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Name	Register Number	Class
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MACPHERSON SECONDARY SCHOOL

MATHEMATICS

Paper 1

October 2012

1 hour

END-OF-YEAR EXAMINATION

SECONDARY TWO EXPRESS

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

Show all your workings on the same page as the rest of the answer.

Omission of essential working will result in loss of marks.

The total marks for this paper is 50.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

For Examiner's Use	
Total	50

This question paper consists of 9 printed pages including this page.

[Turn over

1 Factorise each of the following completely

(a) $16x^2 - y^2$

(b) $3x^2 + 4x - 15$

Answers: (a) [2]

(b) [2]

2 Simplify the following:

(a) $\frac{1}{x-1} - \frac{2x}{x^2-1}$

(b) $\frac{x^2-36y^2}{3x+4y} \div \frac{x-6y}{12x+16y}$

Answers: (a) [2]

(b) [2]

- 3 The scale of a map is 1 : 40 000 . A rectangular plot of land is represented by a rectangle of width 4 cm and length 8 cm on the map.

(a) Find, in km, the actual length of the land.

[Hint: 100 000 cm = 1 km]

(b) Find, in km^2 , the actual area of the land.

Answers: (a) [2]

(b) [2]

4

It is given that $s = ut + \frac{at^2}{2}$.

(a) Express a in terms of s , u and t .

(b) Find the value of a if $s = 200$, $u = 5$ and $t = 4$.

Answers: (a) [2]

(b) [2]

[Turn over

- 5 Solve the simultaneous equations $5x + 3y = 14$ and $3x + 5y = 18$.

Answer: [2]

- 6 It is given that y is inversely proportional to $(x^2 + 3)$. When $x = 2$, $y = 1$.
- (a) Find the equation connecting x and y .
- (b) Find the value of x when $y = 2\frac{1}{3}$.

Answers: (a) [2]

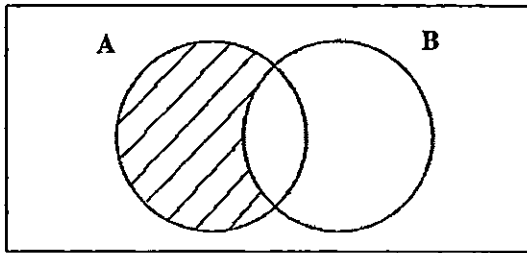
(b) [1]

[Turn over

- 7 In the Venn diagram below, state the region represented by the shaded area.

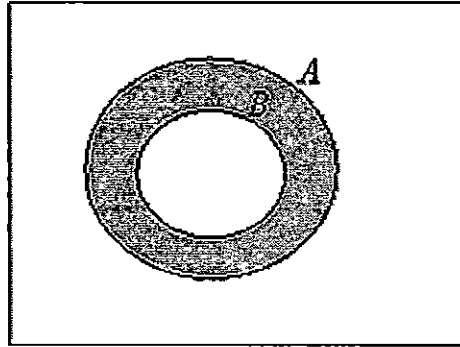
(a)

$\mathcal{E}$



(b)

$\mathcal{E}$



Answer: (a) [1]

(b) [1]

- 8 Solve the following:

(a) $(x - 2)^2 = 49$

(b) $(3x + 7)(2 - x) = 0$

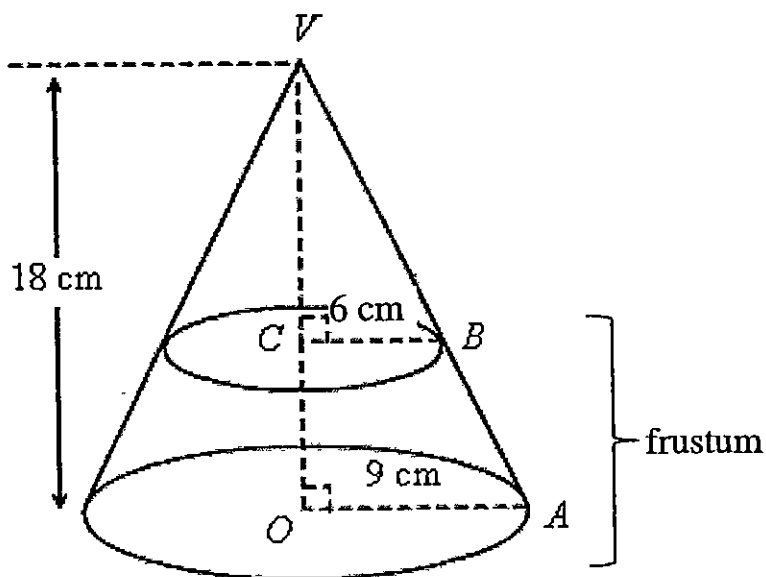
Answers: (a) [2]

(b) [1]

[Turn over

- 9 The upper part of a solid wooden right circular cone is cut off leaving the frustum shown in the diagram. The height of the original cone, VO is 18 cm and the base radius, OA is 9 cm. The radius, BC , of the base of the upper part is 6 cm. Take π to be 3.142.

- (a) Name the two similar triangles in the figure below.
 (b) Hence, show that $VC = 12$ cm.
 (c) Hence, calculate the volume of the frustum, correct to 3 significant figures.



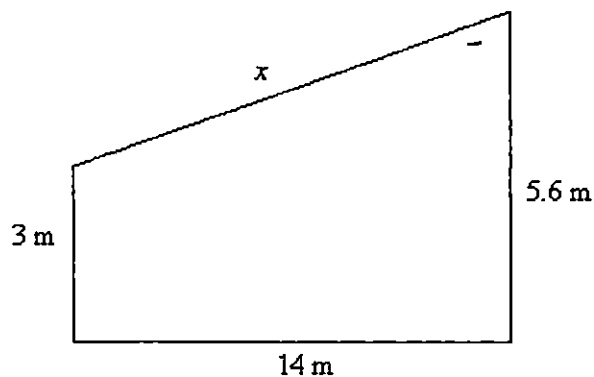
Answers: (a) $\Delta \dots\dots\dots$ and $\Delta \dots\dots\dots$ [1]

(b) Show working above [2]

(c) Volume = $\dots\dots\dots \text{cm}^3$ [3]

[Turn over

- 10 Two vertical posts are 14 m apart. One is 3 m high and the other 5.6 m high. Find x , the distance between the tops of the two posts, giving your answer in metres, correct to 3 significant figures.

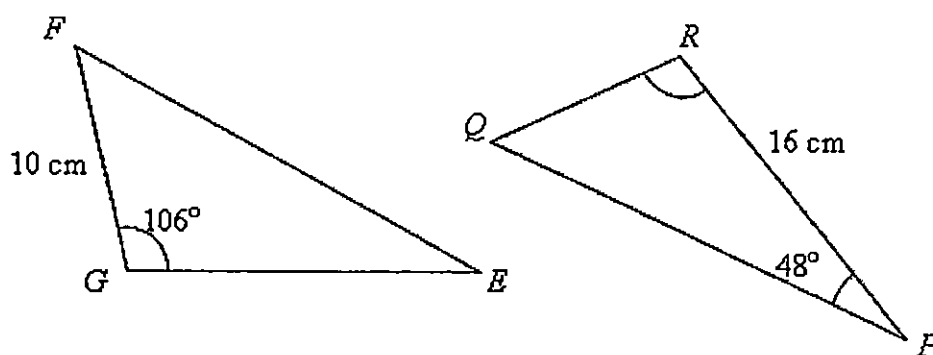


Answer: [3]

- 11 In the figure below, $\triangle EFG$ is congruent to $\triangle PQR$.
Given that $FG = 10$ cm, $PR = 16$ cm, $\angle G = 106^\circ$ and $\angle P = 48^\circ$, find

(a) QR

(b) $\angle Q$



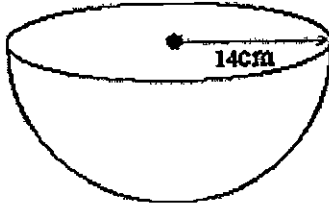
Answers: (a) [1]

(b) [2]

[Turn over

- 12 A solid hemisphere has a radius of 14 cm. Taking $\pi = \frac{22}{7}$, find
- (a) its volume,
 (b) its total surface area.

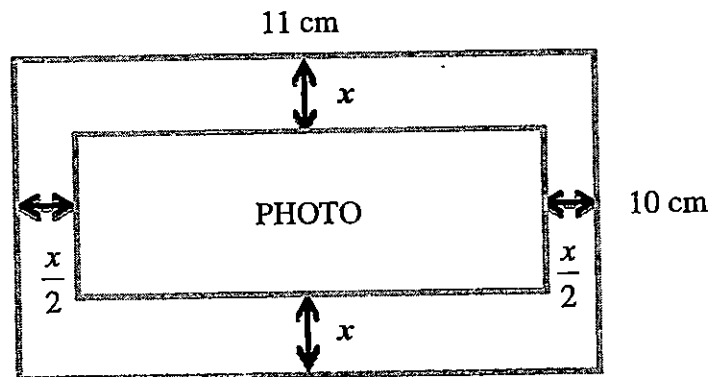
$$\left[\text{Volume of a Sphere} = \frac{4}{3}\pi r^3, \quad \text{Surface Area of a Sphere} = 4\pi r^2 \right]$$



Answers: (a) [2]

(b) [2]

- 13 A photo of area 54 cm^2 is placed on a photo frame of 10 cm by 11 cm with a border shown in the diagram below.



- (a) Write down an expression, in terms of x , for the area of the photo.
 (b) Show that $x^2 - 16x + 28 = 0$.

Answers: (a) [2]

(b) Show working above [2]

[Turn over

- 14 The stem-and-leaf diagram shows the mathematics test marks of 9 students.

Stem and Leaf Diagram				
4	1	3	6	
5	5	5	8	9
6	0	8		

Key : 4 | 9 represent 49 marks

Find

- (a) the modal mark,
- (b) the median mark.
- (c) the mean mark

Note: Leaving out important working will result in loss of marks.

Answers: (a) [1]

(b) [1]

(c) [2]

End of Paper 1

Solutions to 2012 2E Maths EOY P1

Qn	Solution	Marks	Remarks
1a	$(4x)^2 - y^2$ $(4x+y)(4x-y)$	M1 A1	Allow A2 if only last step is given.
1b	$ \begin{array}{r l} \begin{array}{rr} 3x & -5 \\ x & +3 \\ \hline 3x^2 & -15 \end{array} & \begin{array}{l} -5x \\ 9x \\ \hline +4x \end{array} \end{array} $ $3x^2 + 4x - 15 = (3x - 5)(x + 3)$	M1 A1	
2a	$ \begin{aligned} &= \frac{x+1-2x}{x^2-1} = \frac{-(x-1)}{(x-1)(x+1)} \\ &= \frac{-1}{x+1} \end{aligned} $	M1 A1	For correct first step in converting fractions to denominator of $\frac{1}{x^2-1}$ including expansion if any in the individual numerators.
2b	$ \begin{aligned} &= \frac{x^2 - 36y^2}{3x + 4y} \times \frac{12x + 16y}{x - 6y} \\ &= \frac{(x - 6y)(x + 6y)}{3x + 4y} \times \frac{4(3x + 4y)}{x - 6y} \\ &= 4(x + 6y) \text{ or } 4x + 24y \end{aligned} $	M1 A1	
3a	1cm : 40 000cm 1cm : 0.4 km Actual Length = 8 x 0.4 = 3.2 km	M1 A1	
3b	$1\text{cm}^2 : 0.16 \text{ km}^2$ Actual Area = (4x8) x 0.16 = 5.12 km ²	M1 M0 A1	Awarded provided working above presented.

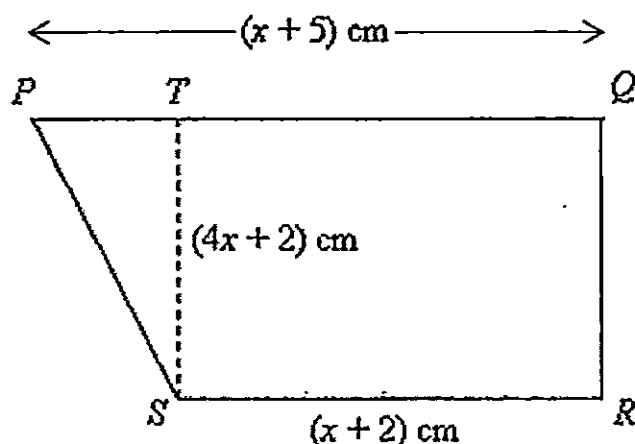
4a	$s - ut = at^2/2$ $2(s - ut) = at^2$ $a = 2(s - ut)/t^2$	M1 A1	
4b	$a = 2(200 - 5(4))/4^2 \leftarrow \text{allow ecf from 4a}$ $= 22.5$	M1 A1	Alternative method: substitution into $s = ut + \frac{at^2}{2}$ and solving for a .
5	$5x + 3y = 14$ ----- (1) $3x + 5y = 18$ ----- (2) (1) $\times 3: 15x + 9y = 42$ ----- (3) (2) $\times 5: 15x + 25y = 90$ ----- (4) (4) - (3): $16y = 48$ $y = 3$ Substituting $y = 3$ into (2), $3x + 5(3) = 18$ $3x = 3$ $x = 1$ $\therefore x = 1, y = 3$	M1 A1	Alternatively, if x is solved first – that becomes the M1 mark. Same mark allocation for substitution method.
6a	$y = \frac{k}{x^2 + 3}$ when $x = 2, y = 1$ thus: $1 = \frac{k}{2^2 + 3}$ $k = 1(4 + 3) = 7$ $y = \frac{7}{x^2 + 3}$	M1 A1	
6b	when $y = 2\frac{1}{3} = \frac{7}{3}$, $\frac{7}{3} = \frac{7}{x^2 + 3}$ Hence $x = 0$	B1	
7a	$A \cap B'$	B1	

7b	$A \cap B'$	B1	
8a	Method 1 $x - 2 = \pm 7$ $x = 2 \pm 7$ $x = -5 \text{ or } 9$	M0 A2	
	Method 2 $x^2 - 4x + 4 = 49$ $x^2 - 4x - 45 = 0$ $(x + 5)(x - 9) = 0$ Either $x = -5$ or 9	M0 A2	
8b	Either $3x + 7 = 0$ or $2 - x = 0$ $x = -7/3$ or $x = 2$	B1	No ½ mark awarded.
9a	ΔVCB and ΔVOA	B1	
9b	$\frac{VC}{VO} = \frac{BC}{AO}$ $VC = \frac{BC}{AO} \times VO$ $= \frac{6}{9} \times 18$ $= 12$	M1 A1	*order of alphabets can be ignored.
9c	Volume of frustum $= \frac{1}{3} \pi (9)^2 (18) - \frac{1}{3} \pi (6)^2 (12)$ $= 1074.564$ $\approx 1070 \text{ cm}^3 \text{ (to 3 s.f.)}$	M2 A1	M1: correct equation M1: correct substitution
10	Relevant height = $5.6 - 3$ $= 2.6 \text{ m}$ $x = \sqrt{14^2 + 2.6^2}$ $= \sqrt{202.76}$ $= 14.239$ $\approx 14.2 \text{ m}$	M1 M1 A1	Any statement of Pythagoras theorem with substitution.
11a	$QR = 10 \text{ cm}$	B1	
11b	$\angle Q = 180 - 106 - 48$ $= 26^\circ$	M1 A1	
12a	Volume $= \frac{1}{2} \times \frac{4}{3} \times \frac{22}{7} (14)^3$ $= 5749 \frac{1}{3} \text{ cm}^3$	M1 A1	5750 cm ³ (to 3sf accepted)
12b	Total Surface Area $= \frac{1}{2} \times 4 \times \frac{22}{7} (14)^2 + \frac{22}{7} (14)^2$	M1	M1 awarded for first term.

	$= 1848 \text{ cm}^2$	A1	
13a	$(11-x)(10-2x)$	B2	B1 awarded per factor. [-1] if equation written instead of expression and/or multiplication sign not omitted.
13b	$110 - 22x - 10x + 2x^2 = 54$ $2x^2 - 32x + 110 = 54$ $2x^2 - 32x + 110 - 54 = 0$ $2x^2 - 32x + 56 = 0$ $x^2 - 16x + 28 = 0$	M1	Expansion
		A1	Division by 2 of correct expn.
14a	55	B1	
14b	55	B1	
14c	$\frac{41 + 43 + 46 + 55 + 55 + 58 + 59 + 60 + 68}{9}$ $= 53.9 \text{ (to 3 s.f.)}$	M1	Or any indication of <u>sum of marks</u> <u>no. of students</u>
		A1	

Answer all the questions

- 1 The diagram shows a trapezium $PQRS$ where $PQ = (x + 5)$ cm, $SR = (x + 2)$ cm, $TS = QR$ and $TS = (4x + 2)$ cm.



- (a) Write down an expression, in terms of x , for the area of the trapezium $PQRS$. Show that it simplifies to $(2x + 7)(2x + 1)$. [2]
- (b) Given that the area of the trapezium is 27 cm^2 , show that it simplifies to $x^2 + 4x - 5 = 0$ [2]
- (c) Hence, find the value(s) of x which gives reasonable lengths for the trapezium. [2]

- 2 The speed of a bullet fired from a rifle is inversely proportional to the square root of its mass. When the mass is 49 g, the speed is 640 m/s.

- (a) Express the speed, S of the bullet in terms of its mass m . [2]
- (b) Find, correct to 3 significant figures,
- (i) the speed when its mass is 45 g. [1]
- (ii) the mass when the speed is 360 m/s. [2]

- 3 A survey was conducted to find out the number of people living in each of 50 flats in an HDB block. The results are shown in the table below.

No. of people	1	2	3	4	5	6	7	8
No. of flats	2	5	8	15	x	7	3	1

- (a) If the mean of the distribution is 4.24, show that the value of x is 9. [3]
- (b) Find the median of the distribution. [2]

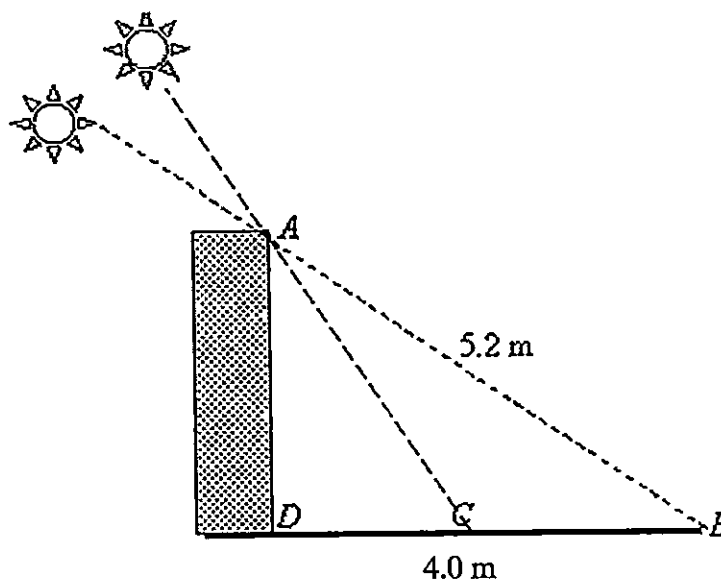
[Turn over

- 4 If $a^2 + b^2 = 13$ and $ab = 6$, find the value of

(a) $(a + b)^2$ [2]

(b) $(a - b)^2$ [2]

- 5 The diagram shows a vertical wall AD and its shadow DC at 1 pm in the afternoon. At 2 pm, it was found that the shadow has lengthened to 4.0 m which was twice its length at 1 pm. The length AB is 5.2 m.



Find, correct to 3 significant figures,

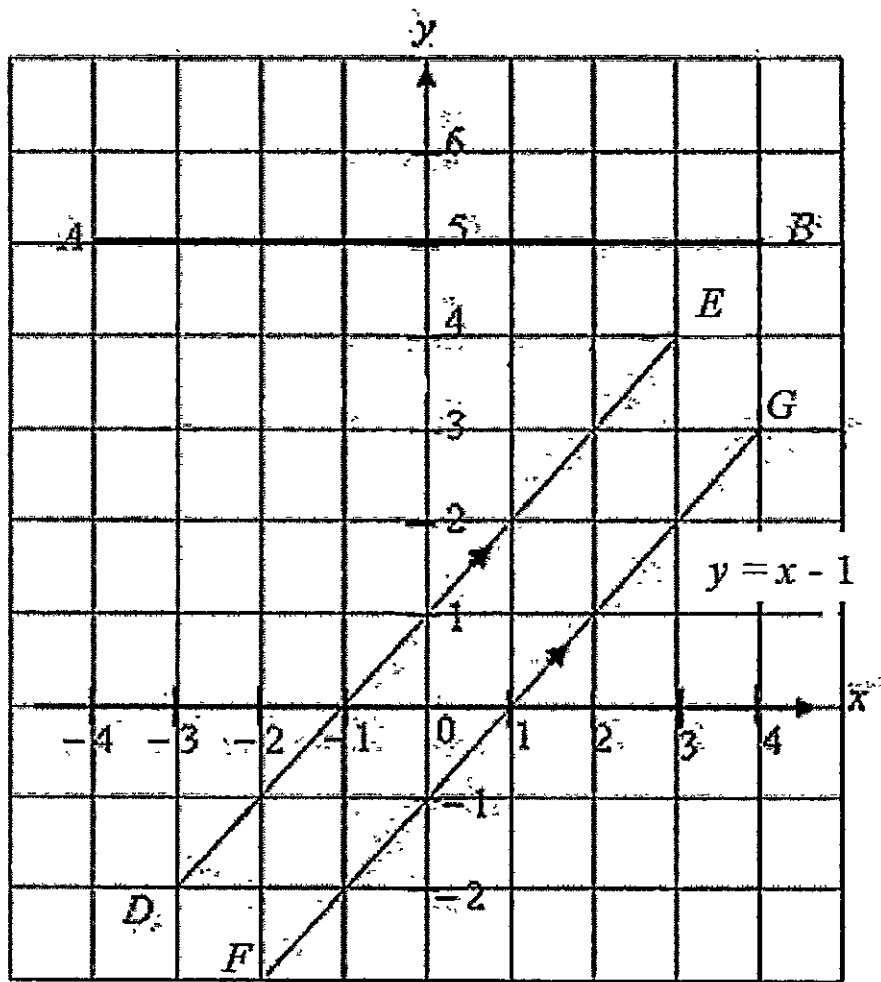
- (a) the height of the wall AD , [2]
 (b) the length AC . [3]

- 6 $\mathcal{E} = \{x : x \text{ is an integer and } 1 < x \leq 13\}$
 $S = \{x : x \text{ is a multiple of } 2\}$
 $T = \{2, 5, 10\}$

- (a) Draw a Venn Diagram to represent the relationship of $\mathcal{E}$, S and T and their elements. [3]
 (b) Hence by shading in the diagram the area representing $(S \cap T')$, find $n(S \cap T')$. [2]

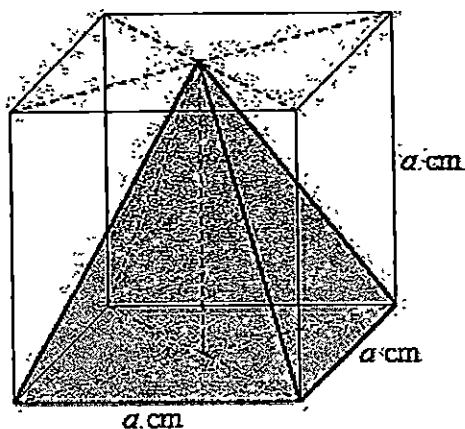
[Turn over]

- 7 In the diagram below, line DE is parallel to line FG.



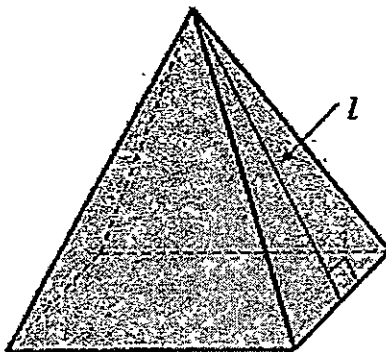
- (a) Write down the equation of the line AB. [1]
- (b) It is given that the equation of line FG is $y = x - 1$.
- (i) State the gradient of line DE. [1]
- (ii) State the y-intercept of line DE. Hence, write down the equation of the line. [2]

- 8 A solid square pyramid toy fits exactly into a cubic gift box, measuring a cm by a cm by a cm, as shown in the diagram.



Given that the volume of the pyramid toy is 243cm^3 .

- (a) Find the value of a . [3]
- (b) Find the slant height l of the toy pyramid. Hence, find the total surface area of the pyramid. [4]



9 **Answer the whole of this question on a sheet of graph paper.**

A ball is thrown upwards from the edge of the top of a vertical building. Its position during its flight is represented by the equation $h = -x^2 + 9x + 12$, where h metres is the height of the ball above the ground and x metres is its horizontal distance from the foot of the building.

- (a) Some corresponding values of x and h are given in the following table.

x	0	1	2	3	4	5	6	7	8	9
h	12	20	26	30	p	32	30	26	20	12

- (i) Find the value of p . [2]

- (ii) Using a scale of 2 cm to 1 unit,
draw a horizontal x -axis for $0 \leq x \leq 9$.
Using a scale of 4 cm to 5 units,
draw a vertical h -axis for $10 \leq h \leq 35$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

- (b) Use your graph to find

- (i) the greatest height reached by the ball, [1]

- (ii) how far the ball has travelled horizontally when it **first** reaches the height of 25 metres. [1]

- (c) A tree of height k metres is located 8.4 metres from the foot of the building. Given that the ball passes 3 metres above the top of the tree, use your graph to estimate the value of k . [2]

End of paper

Solutions to 2012 2E Maths EOY P2

Qn	Solution	Marks	Remarks
1a	$\text{Area} = \frac{1}{2}(x+5+x+2)(4x+2)$ $= (2x+7)\frac{4x+2}{2}$ $= (2x+7)(2x+1)$	M1 A1	Any correct attempt at first formulation of area.
1b	$(2x+7)(2x+1) = 27$ $4x^2 + 2x + 14x + 7 = 27$ $4x^2 + 16x - 20 = 0$ $x^2 + 4x - 5 = 0$	M1 A1	
1c	$(x+5)(x-1) = 0$ Either $x+5 = 0$ or $x-1 = 0$ Either $x = -5$ (rejected) or $x = 1$	M1 A1	With cross method shown 1 mark for both answers. Not awarded if $x = -5$ not rejected.
2a	$S = \frac{k}{\sqrt{m}}$ $640 = \frac{k}{\sqrt{49}}$ $k = 640 \times 7$ $= 4480$ $S = \frac{4480}{\sqrt{m}}$	M1 A1	Allow for "reversed awarding" of marks if this equation is ever expressed in part (b).
2b(i)	$S = \frac{4480}{\sqrt{45}} = 667.8 \approx 668 \text{ m/s}$	B1	
2b(ii)	$m = \left(\frac{4480}{S} \right)^2$	M1	Either symbolic or numerical methods of making m the subject of formula

	$= \left(\frac{4480}{360} \right)^2$ $= 154.9 \text{ g}$ $\approx 155 \text{ g}$	A1	accepted.
3a	$\frac{1 \times 2 + 2 \times 5 + 3 \times 8 + 4 \times 15 + 5 \times x + 6 \times 7 + 7 \times 3 + 8 \times 1}{50} = 4.24$ $\frac{167 + 5x}{50} = 4.24$ $167 + 5x = 212$ $x = \frac{212 - 167}{5}$ $= 9$	M1 M1 A1	
3b	Middle position = $\frac{50+1}{2} = 25.5$ Median = Mean of 25th and 26th position $= \frac{4+4}{2} = 4$	M1 A1	
4a	$(a+b)^2$ $= a^2 + 2ab + b^2$ $= 13 + 2(6)$ $= 25$	M1 A1	For expansion without substitution
4b	$(a+b)^2$ $= a^2 - 2ab + b^2$ $= 13 - 2(6)$ $= 1$	M1 A1	For expansion without substitution
5a	$AD = \sqrt{5.2^2 - 4.0^2}$ $= 3.323 \approx 3.32 \text{ m}$	M1 A1	
5b	$DC = 2.0 \text{ m}$ $AC = \sqrt{3.32^2 + 2.0^2}$ $= 3.878 \approx 3.88 \text{ m}$	M1 M1 A1	

	$l^2 = \frac{5(9)^2}{4}$ $l = \sqrt{101.25} = 10.0623... \approx 10.1 \text{ cm (to 3sf)}$ Surface area of pyramid = $9^2 + 4(1/2 \times 9 \times 10.1)$ $= 262.12$ $\approx 262 \text{ cm}^2$	M1 M1 A1	Awarded with/without rounding to 3sf Awarded with/without rounding to 3sf
9ai	$p = -(4)^2 + 9(4) + 12$ $= 32$	M1 A1	
	The rest of Q9 pls see attached.		