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- 1 Find the value of x for which $\frac{3^{2x-1}}{\sqrt{9^x}} = \frac{1}{27}$. [2]
- 2 Find the range of values of k for which $y = (k+2)x^2 - 12x + 2(k-1)$ cuts the x -axis at two distinct points and has a minimum point. [4]
- 3 The expression $x^3 + ax^2 + bx - 6$ is divisible by $(x+3)$ but leaves a remainder of -8 when divided by $(x-1)$.
- (i) Find the value of a and of b . [4]
- (ii) Factorise the expression completely. [2]
- 4 (i) Given that $(3-\sqrt{7})x = 3+\sqrt{7}$, express x in the form $a+b\sqrt{7}$ where a and b are constants. [3]
- (ii) Hence, evaluate $x + \frac{1}{x}$ without using a calculator. [3]
- 5 (i) Solve the simultaneous equations
 $x = 1 + 2y$,
 $x = 6xy - 2y - 3$. [5]
- (ii) Explain the geometrical meaning of the solutions. [1]
- 6 Given that $\frac{4x^3 + 7x^2 - 13x - 2}{x^2 + 2x - 3} = ax + b + \frac{x+c}{x^2 + 2x - 3}$,
- (i) find the value of each of the integers a , b and c . [4]
- Hence, using the values of a , b and c obtained in part (i),
- (ii) express $\frac{4x^3 + 7x^2 - 13x - 2}{x^2 + 2x - 3}$ in partial fractions. [3]

[Turn over]

- 7 The roots of the equation $2x^2 - 4x + 3 = 0$ are α and β .
- (i) Find in the form $ax^2 + bx + c = 0$, where a , b and c are integers, the equation whose roots are $\alpha\beta^2$ and $\alpha^2\beta$. [5]
- (ii) Find the exact value of $\alpha^3 + \beta^3$ without finding α and β . [3]
- 8 By using the substitution $u = 2^x$ or otherwise,
- (i) find the values of x such that $2^{2x+1} - 7 \times 2^x + 3 = 0$, [5]
- (ii) prove that $2^{x+3} - 2^x$, is exactly divisible by 7. [2]
- 9 Given that $y = mx + c$ is a tangent to the curve $y^2 = 10x$, find the relationship between m and c . [4]
- 10 (a) Given that $\frac{1}{\log_a ab} + \frac{1}{\log_b ab} = a - b$, express b in terms of a . [3]
- (b) Solve the equation $\lg(20 + 5x) - \lg(10 - x) = 1$. [3]
- 11 (a) Sketch the graph of $y = \log_3 x$ [2]
- (b) It is observed that the population of a city is growing in such a way that its population, P , satisfies the formula $P = 500\,000 e^{kt}$, where k is a constant and t is the time in years from beginning of 1990.
- (i) If the population of the city was 700 000 in the beginning of 2005, find the value of k . [1]
- (ii) Find the population in the beginning of 2010, giving your answer correct to the nearest 10 000. [2]
- (iii) Find the year in which the population will first reach 1.5 million. [2]

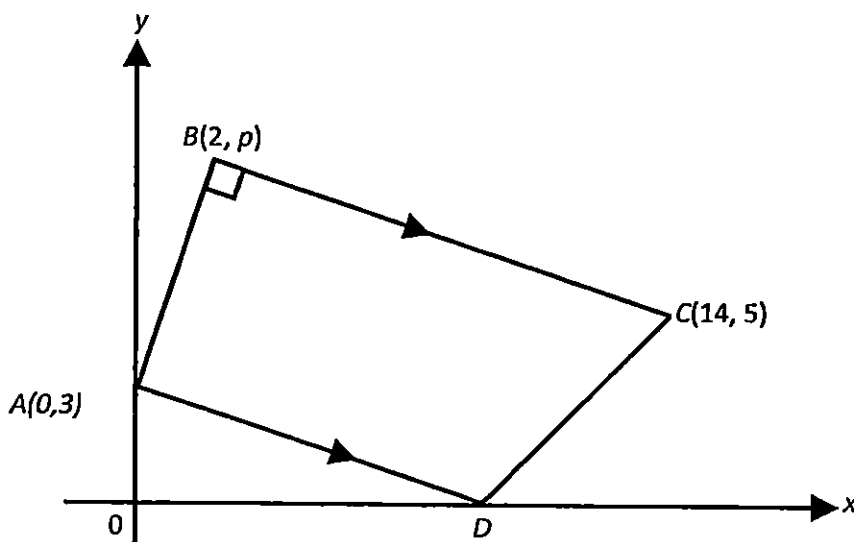
- 12** A piece of wire, 360 cm long, is used to make the twelve edges of a rectangular box in which the length is twice the breadth.

(i) Denoting the breadth of the box by x cm and the height by h cm, express h in terms of x and show that the volume of the box, $V \text{ cm}^3$, is given by

$$V = 180x^2 - 6x^3 \quad [3]$$

(ii) Determine, by solving a cubic equation, the dimensions of the box so that the volume is 24000 cm^3 . [4]

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The diagram shows a trapezium $ABCD$. The coordinates of A , B and C are $(0, 3)$, $(2, p)$ and $(14, 5)$ respectively. AB is perpendicular to BC and AD is parallel to BC .

- (i) Show that $p = 9$. [2]
- (ii) Find the equation of the perpendicular bisector of AB . [4]
- (iii) Find the coordinates of D . [2]
- (iv) Calculate the area of trapezium $ABCD$. [2]

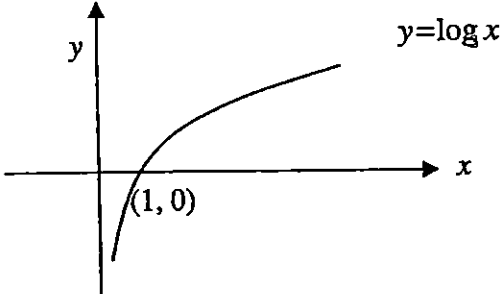
3Exp AM (4047) - Marking Scheme

Question	Solution	Marks
1	$\frac{3^{2x-1}}{3^x} = \frac{1}{3^3}$ $3^{x-1} = 3^{-3}$ $x-1 = -3$ $x = -2$	M1A1[2]
2	$(-12)^2 - 4(k+2)(2k-2) \geq 0$ $144 - 8k^2 - 8k + 16 \geq 0$ $-8k^2 - 8k + 160 \geq 0$ $k^2 + k - 20 \geq 0$ $(k+5)(k-4) \geq 0$ $-5 \leq k \leq 4$ $k+2 \geq 0 \text{ (minimum point)}$ $k \geq -2$ $\therefore -2 \leq k \leq 4$	M1 M1A1 A1 [4]
3	(i) Let $f(x) = x^3 + ax^2 + bx - 6$ $f(-3) = 0 \Rightarrow (-3)^3 + a(-3)^2 + b(-3) - 6 = 0$ $-27 + 9a - 3b - 6 = 0$ $3a - b = 11 \quad \text{--- (1)}$ $f(1) = -8 \Rightarrow 1 + a + b - 6 = -8$ $a + b = -3 \quad \text{--- (2)}$ $(1) + (2) \Rightarrow 4a = 8$ $a = 2$ $b = -5$	M1 M1 A2 [4]

	(ii) $ \begin{array}{r} x^2 - x - 2 \\ x+3 \overline{) x^3 + 2x^2 - 5x - 6} \\ \underline{x^3 + 3x^2} \\ -x^2 - 5x \\ \underline{-x^2 - 3x} \\ -2x - 6 \\ \underline{-2x - 6} \\ 0 \end{array} $ $ \begin{aligned} x^3 + 2x^2 - 5x - 6 &= (x+3)(x^2 - x - 2) \\ &= (x+3)(x-2)(x+1) \end{aligned} $	M1 A1 [2]
4	(i) $ \begin{aligned} x &= \frac{3+\sqrt{7}}{3-\sqrt{7}} \\ &= \frac{(3+\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})} \\ &= \frac{9+6\sqrt{7}+7}{3^2-(\sqrt{7})^2} \\ &= \frac{16+6\sqrt{7}}{2} \\ &= 8+3\sqrt{7} \end{aligned} $ (ii) $ \begin{aligned} x + \frac{1}{x} &= 8+3\sqrt{7} + \frac{1}{8+3\sqrt{7}} \\ &= 8+3\sqrt{7} + \frac{8-3\sqrt{7}}{8^2-(3\sqrt{7})^2} \\ &= 8+3\sqrt{7} + \frac{8-3\sqrt{7}}{64-63} \\ &= 8+3\sqrt{7} + 8-3\sqrt{7} \\ &= 16 \end{aligned} $	B1 M1 A1 [3] M1 M1A1[3]

6(ii)	$x^2 + 2x - 3 = (x+3)(x-1)$ $\text{Let } \frac{x-5}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$ $x-5 = A(x-1) + B(x+3)$ $x=1$ $1-5=4B$ $B=-1$ $x=-3$ $-3-5=-4A$ $A=2$ $\therefore \frac{4x^3 + 7x^2 - 13x - 2}{x^2 + 2x - 3} = 4x - 1 + \frac{2}{x+3} - \frac{1}{x-1}$	M1 M1 A1 [3]
7(i)	$\alpha + \beta = 2 \quad \alpha\beta = \frac{3}{2}$ $\alpha\beta^2 + \alpha^2\beta = \alpha\beta(\alpha + \beta)$ $= 2 \times \frac{3}{2}$ $= 3$ $(\alpha\beta^2)(\alpha^2\beta) = (\alpha\beta)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$ $\therefore \text{The equation is}$ $x^2 - 3x + \frac{27}{8} = 0$ $8x^2 - 24x + 27 = 0$	B1 B1 M1 M1 A1 [5]
7(ii)	$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= (\alpha + \beta)[(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta]$ $= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$ $= 2 \left[2^2 - 3 \times \frac{3}{2} \right]$ $= 2 \left(4 - \frac{9}{2} \right)$ $= -1$	B1 M1 A1[3]

8(i)	$2^{2x+1} - 7 \times 2^x + 3 = 0$ $2^{2x} \times 2^1 - 7 \times 2^x + 3 = 0$ Let $u = 2^x$ $2u^2 - 7u + 3 = 0$ $(2u - 1)(u - 3) = 0$ $u = \frac{1}{2} \quad \text{or} \quad u = 3$ $2^x = 2^{-1} \quad \quad \quad 2^x = 3$ $x = -1 \quad \quad \quad x \lg 2 = \lg 3$ $\quad \quad \quad x = \frac{\lg 3}{\lg 2} = 1.58$ $x = -1, 1.58$	M1A1 M1 A2 [5]
8(ii)	$2^{x+3} - 2^x = 2^x \times 2^3 - 2^x$ $= 2^x(2^3 - 1)$ $= 2^x \times 7$ $\therefore 2^{x+3} - 2^x \text{ is exactly divisible by } 7$	M1 A1 [2]
9	$(mx + c)^2 = 10x$ $m^2x^2 + 2mxc + c^2 - 10x = 0$ $m^2x^2 + (2mc - 10)x + c^2 = 0$ $(2mc - 10)^2 - 4m^2c^2 = 0$ $4m^2c^2 - 40mc + 100 - 4m^2c^2 = 0$ $40mc = 100$ $mc = \frac{5}{2}$	M1 M1 M1A1[4]
10(a)	$\frac{1}{\log_a ab} + \frac{1}{\log_b ab} = a - b$ $\log_{ab} a + \log_{ab} b = a - b$ $\log_{ab} ab = a - b$ $1 = a - b$ $b = a - 1$	B1 B1 A1

10(b)	$\lg(20+5x) - \lg(10-x) = 1$ $\lg\left(\frac{20+5x}{10-x}\right) = \lg 10$ $\frac{20+5x}{10-x} = 10$ $20+5x = 100-10x$ $15x = 80$ $x = \frac{16}{3}$	B1 M1 A1 [3]
11(a)	 $y = \log x$	B210
11(b)	(b) (i) $700000 = 500000e^{15k}$ $\ln \frac{7}{5} = 15k$ $k = 0.022431482$ $k = 0.0224 \text{ (3 s.f.)}$ (ii) population in the beginning of 2010 $= 500000e^{0.022431482 \times 20}$ $= 783082.2525$ $= 780000 \text{ (correct to the nearest 10000)}$ (iii) $1.5 = 0.5e^{kt}$ $3 = e^{0.022431482t}$ $\ln 3 = 0.022431482t$ $t = \frac{\ln 3}{0.022431482} = 48.976$ $\therefore$ The year is 2038 (2039 also accepted in this case as the value of t is very close to 49)	A1 [1] M1 A1 [2] M1 A1 [2]

12(i)	$4(2x + x + h) = 360$ $3x + h = 90$ $h = 90 - 3x$ $V = (2x)(x)(90 - 3x)$ $= 2x^2(90 - 3x)$ $= 180x^2 - 6x^3$	B1 M1 A1[3]
12(ii)	$180x^2 - 6x^3 = 24000$ $-6x^3 + 180x^2 - 24000 = 0$ $x^3 - 30x^2 + 4000 = 0$ Let $f(x) = x^3 - 30x^2 + 4000$ $f(20) = 20^3 - 30(20)^2 + 4000 = 0$ $(x - 20)$ is a factor $ \begin{array}{r} x^2 - 10x - 200 \\ x - 20 \overline{) x^3 - 30x^2 + 0x + 4000} \\ \underline{x^3 - 20x^2} \\ -10x^2 + 0x \\ \underline{-10x^2 + 200x} \\ -200x + 4000 \\ \underline{-200x + 4000} \\ 0 \end{array} $ $(x - 20)(x^2 - 10x - 200) = 0$ $(x - 20)(x - 20)(x + 10) = 0$ $x = 20, 20, -10 \text{ (NA)}$ $\therefore x = 20$ Length = 40 cm, Breadth = 20 cm and Height = 30 cm	M1 M1 A1 A1 [4]

13(i)	$\frac{5-p}{12} \times \frac{p-3}{2} = -1$ $(5-p)(p-3) = -24$ $5p - p^2 - 15 + 3p = -24$ $p^2 - 8p - 9 = 0$ $(p+1)(p-9) = 0$ $p = -1 \quad \text{or} \quad p = 9$ (NA $\therefore P = 9$)	M1 A1[2]
13(ii)	A (0,3) and B (2,9) Midpoint of AB is (1,6) $m_{AB} = \frac{9-3}{2-0} = 3$ Gradient of perpendicular bisector = $-\frac{1}{3}$ Equation of the perpendicular bisector is $y - 6 = -\frac{1}{3}(x - 1)$ $3y - 18 = -x + 1$ $x + 3y = 19$	B1 M1 M1 A1[4]
12(iii)	Equation of AD is $y = -\frac{1}{3}x + 3$ When $y = 0$ $0 = -\frac{1}{3}x + 3$ $x = 9$ $\therefore D(9,0)$	M1 A1[2]
12(iv)	Area of ABCD = $\frac{1}{2} \begin{vmatrix} 0 & 9 & 14 & 2 & 0 \\ 3 & 0 & 5 & 9 & 3 \end{vmatrix}$ $= \frac{1}{2} [(0 + 45 + 126 + 6) - (27 + 0 + 10 + 0)]$ $= 70 \text{ units}^2$	M1 A1 [2]