuts left arm at only 1 point. 1 solution	 B1 B1

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SWISS COTTAGE SECONDARY SCHOOL
SECONDARY THREE EXPRESS
SECOND SEMESTRAL EXAMINATIONS

Name: _____ () Class: Sec _____

ADDITIONAL MATHEMATICS

Paper 2

4047/02

Thursday 8 October 2015

- 2 hours

Additional materials: Answer paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

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The use of a scientific calculator is expected, where appropriate.

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The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

Submit Sections A and B separately

This question paper consists of 5 printed pages.

Setter: Mrs Chen Yen Wah

Vetter: Ms Zoe Pow

[Turn over

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*We Nurture Students to **Think, Care and Lead** with P.R.I.D.E.*

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer all questions.

Section A (42 marks)

- 1 Find the range of values of x for which $(2x-3)^2 > x$. [3]
- 2 Find the term independent of x in the expansion of $\left(2x - \frac{1}{x^2}\right)^9$. [4]
- 3 Solve the equation $\log_3 x^2 - 1 = 3 \log_x 3$. [4]
- 4 Without using a calculator, solve, for x and y , the simultaneous equations

$$3^{x+1} = 27(3^{y-1}),$$

$$\log_2 6 + 1 = \log_2 (6x + 3y).$$
 [5]
- 5 Given that the expansion of $(k+x)(1-3x)^n$ in ascending powers of x is $3 - 44x + px^2 + \dots$ find the values of the constants k , n and p . [6]
- 6 (i) Show that $\frac{\tan A}{\sec A + 1} + \frac{\tan A}{\sec A - 1} = 2 \operatorname{cosec} A$. [4]
- (ii) Hence, find all the angles between 0 and 2π which satisfy the equation

$$\frac{\tan A}{\sec A + 1} + \frac{\tan A}{\sec A - 1} = 5$$
 [3]
- 7 (a) Find the value of k for which the line $y = 2x + k$ is a tangent to the curve $y = x^2 - 3x + 4$. [3]
- (b) Find the range of values of k for which $x^2 + 12x + 9$ is always greater than $4x + k$. [3]
- 8 The roots of the quadratic equation $2x^2 - 3x + 1 = 0$ are α and β .
 - (i) Show that $\alpha^2 - \alpha\beta + \beta^2 = (\alpha + \beta)^2 - 3\alpha\beta$. [1]
 - (ii) Find the quadratic equation whose roots are α^3 and β^3 . [6]

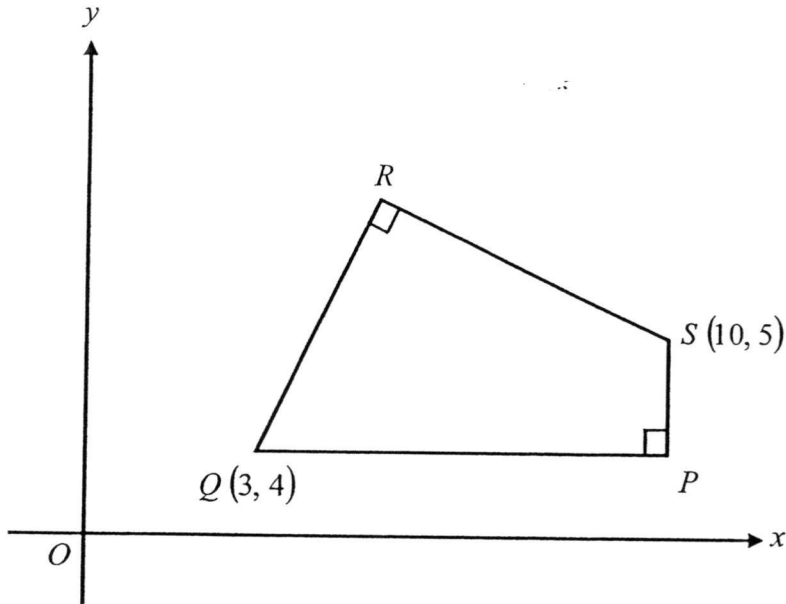
Begin Section B on a fresh sheet of paper

Section B (38 marks)

- 9 The function f is defined by $f(x) = 3 \cos 2x$ for $0 \leq x \leq 2\pi$.
- (i) State the amplitude and period of f . [2]
 - (ii) Find the x -coordinate(s) of the points whereby $f(x) = 3 \cos 2x$ meets the x -axis. [2]
 - (iii) Sketch the graph of $y = f(x)$ for $0 \leq x \leq 2\pi$. [3]
 - (iv) On the diagram drawn in part (iii), sketch the graph of $y = \frac{2x}{3\pi}$ for $0 \leq x \leq 2\pi$. [1]
 - (v) State the number of solutions, for $0 \leq x \leq 2\pi$, of the equation $9\pi \cos 2x = 2x$. [2]
- 10 A circle passes through the points $A(4, 11)$ and $B(6, 9)$.
Its centre lies on the line $y = 2x$. Find
- (i) the equation of the perpendicular bisector of AB , [3]
 - (ii) the coordinates of the centre of the circle, [2]
 - (iii) the equation of the circle, [2]
 - (iv) the equation of the circle reflected about the line $y = x$. [2]
- 11 (a) Solve the equation $2x^3 - 9x^2 + 3x + 4 = 0$. [5]
- (b) The expression $6x^3 + px^2 + qx + 10$, where p and q are constants, has a factor of $2x - 1$ and leaves a remainder of -20 when divided by $x + 2$. Find the value of p and of q . [4]

[Turn Over]

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The diagram shows a quadrilateral $PQRS$ in which SR is perpendicular to RQ and QP is perpendicular to PS . The point Q is $(3, 4)$ and the point S is $(10, 5)$.

Given that QR is parallel to the line $6x - 2y = 13$, find

(i) the equation of QR , [2]

(ii) the coordinates of R , [4]

(iii) the area of the quadrilateral $PQRS$. [2]

T is a point on the line SR such that the area of $\triangle QTR$: area of $\triangle QTS = 3 : 2$.

(iv) Find the coordinates of the point T . [2]

End of Paper

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Sec 3E AM P2 Answer Key

1. $x > \frac{9}{4} = 2\frac{1}{4}$ or $x < 1$
2. -5376
3. $x = 3\sqrt{3} = 5.20$
4. $x = 1\frac{2}{3}$ and $y = \frac{2}{3}$
5. $k = 3$, $n = 5$ and $p = 255$
6. $A = 0.412 \text{ rad (3sf)}$ or $A = 2.73 \text{ rad (3sf)}$
7. a) $k = -\frac{9}{4} = -2\frac{1}{4}$, b) $k < -7$
8. ii) $8x^2 - 9x + 1 = 0$
9. i) amplitude = 3, period = π , ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$, v) 4
10. i) $y = x + 5$, ii) $(5, 10)$, iii) $(x - 5)^2 + (y - 10)^2 = 2$, iv) $(x - 10)^2 + (y - 5)^2 = 2$
11. a) $x = 1$, $x = -\frac{1}{2}$ or $x = 4$, b) $q = -19$, $p = -5$
12. i) $y = 3x - 5$, ii) $R(4, 7)$, iii) 13.5 units^2 , iv) $T = \left(7\frac{3}{5}, 5\frac{4}{5}\right)$

Sec 3E AM P2 Solution

Qn. #	Solution	Mark Allocation	Markers' Report
1	$(2x-3)^2 > x$ $4x^2 - 12x + 9 > x$ $4x^2 - 13x + 9 > 0$ $(4x-9)(x-1) > 0$ $x > \frac{9}{4} = 2\frac{1}{4}$ or $x < 1$	M1 – Simplification M1 – Factorisation A1	
2	<u>Method 1</u> $T_{r+1} = \binom{9}{r} (2x)^{9-r} \left(-\frac{1}{x^2}\right)^r$ $= \binom{9}{r} (2)^{9-r} (x)^{9-r} (-1)^r (x^{-2})^r$ $= \binom{9}{r} (2)^{9-r} (-1)^r (x)^{9-r} (x)^{-2r}$ $= \binom{9}{r} (2)^{9-r} (-1)^r (x)^{9-r-2r}$ $(x)^{9-3r} = x^0$ $9-3r = 0$ $r = 3$ $T_4 = \binom{9}{3} (2)^{9-3} (-1)^3$ $= -5376$ <u>Method 2</u> $\left(2x - \frac{1}{x^2}\right)^9$ $= (2x)^9 + \binom{9}{1} (2x)^8 \left(-\frac{1}{x^2}\right) + \binom{9}{2} (2x)^7 \left(-\frac{1}{x^2}\right)^2 + \binom{9}{3} (2x)^6 \left(-\frac{1}{x^2}\right)^3 + \dots$ By inspection, the 4 th term is the term independent of x .	M1 – Simplification M1 – Find r M1 A1 M1 – Pattern clear M1 – Stating term M1 A1	112

SECOND SEMESTRAL EXAMINATIONS 2015
Sec 3E AM P2 Solution

Qn. #	Solution	Mark Allocation	Markers' Report
	$T_4 = \binom{9}{3}(2)^6(-1)^3$ $= -5376$		
3	$\log_3 x^2 - 1 = 3 \log_x 3$ $2 \log_3 x - 1 = \frac{3}{\log_3 x}$ Let $u = \log_3 x$. $2u - 1 = \frac{3}{u}$ $2u^2 - u - 3 = 0$ $(2u - 3)(u + 1) = 0$ $u = \frac{3}{2} \quad \text{or } u = -1$ $\log_3 x = \frac{3}{2} \quad \text{or } \log_3 x = -1$ $x = 3^{\frac{3}{2}} \quad \text{or } x = 3^{-1}$ $x = \sqrt{3^3} \quad \text{or } x = \frac{1}{3}$ $x = \sqrt{27} \quad \text{or } x = \frac{1}{3}$ $x = 3\sqrt{3} = 5.20 \text{ (3sf)}$	M1 – Convert base M1 – Factorisation M1 – Log form to index form A1 – Both correct	
4	$3^{x+1} = 27(3^{y-1})$ Eq (1) $\log_2 6 + 1 = \log_2 (6x + 3y)$ Eq (2) From (1), $3^{x+1} = 3^3(3^{y-1})$ $x + 1 = 3 + y - 1$ $x - y = 1$ Eq (3) From (2), $\log_2 6 + \log_2 2 = \log_2 (6x + 3y)$ $\log_2 6 \times 2 = \log_2 (6x + 3y)$ $\log_2 12 = \log_2 (6x + 3y)$ $6x + 3y = 12$ $2x + y = 4$ Eq (4)	- - - -	M1 – Change of base and laws of indices M1 – Logarithmic laws M1 – substitution or elimination

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Sec 3E AM P2 Solution

Qn. #	Solution	Mark Allocation	Markers' Report
6i	$\begin{aligned} \text{LHS} &= \frac{\tan A}{\sec A + 1} + \frac{\tan A}{\sec A - 1} \\ &= \frac{\tan A(\sec A - 1) + \tan A(\sec A + 1)}{(\sec A + 1)(\sec A - 1)} \\ &= \frac{\tan A[(\sec A - 1) + (\sec A + 1)]}{(\sec A + 1)(\sec A - 1)} \\ &= \frac{\tan A(\sec A - 1 + \sec A + 1)}{\sec^2 A - 1^2} \\ &= \frac{\tan A(2 \sec A)}{\sec^2 A - 1^2} \\ &= \frac{\tan A(2 \sec A)}{\tan^2 A} \\ &= \frac{(2 \sec A)}{\tan A} \\ &= \frac{2}{\cos A} \div \frac{\sin A}{\cos A} \\ &= \frac{2}{\cos A} \times \frac{\cos A}{\sin A} \\ &= \frac{2}{\sin A} \\ &= 2 \operatorname{cosec} A \\ &= \text{RHS (proven)} \end{aligned}$	M1 – Trigo Identity $(\sec A + 1)(\sec A - 1)$ $= \sec^2 A - 1$ M1 – Trigo Identity $(\sec^2 A - 1 = \tan^2 A)$ M1 – $\sec A = \frac{1}{\cos A}$ A1 – Simplification	
6ii	$\begin{aligned} \frac{\tan A}{\sec A + 1} + \frac{\tan A}{\sec A - 1} &= 5 \\ 2 \operatorname{cosec} A &= 5 \\ \frac{2}{\sin A} &= 5 \\ \sin A &= \frac{2}{5} \\ \text{Basic angle, } \alpha &= \sin^{-1} \frac{2}{5} = 0.412 \text{ rad} \\ A &= 0.412 \text{ rad (3sf) or } A = 2.73 \text{ rad (3sf)} \end{aligned}$	M1 – Simplification M1 – Basic angle A1	
7a)	$\begin{aligned} y &= 2x + k && \text{- Eq 1} \\ y &= x^2 - 3x + 4 && \text{- Eq 2} \end{aligned}$ Sub (1) into (2), $x^2 - 3x + 4 = 2x + k$ $x^2 - 5x + 4 - k = 0$	M1 – Simplifying	

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Sec 3E AM P2 Solution

Qn. #	Solution	Mark Allocation	Markers' Report
	For line to be tangent to curve, $b^2 - 4ac = 0$. $(-5)^2 - 4(1)(4-k) = 0$ $25 - 16 + 4k = 0$ $k = -\frac{9}{4} = -2\frac{1}{4}$	M1 – Discriminant = 0 A1	
7b)	$x^2 + 12x + 9 > 4x + k$ $x^2 + 8x + 9 - k > 0$ $b^2 - 4ac < 0$ $8^2 - 4(1)(9-k) < 0$ $64 - 36 + 4k < 0$ $4k < -28$ $k < -7$	M1 – Simplifying M1 – Discriminant < 0 A1	
8i)	$\alpha^2 - \alpha\beta + \beta^2$ $= \alpha^2 + 2\alpha\beta + \beta^2 - 3\alpha\beta$ $= (\alpha + \beta)^2 - 3\alpha\beta$ (shown)	B1	
8ii)	$2x^2 - 3x + 1 = 0$ $\alpha + \beta = \frac{3}{2}$ $\alpha\beta = \frac{1}{2}$ $\alpha^3 + \beta^3$ $= (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$ $= \left(\frac{3}{2}\right)\left[\left(\frac{3}{2}\right)^2 - 3\left(\frac{1}{2}\right)\right]$ $= 1\frac{1}{8}$ $\alpha^3\beta^3 = (\alpha\beta)^3$ $= \left(\frac{1}{2}\right)^3$ $= \frac{1}{8}$	M1 M1 M1 – factorise using sum of cube $\alpha^3 + \beta^3$ M1 M1 A1	114

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Qn. #	Solution	Mark Allocation	Markers' Report
	New equation is $x^2 - \frac{9}{8}x + \frac{1}{8} = 0$ $8x^2 - 9x + 1 = 0$		
9i)	amplitude = 3 period = π	B1 B1	
9ii)	When $y = 0$, $3\cos 2x = 0$ $\cos 2x = 0$ $2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$	A1/A1 (1 mark for every 2 correct answers)	
9iii) 9iv)		Shape – B1 Period – B1 Coordinates – B1 $y = \frac{2x}{3\pi} = \text{B1}$ $x = \pi, y = \frac{2}{3}$ $x = 2\pi, y = \frac{4}{3}$	
9v)	$9\pi \cos 2x = 2x$ $3\cos 2x = \frac{2x}{3\pi}$ No. of solutions = 4	M1 A1	
10i)	Let equation of perpendicular bisector of AB be $y = mx + c$. $m_{AB} = \frac{11-9}{4-6} = -1$ Since $m_{AB} = -\frac{1}{m}$, $m = 1$. Therefore, $y = x + c$	M1 – gradient of m M1 – midpoint of AB	

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Sec 3E AM P2 Solution

Qn. #	Solution	Mark Allocation	Markers' Report
	Midpoint, M of AB $= \left(\frac{4+6}{2}, \frac{11+9}{2} \right) = (5, 10).$ $M(5, 10)$ lies on perpendicular bisector, hence $y - 10 = x - 5$ $y = x + 5$	A1	
10ii)	Centre passes through perpendicular bisector as well as line $y = 2x$, hence, $y = x + 5 \quad \text{- eq (1)}$ $y = 2x \quad \text{- eq (2)}$ Sub (2) into (1), $2x = x + 5$ $x = 5$ $y = 10$ Centre = $(5, 10)$	M1 – Substitution A1	
10iii)	Let equation of circle be $(x - 5)^2 + (y - 10)^2 = r^2$ Using $C(5, 10)$ and $A(4, 11)$, $r^2 = (11 - 10)^2 + (4 - 5)^2 = 2$ Therefore, $(x - 5)^2 + (y - 10)^2 = 2$	M1 – Find r^2 A1	
10iv)	Centre becomes $(10, 5)$ Therefore, new equation is $(x - 10)^2 + (y - 5)^2 = 2$	M1 – new centre A1	
11a)	Let $f(x) = 2x^3 - 9x^2 + 3x + 4$ $f(1) = 0$, therefore $(x - 1)$ is a factor of $f(x)$. $2x^3 - 9x^2 + 3x + 4 = (x - 1)(ax^2 + bx + c)$ Comparing coefficient of x^3, $a = 2.$ Comparing constant, $c = -4.$ Comparing coefficient of x, $3 = c - b$ $3 = -4 - b$	M1 (find factor) M1 M1	115

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Sec 3E AM P2 Solution

Qn. #	Solution	Mark Allocation	Markers' Report
	$b = -7$ $2x^3 - 9x^2 + 3x + 4 = (x-1)(2x^2 - 7x - 4)$ $2x^3 - 9x^2 + 3x + 4 = (x-1)(2x+1)(x-4)$ $(x-1)(2x+1)(x-4) = 0$ $x = 1, x = -\frac{1}{2} \text{ or } x = 4$	M1 A1	
11b)	Let $f(x) = 6x^3 + px^2 + qx + 10$ Since $f\left(\frac{1}{2}\right) = 0$, $6\left(\frac{1}{2}\right)^3 + p\left(\frac{1}{2}\right)^2 + q\left(\frac{1}{2}\right) + 10 = 0$ $\frac{3}{4} + \frac{p}{4} + \frac{q}{2} + 10 = 0$ $\frac{p}{4} + \frac{q}{2} + 10\frac{3}{4} = 0$ $p + 2q = -43$ Eq (1) Since $f(-2) = -20$, $6(-2)^3 + p(-2)^2 + q(-2) + 10 = -20$ $-48 + 4p - 2q + 10 = -20$ $4p - 2q = 18$ $2p - q = 9$ Eq (2) From (1), $p = -43 - 2q$ Eq (3) Sub (3) into (2), $2(-43 - 2q) - q = 9$ $-86 - 4q - q = 9$ $-5q = 95$ $q = -19$ $p = -5$	- M1 – Form Eq (1) M1 – Form Eq (2) - M1 – Simplification A1 A1	
12i)	Let the equation of QR be $y = mx + c$.		

SECOND SEMESTRAL EXAMINATIONS 2015

Sec 3E AM P2 Solution

Qn. #	Solution	Mark Allocation	Markers' Report
	$m = \frac{6}{2} = 3$ Sub (3, 4), $4 = 3(3) + c$ $c = -5$ $y = 3x - 5$	M1 – Gradient A1	
12ii)	Let equation of RS be $y = mx + c$. $m = -\frac{1}{3}$ Sub (10, 5), $5 = \left(-\frac{1}{3}\right)(10) + c$ $c = 8\frac{1}{3}$ $y = -\frac{1}{3}x + 8\frac{1}{3}$ To find R, $y = 3x - 5$ - Eq (1) $y = -\frac{1}{3}x + 8\frac{1}{3}$ - Eq (2) Sub (1) into (2), $3x - 5 = -\frac{1}{3}x + 8\frac{1}{3}$ $9x - 15 = -x + 25$ $10x = 40$ $x = 4$ $y = 3(4) - 5 = 7$ R (4, 7)	M1 – Gradient M1 – Equation of RS M1 – Simplification A1	
12iii)	P (10, 4) Area of PQRS $= \frac{1}{2} \times \begin{vmatrix} 10 & 10 & 4 & 3 & 10 \\ 4 & 5 & 7 & 4 & 4 \end{vmatrix}$	M1	116

Sec 3E AM P2 Solution

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